# On the irreversible susceptibility of a textured Stoner–Wohlfarth ensemble ☆

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## Abstract

For models containing both reversible and irreversible magnetization processes the time-dependent irreversible susceptibility and the differentiated remanence curve differ from each other. To demonstrate this, both quantities were calculated for a non-interacting Stoner–Wohlfarth ensemble in dependence on the degree of alignment, the magnitude and the application time of an opposite field. For the differentiated remanence curve at $t = 0$ s an analytical expression was derived. For general times the calculation was carried out numerically.

## 1. Introduction

Measurements of the magnetic viscosity, i.e. $S = \partial M / \partial \ln(t)$, with $M$ being the magnetisation and $t$ the time, are interesting for both scientific and practical reasons, the latter mainly since the long-term behaviour of magnetic materials is a crucial point regarding the utilization of these materials for technological purposes. The scientific interest is mainly due to the fact that one can get insight into the magnetic reversal mechanisms on the one hand and on the other hand it helps to define appropriate micromagnetic models. In connection with magnetic viscosity measurements, the irreversible part of the susceptibility $\chi_{irr}$ attracted much attention, since it

has to be measured if the so-called fluctuation field, $S_v$, should be determined from the ratio of the magnetic viscosity over the irreversible susceptibility [1]. Whereas both $S$ and $\chi_{irr}$ depend on the shape of the specimens, $S_v$ does not and is therefore a characteristic of the material. Adopting the activation volume approach to the magnetic viscosity, it can be shown that $S_v$ is directly related to the activation volume $v$ [5,6] so that the latter can be estimated by measuring $S$ and $\chi_{irr}$. In the literature two different methods to measure it are reported. The first way is to differentiate the remanence curve [2]. The second way is to measure the difference between the total and the reversible change of magnetization [3]. If both reversible and irreversible processes are involved in magnetization changes the two methods measure different quantities. It is the aim of this paper to make this point more explicit. Therefore both the irreversible susceptibility and the differentiated remanence curve and their time and field dependence are

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calculated for the Stoner–Wohlfarth model [4], since it is the simplest model containing reversible and irreversible changes.

## 2. The difference between $\chi_{irr}$ and $\chi_R$

For the Stoner–Wohlfarth model [4] the time-dependent magnetization is

$$\left\langle \frac{M(t, h)}{M_S} \right\rangle = \langle \cos(\vartheta - \varphi_1(h)) p(t, h) \rangle$$
$$+ \langle \cos(\vartheta - \varphi_2(h))(1 - p(t, h)) \rangle. \tag{1}$$

where $\vartheta$ is the angle between the magnetic field direction and the easy axis of a grain, $\varphi_1$ is the angle between the metastable magnetization direction and the easy axis. $\varphi_2$ between the stable magnetization direction and easy axis. $h$ is the internal magnetic field measured in units of the anisotropy field, $H_A = 2K_1/M_S$. The probability, $p(t, h)$ to find a grain in its metastable state at time $t$, if it was initially in that state. is

$$p(t) = \Theta(h_S - h)\left\{ \frac{w_{21}}{w_{12} + w_{21}} \right.$$
$$\left. + \frac{w_{12}}{w_{12} + w_{21}} e^{-(w_{12} + w_{21})t} \right\}. \tag{2}$$

The step function ensures that the probability vanishes if the magnetic field is higher than the switching field $h_S = (\cos^{2/3}\vartheta + \sin^{2/3}\vartheta)^{-3/2}$ for a given $\vartheta$. The rates $w_{12}$ and $w_{21}$ are

$$w_{12} = \Gamma_0 \exp\left( -\frac{E_M - E_1}{k_B T} \right),$$
$$w_{21} = \Gamma_0 \exp\left( -\frac{E_M - E_2}{k_B T} \right). \tag{3}$$

where $\Gamma_0$ is typically $10^9$–$10^{12}$ Hz [5]. The energies $E_1$, $E_2$, $E_M$ correspond to the metastable state, the stable state, and to the maximum energy state in between. $\langle \ldots \rangle$ denotes the average with respect to an easy-axis distribution, $f(\vartheta)$. Here the easy-axis distribution is assumed to be rotational symmetric with the axis of symmetry parallel to the field.

The total susceptibility after a waiting time $t$ is found by differentiation:

$$\frac{\partial \langle m \rangle}{\partial h} = \left\langle p \frac{\partial}{\partial h} \cos(\vartheta - \varphi_1) \right\rangle$$
$$+ \left\langle (1 - p) \frac{\partial}{\partial h} \cos(\vartheta - \varphi_2) \right\rangle$$
$$+ \left\langle \cos(\vartheta - \varphi_1) \frac{\partial}{\partial h} p \right\rangle$$
$$+ \left\langle \cos(\vartheta - \varphi_2) \frac{\partial}{\partial h}(1 - p) \right\rangle. \tag{4}$$

The first two terms of the RHS of Eq. (4) are the change in magnetization due to the reversible rotation process. Therefore the remaining two terms constitute the irreversible susceptibility

$$\chi_{irr} = \left\langle (\cos(\vartheta - \varphi_1) - \cos(\vartheta - \varphi_2)) \frac{\partial}{\partial h} p(t, h) \right\rangle. \tag{5}$$

If the opposite magnetic field is switched off after a waiting time $t$, one gets the remanence $m_R(t, h)$. It can be calculated according to

$$m_R(t, h) = \langle \cos(\vartheta - \pi) p(t, h) \rangle$$
$$+ \langle \cos(\vartheta)(1 - p(t, h)) \rangle \tag{6}$$

The differentiated remanence should be called $\chi_R$. It is

$$\chi_R = \left\langle \cos(\vartheta - \pi) \frac{\partial}{\partial h} p(t, h) \right\rangle$$
$$+ \left\langle \cos(\vartheta) \frac{\partial}{\partial h}(1 - p(t, h)) \right\rangle. \tag{7}$$

Comparing Eq. (5) with Eq. (7) the difference is evident. For the differentiated remanence for a waiting time $t = 0$ an analytical expression can be derived. The details of the calculation will be given elsewhere. One finds

$$\chi_R = 2 \sum_{i=1,2} \sin \vartheta_i f(\vartheta_i)$$
$$\times \cos \vartheta_i \frac{(\cos^{2/3}\vartheta_i + \sin^{2/3}\vartheta_i)^{5/2}}{|\cos^{-1/3}\vartheta_i \sin \vartheta_i - \sin^{-1/3}\vartheta_i \cos \vartheta_i|}, \tag{8}$$
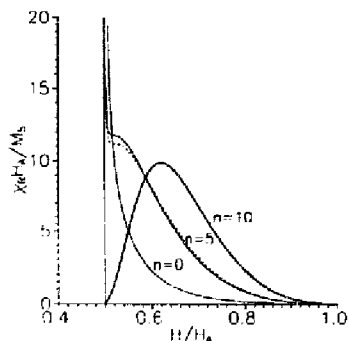
Fig. 1. $\chi_R(t = 0)$ for different values of texture parameter, $n$, of the cosine texture, $f(\vartheta) = (2n + 1)\cos^{2n}\vartheta$. The dotted curve is for a Gaussian texture $f(\vartheta) \simeq \exp-(\vartheta / \sigma)^2$ with $\sigma = 0.4203$ which gives the same alignment $\langle \cos \vartheta \rangle$ as $n = 5$.
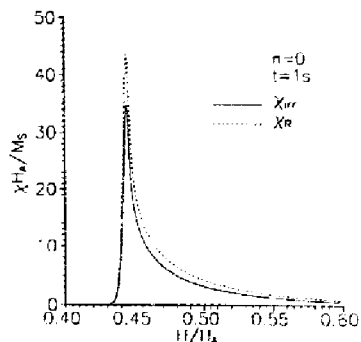


Fig. 2. The irreversible suscebtibility, $\chi_{irr}$, and the differentiated remanence, $\chi_R$, for an isotropic ensemble and an application time of the opposite field of 1 s.

where the $\vartheta_i$ are calculated from

$$\cos \vartheta_{1,2} = \frac{1}{\sqrt{2}} \sqrt{1 \pm \sqrt{1 - \frac{4}{27} \frac{(1-h^2)^2}{h^4}}} \,. \qquad (9)$$

The texture dependence of $\chi_R(t = 0)$ is shown in Fig. 1. The difference between Gaussian-textured and cos-textured ensembles is negligible if the two texture functions give the same remanence ratio. For waiting times $t > 0$, both $\chi_R$ and $\chi_{irr}$ have to be calculated numerically from Eqs. (7) and (5). For that the parameter $E_{12}/k_B T$ determining the barrier height has to be selected. To have a remarkable effect during experiments lasting typically from 1·s to $10^4$ s this parameter should lie between 25 and 33 [6]. If all the energies are measured in units of $2VK_1$, with $V$ being the volume of the grains, then the value $25 k_B T / 2VK_1$ should be between 0 and 0.5. We choose $k_B T / 2VK_1$ to be 0.001. This is at least a choice of an effective particle volume $V$. For Ba-ferrite one gets at room temperature e.g. 22 nm diameter, $SmCo_5$ 8 nm, and for the other hard magnetic materials in between, for the magnetite fine particle system used in Ref. [7] ($K_1 = 0.44$ MJ/$m^3$, $d = 7.8$ nm, $T = 10$ K) one finds a value of about 0.006. For the frozen cobalt ($K_1 = 0.2$ MJ/$m^3$) dispersions used in Ref. [8] the values are between 0.003 corresponding to a particle diameter of $d = 12$ nm and a temperature of $T = 80$ K and 0.158 for $d = 5$ nm and $T = 300$ K. Fig. 2 demonstrates the difference between the two quantities. In Fig. 3 $\chi_{irr}$ is shown for different application times of the opposite field. There is also a singularity at $h = 0.5$ for $t = 0$ which is transformed into a peak, shifting to lower values of the field with increasing $t$.

## 3. Conclusion

Our results demonstrate that the irreversible susceptibility and the differentiated remanence curve should not be used as synonyms to prevent confusion. Regarding the determination of the fluctuation field in viscosity studies it is $\chi_{irr}$ instead $\chi_R$ that should be measured. Nevertheless the difference is only important if reversible processes are not negligible. That is in fact the case for the Stoner–Wohlfarth ensembles as can be seen from Fig. 4, where
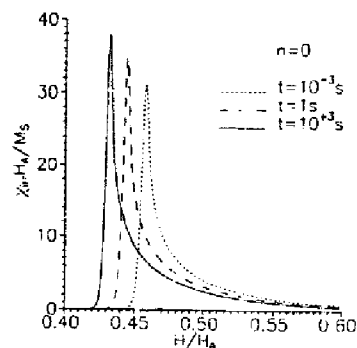


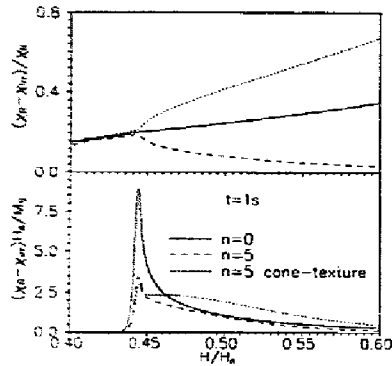Fig. 3. $\chi_{irr}$ for different times of application of the opposite field.

Fig. 4. The absolute difference, $\chi_R - \chi_{irr}$, and the relative difference $(\chi_R - \chi_{irr})/\chi_R$, for an isotropic ($n = 0$), a well textured ($n = 5$), and a cone-textured ($n = 5$ cone-textured) ensemble for an application time of the opposite field of 1 s.

the absolute and the relative differences between $\chi_R$ and $\chi_{irr}$ are depicted. For the isotropic ensemble the effect is about 20% in the vicinity of the coercivity field. While the absolute value of the difference decreases rapidly for fields above $H_A/2$ the relative value increases. The reversible change of the magnetisation becomes larger with increasing angle between the field direction and the easy axis. Therefore for a well textured ensemble ($n = 5$) the relative difference decreases to some per cent for fields

above the coercivity field. Otherwise, when a cone-bag-shaped texture function ($f(\vartheta) = (2n + 2)\sin^{2n}\vartheta \cos \vartheta$) is chosen, the relative difference increases with the field. For the cone-shaped texture with $n = 5$, for instance, the maximum number of grains has the misalignment angle $\vartheta = 72.5°$.

In usual magnetometers the charactistic measuring time is much to long for the singularity at $t = 0$ but also the sharp peak at finite times will be smeared out if broad distributions of other intrinsic parameters have to be taken into account. Especially a distribution of the parameter $K_1V$ may be more important than the easy-axis distribution [8].

### References

[1] R. Street and J.C. Woolley, Proc. Phys. Soc. A62 (1949) 562.
[2] R.W. Chantrell, M. Fearon and E.P. Wohlfarth, Phys. Stat. Sol. (a) 97 (1986) 213.
[3] H. Nishio, IEEE Transl. J. Magn. Jpn. 5 (1990) 374.
[4] E.C. Stoner and E.P. Wohlfarth, Philos. Trans. R. Soc. A240 (1948) 599.
[5] L. Néel, Adv. Phys. 4 (1955) 191.
[6] P. Gaunt, J. Appl. Phys. 59 (1986) 4129.
[7] M. El-Hilo, K. O'Grady, R.W. Chantrell and D.P.E. Dickson, J. Magn. Magn. Mater. 123 (1993) 30.
[8] K. O'Grady, A. Bradbury, J. Popplewells, S.W. Charles, R. Chantrell, J. Magn. Magn. Mater. 49 (1985) 106.