Archive:

Second Interdisciplinary and Research Alumni Symposium - iJaDe2018

Wednesday 5th and Thursday 6th of September 2018

The workshop will focus on stochastic analysis and applications of stochastics to analysis. We are particularly interested in non-local and jump-driven phenomena that are decisive for the modelling of extremes (e.g. in finance, weather etc.), branching phenomena that describe the evolution of ecological systems and are related to nonlinear partial differential equations, as well as multi-scale problems from the material sciences (microstructures, heterogeneous materials) where elliptic operators with random coefficients occur.

Organizers: The workshop is organized by research teams at
TU Dresden (Germany) headed by Prof. R. SCHILLING,
Kansai University (Japan) headed by Prof. T. UEMURA and
Osaka University (Japan) headed by Prof. Y. SHIOZAWA.

Location:
Osaka University, Graduate School of Science, Osaka, Japan
Building B, Room B308 (3rd floor)

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Abstracts (page 5-23)

SECOND INTERDISCIPLINARY AND RESEARCH ALUMNI SYMPOSIUM

iJaDe2018   Physics Programme

Funded by the Alexander von Humboldt Foundation and
the Excellence Initiative of the German Federal and State Governments

Workshop Programme: Mathematics

Wednesday 5th and Thursday 6th of September 2018
Osaka University, Graduate School of Science, Osaka, Japan
Building B, Room B308 (3rd floor)
Campus Map online   Campus Map (PDF)   Practical Information

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We greatfully acknowledge advice by and cooperation with Prof. M. Fukushima (Osaka), Prof. M. Takeda (Tohoku), Prof. T. Kumagai (RIMS Kyoto) and Prof. J. Masamune (Hokkaido).

Please contact the Organizers for further information:
Prof. Dr. Toshihiro Uemura (Kansai University, Osaka, Japan)
Prof. Dr. Yuichi Shiozawa (Osaka University, Osaka, Japan)
Prof. Dr. René Schilling (TU Dresden, Dresden, Germany)

The following colleagues have agreed to deliver a talk:

Hiroaki Aikawa (Hokkaido University)
Jiro Akahori (Ritsumeikan University)
Björn Böttcher (TU Dresden)
Wojciech Cygan (TU Dresden)
Kenji Handa (Saga University)
Yuri Imamura (Tokyo University of Science)
Yu Ito (Kyoto Sangyo University)
Kamil Kaleta (Wroclaw University of Science and Technology)
Viktoriya Knopova (TU Dresden)
Masaharu Nishio (Osaka City University)
Liang Song (Tsukuba University)
Tomoko Takemura (Nara Women's University)
Atsushi Takeuchi (Osaka City University)
Yoshihiro Tawara (Nagaoka National College of Technology)
Kazutoshi Yamazaki (Kansai University)
Kouji Yano (Kyoto University)

Schedule and Abstracts:
Schedule (PDF)   Abstracts (PDF)

2nd Interdisciplinary & Research Alumni Symposium iJaDe2018

Wednesday, September 5th

09.30 – 10.30 **Nambu Yoichiro Hall**	**Joint opening of the Mathematics and Physics workshops** Prof. S. Tajima (Dean of Graduate School of Science, Osaka U) Prof. K. Yamanoi (Dean of Mathematics, Osaka U) Prof. M. Asakawa (Dean of Physics, Osaka U) Prof. R. L. Schilling (Mathematics, TU Dresden) Prof. K. Zuber (Physics, TU Dresden)
10.30 – 11.00	**Relocation to Workshop Venue & Coffee Break (B 302)**
11.00 – 11.35	**Hiroaki Aikawa** (Hokkaido U) Global integrability of supertemperatures and related notions in potential theory
11.40 – 12.15	**Kamil Kaleta** (Wroclaw University of Science and Technology) Progressive intrinsic ultracontractivity for nonlocal Schrödinger operators
	Lunch Break
14.00 – 14.35	**Masaharu Nishio** (Osaka City U) Hilbert spaces and reproducing kernels for parabolic operators of fractional order
14.40 – 15.15	**Yoshihiro Tawara** (National Institute of Technology, Nagaoka College) Compactness of Markov and Schrödinger semi-groups
	Coffee Break (B 302)
15.30 – 16.05	**Kazutoshi Yamazaki** (Kansai U) Periodic observations of spectrally one-sided Lèvy processes and applications to inventory control
16.10 – 16.45	**Victoria Knopova** (TU Dresden) On the construction of a general stable-like Markov process
16.50 – 17.25	**Kouji Yano** (Kyoto U) Generalized arcsine laws for null recurrent diffusions and for infinite ergodic transformations

Thursday, September 6th

09.30 – 10.05	**Song Liang** (University of Tsukuba) A mechanical model of Brownian motion
10.10 – 10.45	**Kenji Handa** (Saga U) Coagulation-fragmentation equations and underlying stochastic dynamics
	Coffee Break (B 302)
11.00 – 11.35	**Wojciech Cygan** (TU Dresden) A class of random perturbations of hierarchical Laplacians and statistics of their spectra
11.40 – 12.15	**Jiro Akahori** (Ritsumeikan U) Another View of the Riccatti equations arising in Affine Class in Finance
	Lunch Break
14.00 – 14.35	**Yuri Imamura** (Tokyo University of Science) A Discrete Scheme of Static Hedging of Barrier Options
14.40 – 15.15	**Yu Ito** (Kyoto Sangyo U) A fractional calculus approach to rough path integration
	Coffee Break (B 302)
15.30 – 16.05	**Tomoko Takemura** (Nara Women's U) Convergence of diffusion processes in a tube and Dirichlet forms of limit processes
16.10 – 16.45	**Björn Böttcher** (TU Dresden) Distance Multivariance - measuring & detecting multivariate dependence
16.50 – 17.25	**Atsushi Takeuchi** (Osaka City U) Density for the solution to stochastic functional differential equations

Venue: **opening ceremony only:** Osaka University, Nambu Yoichiro Hall: http://www.sci.osaka-u.ac.jp/en/access-maps/campus-map/
workshop: Osaka University, Graduate School of Science, Building B, Room B308 (3rd floor), http://www.sci.osaka-u.ac.jp/en/access-maps/campus-map/

Organizers: Prof. Dr. Toshihiro Uemura (Kansai University, Osaka, Japan), Prof. Dr. Yuichi Shiozawa (Osaka University, Osaka, Japan), Prof. Dr. René L. Schilling (TU Dresden, Dresden, Germany)

GLOBAL INTEGRABILITY OF SUPERTEMPERATURES ON A JOHN CYLINDER

HIROAKI AIKAWA

ABSTRACT

Ever since Armitage showed that every nonnegative superharmonic function on a bounded domain of bounded curvature ($= C^{1,1}$ domain) in $\mathbb{R}^n$ is L^p-integrable up to the boundary for $0 < p < n/(n-1)$, the global integrability of nonnegative supersolutions has attracted many mathematicians.

In this talk we consider a parabolic counterpart. We study the global integrability of nonnegative supertemperatures on the cylinder $D \times (0,T)$, where D is a Lipschitz domain or a John domain. We show that the integrability depends on the lower estimate of the Green function for the Dirichlet Laplacian on D.

In particular, if D is a bounded C^1-domain, then every nonnegative supertemperature on $D \times (0,T)$ is L^p-integrable over $D \times (0,T')$ for any $0 < T' < T$, provided $0 < p < (n+2)/(n+1)$. The bound $(n+2)/(n+1)$ is sharp.

We employ different arguments for a Lipschitz cylinder and for a John cylinder. While heat kernel estimates are crucial for a Lipschitz cylinder, they are not available for a John cylinder. A parabolic box argument related to intrinsic ultracontractivity plays a crucial role instead.

Joint work with Hara and Hirata.

Department of Mathematics, Hokkaido University, Sapporo 060-0810, Japan

E-mail address: aik@math.sci.hokudai.ac.jp

Kamil Kaleta

Progressive intrinsic ultracontractivity for nonlocal Schrödinger operators

We give sharp two-sided large time estimates of the heat kernel for a large class of non-local Schrödinger operators with confining potentials, which are based on generators of Lévy processes. We identify a new useful regularity property of compact semigroups, which is weaker than asymptotic intrinsic ultracontractivity. It means that the space-time regularity of the semigroup essentially improves as the time parameter diverges to infinity.
This is a joint work with René Schilling.

Kamil Kaleta Wroclaw University of Science and Technology, Poland
`kamil.kaleta@pwr.edu.pl`

Hilbert spaces and reproducing kernels for parabolic operators of fractional order*†‡

Masaharu Nishio§

Hardy space and Bergman space on the unit disc in the complex plane are well-know Hilbert spaces, which have the reproducing kernels. They are function spaces which consist of holomorphic functions.

In this talk, I consider a parabolic operator $L^{(\alpha)} := \partial_t + (-\Delta_x)^\alpha$ and its iterates on the upper half space $\boldsymbol{H} := \{(x,t)|x \in \boldsymbol{R}^n, t > 0\}$, and discuss the analogue of the above Bergman space. I also mention the relation of harmonic and polyharmonic functions with the Poisson operator $L^{(1/2)}$.

Fundamental properties of the harmonic Bergman space on the upper half space are discussed in [5], and my talk is based on results in [1, 2, 3, 4].

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‡Key words and phrases. mean value property, parabolic operators, Bergman spaces.

§Department of Mathematics, Osaka City University, Osaka 558-8585, JAPAN

Compactness of Markov and Schrödinger semi-groups

Yoshihiro Tawara (jointwork with M. Takeda and K. Tsuchida)

Let E be a locally compact separable metric space and m a positive Radon measure on E with full support. Let X be an m-symmetric Markov process on E. We assume that X is irreducible and has strong (resolvent) Feller property. Moreover, we assume that X possesses the *tightness property*, i.e., for any $\epsilon > 0$ there exists a compact set K such that $\sup_{x \in E} R_1 1_{K^c}(x) \leq \epsilon$. Here R_1 is the 1-resolvent of X and 1_{K^c} is the indicator function of the complement of K. When X has these properties, we say in this talk that X belongs to Class (T). Takeda proved that if X belongs to Class (T), its semi-group turns out to be a compact operator on $L^2(E; m)$. In this talk, we apply this criterion to Dirichlet Laplacians Δ_D and Schrödinger operators $\Delta - V$ with positive potential and show probabilistically the compactness of these operators.

Periodic observations of spectrally one-sided Lévy processes and applications to inventory control

Kazutoshi Yamazaki *[1]

[1]Department of Mathematics, Kansai University

Abstract

We consider a version of the stochastic inventory control problem for a spectrally positive Lévy demand process, in which the inventory can only be replenished at independent exponential times. We compute, via the scale function, the fluctuation identities for the controlled process under a periodic barrier policy, which replenishes any shortage below a certain barrier at each replenishment opportunity. These identities can be used to show the optimality of such policy in a straightforward manner. Numerical results are also provided. This is based on a joint work with A. Bensoussan and J.L. Pérez.

Keywords: inventory models; spectrally one-sided Lévy processes; scale functions; periodic observation; resolvents

References

[1] J.L. Pérez, K. Yamazaki and A. Bensoussan (2018), "Optimal periodic replenishment policies for spectrally positive Lévy demand processes," *arXiv*: 1806.09216.

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*E-mail address: kyamazak@kansai-u.ac.jp

On the construction of a general stable-like Markov process

Victoria Knopova (TU Dresden)

We consider an integro-differential operator

$$\begin{aligned} Lf(x) &= b(x)\cdot\nabla f(x) \\ &\quad + \int_{\mathbb{R}^d\setminus\{0\}} \Big(f(x+u)-f(x)-\nabla f(x)\cdot u 1_{|u|\le 1}\Big) N(x,du), \end{aligned}$$

defined on the space $C^2_\infty(\mathbb{R}^d)$ of twice continuously differentiable functions with vanishing at infinity derivatives. The drift $b\in\mathbb{R}^d$ is assumed to be bounded and Hölder continuous, and the Lévy-type kernel $N(x,du)$ is a sum of an α-stable like part and a lower order perturbation.

We show that under certain regularity assumptions the extension of $(L, C^2_\infty(\mathbb{R}^d))$ is the generator of a Feller semigroup $(P_t)_{t\ge 0}$.

The talk is based on the on-going work with A. Kulik and R. Schilling.

Generalized arcsine laws for null recurrent diffusions and for infinite ergodic transformations

Kouji YANO (Kyoto University)

For the simple symmetric random walk $\{X_n\}$ on $\mathbb{Z}$, *Lévy's arcsine law* [3] asserts that

$$\frac{1}{n}\sum_{k=0}^{n-1} 1_{\{X_k>0\}} \xrightarrow[n\to\infty]{\mathrm{d}} \frac{\mathrm{d}x}{\pi\sqrt{x(1-x)}} \text{ on } (0,1), \tag{1}$$

whose limit is called the *arcsine distribution*. There have been lots of studies generalizing this result from various aspects.

In this talk we focus on generalizations of Lamperti type. We recall several results of generalizations for null recurrent diffusions. We also recall similar results for infinite ergodic transformations and then we shall mention our recent work [4], jointly with Toru Sera, about generalization of previous results by utilizing the methods which have been employed for diffusions.

Lamperti [2] studied a Markov chain $\{X_n\}$ on $\mathbb{Z}$ which cannot skip 0 when it changes signs and proved that the convergence

$$\frac{1}{n}\sum_{k=0}^{n-1} 1_{\{X_k>0\}} \xrightarrow[n\to\infty]{\mathrm{d}} Z \tag{2}$$

holds for some non-trivial random variable Z only if Z follows the *Lamperti distribution* with parameters $\alpha, p \in (0,1)$:

$$\frac{\sin\alpha\pi}{\pi}\cdot\frac{p(1-p)x^{\alpha-1}(1-x)^{\alpha-1}\mathrm{d}x}{p^2(1-x)^{2\alpha}+(1-p)^2x^{2\alpha}+2p(1-p)x^\alpha(1-x)^\alpha\cos\alpha\pi} \text{ on } (0,1). \tag{3}$$

Barlow–Pitman–Yor [1] studied the class of *skew Bessel diffusions* on *multiray*, i.e., a finite number of half lines called *rays* radiating from the origin, and obtained multiray generalization of the Lamperti distributions. These results have been generalized to null recurrent diffusions by Truman–Williams [7], Watanabe [8], Watanabe–K. Yano–Y. Yano [9] and Y. Yano [10].

Thaler [5] studied a class of infinite ergodic transformations T where the state space is divided into a disjoint union $A_1 + Y + A_2$ and the orbit $\{T^n a\}$ of an initial state a cannot skip the *junction* Y when it changes *rays* A_1 and A_2. A typical example is the *Boole transform* defined by

$$Ta = a - \frac{1}{a} \quad (a \in \mathbb{R}\setminus\{0\}), \quad T0 = 0, \tag{4}$$

which is an ergodic transformation preserving the Lebesgue measure λ. Note that the orbit of any initial state cannot skip the junction $Y = [-1,1]$ when it changes rays $(-\infty,-1]$

and $[1,\infty)$. He proved that, for any probability measure ν absolutely continuous w.r.t. λ, the invariant measure, the following convergence holds:

$$\frac{1}{n}\sum_{k=0}^{n-1} 1_{\{T^k a>0\}} \text{ (under } \nu(\mathrm{d}a)) \xrightarrow[n\to\infty]{\mathrm{d}} \frac{\mathrm{d}x}{\pi\sqrt{x(1-x)}} \text{ on } (0,1), \tag{5}$$

where we note that

$$\frac{1}{n}\sum_{k=0}^{n-1} 1_{\{T^k a\in Y\}} \text{ (under } \nu(\mathrm{d}a)) \xrightarrow[n\to\infty]{\text{a.s.}} 0. \tag{6}$$

The Boole transform can be deformed into an interval map having two indifferent fixed points. He obtained a limit theorem of Lamperti type for infinite ergodic interval maps T assuming certain regular variation conditions on T at the two indifferent fixed points of T. His theory was developed by Thaler–Zweimüller [6] and Zweimüller [11].

Our recent results of Sera–K. Yano [4] is a multiray generalization of the previous results [5], [6] and [11]. Their method was convergence of moments, which seems quite difficult to handle the joint distribution of occupation ratios. Our method is based on the *double Laplace transforms*, which have played a crucial role in the above-mentioned papers about null recurrent diffusions.

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A mechanical model of Brownian motion

Song Liang (University of Tsukuba)

We provide a connection between Brownian motion and a classical mechanical system. Precisely, we consider a system of one massive particle interacting with an idea gas, evolved according to non-random Newton mechanical principle, via a repulsive interaction potential function, and prove that under certain condition, the (position, velocity)-process of the massive particle converges to a diffusion under a certain scaling limit, such that the mass of the light particles converges to 0, while the density and the velocities of them go to infinity.

Coagulation-fragmentation equations and underlying stochastic dynamics

Kenji Handa (Saga U)

We consider stochastic dynamics of interval partitions evolving according to certain split-merge transformations. An asymptotic result for properly rescaled processes is shown to obtain a solution to a nonlinear equation called the coagulation-fragmentation equation.

A CLASS OF RANDOM PERTURBATIONS OF HIERARCHICAL LAPLACIANS AND STATISTICS OF THEIR SPECTRA

Wojciech Cygan
Technische Universität Dresden & Uniwersytet Wrocławski

Let (X, d) be a proper *ultrametric* space. Given a measure m on X and a function $B \mapsto C(B)$ defined on the set of all non-singleton balls $B \subset X$ we consider the hierarchical Laplacian

$$L_C f(x) := \sum_{B \in \mathcal{B}\,:\, x \in B} C(B) \left(f(x) - \frac{1}{m(B)} \int_B f \, dm \right).$$

Choosing a sequence $\{\varepsilon(B)\}$ of i.i.d. random variables we define the perturbed function $C(B, \omega)$ and the perturbed hierarchical Laplacian $L^\omega = L_{C(\omega)}$. In the talk we discuss convergence of the sequence of arithmetic means of the L^ω-eigenvalues to a normal distribution. We also study related point processes built of the eigenvalues and their convergence to a Poisson point process.

Based on a joint project with A. Bendikov (Univ. Wrocław).

Another View of the Riccatti equations arising in Affine Class in Finance

Jiro Akahori (Ritsumeikan U)

Fourier transform of the transition probability of (some) processes in "affine class" is given by solving Riccatti type equation. Recently, a "fractional" extension of this well known fact in quantitative finance are found in the context of "rough volatility". Inspired by the result, I studied the problem in an infinite-dimensional way to see the equation in a different way.

A Discrete Scheme of Static Hedging of Barrier Options

Yuri Imamura *

Abstract

We consider a discrete scheme for static hedging of barrier options, by establishing a discrete version of the transform which Peter Carr and Sergey Nadtochiy (2013) introduced, for a general one dimensional diffusion case. The transform describes the (put type) pay-off which balances at the barrier with a given (call-type) pay-off and hence the plain option with the former pay-off statically hedges a barrier option with the latter pay-off. In this talk I will construct the map for a class of Markov chains, which includes a discretization of Carr-Nadtochiy ' s correspondence, and also its multi-dimensional version. The latter gives a new insight to the literature.

*Dept. of Business Economics, Tokyo University of Science, Japan, (mail: imamuray@rs.tus.ac.jp)

A fractional calculus approach to rough path integration

Yu Ito (Kyoto Sangyo U)

This study is an alternative approach to the fundamental theory of rough path analysis on the basis of fractional calculus. In this talk, using fractional calculus, we will provide alternative expressions of the rough path integrals by T. J. Lyons (1998) and by M. Gubinelli (2004). The expressions are given explicitly by the Lebesgue integrals for fractional derivatives. Our results can be regarded as a generalization of those of Y. Hu and D. Nualart (2009), and one of the key ingredients for our results is a method by M. Zähle (1998).

Convergence of diffusion processes in a tube and Dirichlet forms of limit processes

Tomoko TAKEMURA

July 6, 2018

1 Preliminary

Let s be a continuous increasing function on an open interval (r_1, r_2), where $-\infty \le r_1 < r_2 \le \infty$, m be a right continuous increasing function on (r_1, r_2). If r_i is (s, m)-regular in the sense of Feller, then *absorbing* or *reflecting* boundary condition is posed at r_i. Let $\mathcal{G}_{s,m}$ be a one dimensional generalized diffusion operator on (r_1, r_2) with a scale function s and a speed measure m. We denote by $X_{s,m} = [X(t), P_x^X]$ on (r_1, r_2) the one dimensional diffusion process on (r_1, r_2) whose generator is given by $\mathcal{G}_{s,m}$. Here we consider $X^{(1)} = X_{s^{(1)},m^{(1)}}$ on (r_1, r_2), $\mathrm{R}_n = X_{s_n,m_n}$ on $I_n = (0, l_n)$, and $\mathrm{R} = X_{s,m}$ on $I = (0, l)$, where $-\infty \le r_1 < r_2 \le \infty$ and $0 < l_n,\ l \le \infty$ for $n \in \mathbb{N}$. We assume $|s^{(1)}(r_i)| = \infty,\ i = 1,\ 2$, the left end point 0 is (s_n, m_n)-entrance and (s, m)-entrance, and the absorbing or reflecting boundary condition is posed at l_n[resp. l] whenever it is (s_n, m_n)-regular[resp. (s, m)-regular]. Let $\Theta = [\Theta(t), P_\theta^\Theta]$ be a spherical Brownian motion on S^{d-1}.

Let ν_n be a Radon measure on I_n and assume that the support of ν_n coincides with I_n, $|\int_{(0,c)} s_n(x)\, d\nu_n(r)| = \infty$, $\forall c \in I_n$, $\forall n \in \mathbb{N}$, and $|\int_{(c,l_n)} d\nu_n(r)| < \infty$ if l_n is (s_n, m_n)-regular and reflecting. We set ν and make the assumptions for ν as well as ν_n. We set

$$\mathbf{f}_n(t) = \int_{I_n} l^{\mathrm{R}_n}(t, r)\, d\nu_n(r), \qquad \mathbf{f}(t) = \int_I l^{\mathrm{R}}(t, r)\, d\nu(r), \quad t \ge 0,$$

where $l^{\mathrm{R}_n}(t, r)$[resp. $l^{\mathrm{R}}(t, r)$] is the local time of R_n[resp. R]. Let

$$\mathbb{Y}_n = \Big[\mathbb{Y}_n(t) = \Big(\mathrm{R}_n(t), \Theta\big(\mathbf{f}_n(t)\big), X^{(1)}(t)\Big), P^{\mathbb{Y}_n}_{(r,\theta,x)}\Big],\ \mathbb{Y} = \Big[\mathbb{Y}(t) = \Big(R(t), \Theta\big(\mathbf{f}(t)\big), X^{(1)}(t)\Big), P^{\mathbb{Y}}_{(r,\theta,x)}\Big],$$

where $P^{\mathbb{Y}_n}_{(r,\theta,x)} = P_r^{\mathrm{R}_n} \otimes P_\theta^\Theta \otimes P_x^{X^{(1)}}$, $(r, \theta, x) \in I_n \times S^{d-1} \times (r_1, r_2)$ and $P^{\mathbb{Y}}_{(r,\theta,x)} = P_r^{\mathrm{R}} \otimes P_\theta^\Theta \otimes P_x^{X^{(1)}}$, $(r, \theta, x) \in I \times S^{d-1} \times (r_1, r_2)$. We note that $\big(R(t), \Theta(\mathbf{f}(t))\big)$ is the skew product Ξ of the one dimensional diffusion process R and the spherical Brownian motion Θ with respect to the positive continuous additive functional $\mathbf{f}(t)$. Next we consider the time changed processes. We set

$$\Psi_n(t) = \int_0^t \mu_{1,n}(\mathrm{R}_n(s))\mu_{2,n}\big(\Theta(\mathbf{f}(s))\big)\, ds, \quad \Psi(t) = \int_I l^{\mathrm{R}}(t, r)\, d\mu(r), \quad t > 0,$$

where $\mu_{1,n}(r)\mu_{2,n}(\theta)$ is a bounded measurable function on $I_n \times S^{d-1}$ which is bounded below by a positive constant and $d\mu(r) = 1_\Lambda(r)\, dm(r)$ for $\Lambda \subset I$. We set $\Gamma = \Lambda \times S^{d-1} \times (r_1, r_2)$. We denote by $\Phi_n(t)$[resp. $\Phi(t)$] the right continuous inverse of $\Psi_n(t)$[resp. $\Psi(t)$]. We consider the time changed processes

$$\mathbb{X}_n = \Big[\mathbb{X}_n(t) = Y_n\big(\Phi_n(t)\big), P^{\mathbb{Y}_n}\Big], \quad \mathbb{X} = \Big[\mathbb{X}(t) = Y\big(\Phi(t)\big), P^{\mathbb{Y}}\Big].$$

2 Main theorem

Theorem 1 *Under some assumptions, the time changed process $\mathbb{X}_n$ converge to the time changed process $\mathbb{X}$ in the following sense.*

$$\lim_{n\to\infty} p_t^{\mathbb{X}_n} f(r,\theta,x) = p_t^{\mathbb{X}} f(r,\theta,x)$$

for $t>0$, $(r,\theta,x)\in\Gamma$, and $f\in C_b\big((0,\infty)\times S^{d-1}\times(r_1,r_2)\big)$, where the semi group $p_t^X f(r,\theta,x) = E^{P^X_{(r,\theta,x)}}[f(X_t)]$, and $\tilde{\mathbb{X}}_n$[resp. $\tilde{\mathbb{X}}$] associate with $\tilde{s}_n$, $\tilde{m}_n$, $\tilde{\nu}_n$, and $\tilde{\mu}_{1,n}$[resp. $\tilde{s}$, $\tilde{m}$, $\tilde{\nu}$, and $\tilde{\mu}$].

Let $(\mathcal{E}^\Theta,\mathcal{F}^\Theta)$ be a Dirichlet form on $L^2(S^{d-1},m^\Theta)$ corresponding to Θ on S^{d-1}, and $(\mathcal{E}^{\mathrm{D}},\mathcal{F}^{\mathrm{D}})$ be a Dirichlet form on $L^2((r_1,r_2),m^{\mathrm{D}})$ corresponding to $X^{(1)}$. We denote by $S\times D = S^{d-1}\times S^{d-1}\times(r_1,r_2)\times(r_1,r_2)$ and $\mathcal{M}$ the product measure $m^\Theta\otimes m^\Theta\otimes m^{\mathrm{D}}\otimes m^{\mathrm{D}}$. We note that $I\setminus\Lambda = \cup_{k\in K} I_k$, a finite or a countable disjoint union of open intervals $I_k=(a_k,b_k)$ with the end points belonging to $\Lambda\cup\{0,l\}$.

Theorem 2 *Assume $\Lambda\neq I$, then the Dirichlet form $(\mathcal{E}^{\mathbb{X}},\mathcal{F}^{\mathbb{X}})$ of $\mathbb{X}$ is regular on $L^2(\Gamma,\mu\otimes m^\Theta\otimes m^{\mathrm{D}})$ and has $\mathcal{C}^{\mathbb{Y}}|_\Gamma$ as a core. For $f\in\mathcal{C}^{\mathbb{Y}}|_\Gamma$, the Dirichlet form $(\mathcal{E}^{\mathbb{X}},\mathcal{F}^{\mathbb{X}})$ is given by the following.*

$$\begin{aligned}
\mathcal{E}^{\mathbb{X}}(f,f) &= \int_\Gamma \partial^*_{s^{\mathrm{R}}} f(r,\theta,x)^2\, dm^{\mathrm{R}}(r)dm^\Theta(\theta)dm^{\mathrm{D}}(x) + \int_{\Lambda\times(r_1,r_2)} \mathcal{E}^\Theta(f(r,\cdot,x),f(r,\cdot,x))\, d\nu(r)dm^{\mathrm{D}}(x)\\
&+ \int_{\Lambda\times S^{d-1}} \mathcal{E}^{\mathrm{D}}(f(r,\theta,\cdot),f(r,\theta,\cdot))\, dm^{\mathrm{R}}(r)dm^\Theta(\theta)\\
&+ \frac{1}{2}\sum_{*1}\int_{S\times D}\{f(a_k,\zeta)-f(a_k,\eta)\}^2 J_1^{R_k}(\zeta,\eta:a_k,a_k)\, d\mathcal{M}(\zeta,\eta)\\
&+ \frac{1}{2}\sum_{*2}\int_{S\times D}\{f(b_k,\zeta)-f(b_k,\eta)\}^2 J_2^{R_k}(\zeta,\eta:b_k,b_k)\, d\mathcal{M}(\zeta,\eta)\\
&+ \frac{1}{2}\sum_{*3}\int_{S\times D}\{f(a_k,\zeta)-f(b_k,\eta)\}^2 J_3^{R_k}(\zeta,\eta:a_k,b_k)\, d\mathcal{M}(\zeta,\eta)\\
&+ \frac{1}{2}\sum_{*3}\int_{S\times D}\{f(b_k,\zeta)-f(a_k,\eta)\}^2 J_4^{R_k}(\zeta,\eta:b_k,a_k)\, d\mathcal{M}(\zeta,\eta) + I(f), \qquad (1)
\end{aligned}$$

where $*1 = k\in K,\ 0\le a_k<b_k<l$, $*2 = k\in K,\ 0<a_k<b_k\le l$, and $*3 = k\in K,\ 0<a_k<b_k<l$. Here the last term $I(f)$ should be read as

$$\frac{1}{s^{\mathrm{R}}(l)-s^{\mathrm{R}}(a_k)}\int_{S^{d-1}\times(r_1,r_2)} f(a_k,\theta,x)\, dm^\Theta(\theta)dm^{\mathrm{D}}(x)$$

if $0<a_k<b_k=l$, l is $(s^{\mathrm{R}},m^{\mathrm{R}})$-regular with absorbing, -exit, or -natural and $s^{\mathrm{R}}(l)<\infty$. $I(f)=0$ if $0<a_k<b_k=l$ and $s^{\mathrm{R}}(l)=\infty$. Furthermore the first term of the right hand side of (1) vanishes in case that $\int_\Lambda ds^{\mathrm{R}}(r)=0$.

In this talk we show details of assumptions and proofs and give the jumping measures $J_i^{R_k}(\zeta,\eta: a_k,b_k)$, $i=1,\ 2,\ 3,\ 4$ and the killing part $I(f)$.

Distance Multivariance - measuring and detecting multivariate dependence

Björn Böttcher, Institut für mathematische Stochastik, TU Dresden, Germany, `bjoern.boettcher@tu-dresden.de`

We introduce distance multivariance M and related quantities and highlight some properties. The talk is based on the recent preprints [1,2,3,4].

Let $X_1, \dots, X_n$ be random vectors with possibly distinct dimensions, i.e., with values in $\mathbb{R}^{d_i}$ for $i = 1, \dots, n$. If these vectors are $(n-1)$-independent then $M(X_1, \dots, X_n) = 0$ characterizes the independence of $X_1, \dots, X_n$. This is the basis for the construction of explicit measures of (in)dependence, e.g. total distance multivariance $\overline{M}$ and m-distance multivariance M_m such that

$$\overline{M}(X_1, \dots, X_n) = 0 \text{ if and only if } X_1, \dots, X_n \text{ are independent,}$$
$$M_2(X_1, \dots, X_n) = 0 \text{ if and only if } X_1, \dots, X_n \text{ are pairwise independent.}$$

In addition to the theoretical characterization there exist corresponding sample versions. These are computational efficient and their distributional properties are known. Thus empirical measures and empirical tests of (in)dependence can be constructed and implemented. All functions for the application of distance multivariance are published in the R package `multivariance` [5].

Roughly speaking, distance multivariance is constructed as the distance of characteristic functions in an L^2-space with respect to symmetric Lévy measures, and it has an equivalent representation in terms of expectations of continuous negative definite functions applied to differences of random variables. This yields that the sample version can be calculated using (doubly centered) distance matrices of the samples.

References

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Density for the solution to stochastic functional differential equations

Atsushi TAKEUCHI
(Osaka City University, JAPAN)

Let T and r be constant fixed throughout the talk, and $A : [0,T] \times C([-r,0]\,;\mathbb{R}^d) \to \mathbb{R}^d$ and $B : [0,T] \times C([-r,0]\,;\mathbb{R}^m \otimes \mathbb{R}^d)$, with certain nice conditions on the regularity and the boundedness. For a deterministic path $\eta \in C([-r,0]\,;\mathbb{R}^d)$, we shall consider stochastic functional differential equations of the form:

$$X(t) = \begin{cases} \eta(t) & (-r \le t \le 0), \\ \eta(0) + \displaystyle\int_0^t A(s, X_s)\, ds + \int_0^t B(s, X_s)\, dW(s) & (0 < t \le T), \end{cases}$$

where $W = \{W(t)\,;\, 0 \le t \le T\}$ is the m-dimensional Brownian motion starting at the origin, and $X_t = \{X(t+u)\,;\, -r \le u \le 0\}$ is the segment of the process X, while $X(t)$ is $\mathbb{R}^d$-valued at time t. Since the coefficients depend on the past histories of the process, the solution process is non-Markovian, and we cannot use any fruitful techniques of partial differential equations and potential theory. In this talk, we shall study some properties on the density for the solution under the condition that the coefficients of the diffusion terms satisfy the uniformly elliptic condition, via the Malliavin calculus.