Archive:

Workshop on Stochastic Analysis and related topics 2015

Thursday 5th and Friday 6th of November 2015

This is a workshop jointly organized by TU Dresden and our DAAD PaJaKo partners in Japan Kansai University (Osaka, Japan) and Korea Seoul National University (Seoul, South Korea). The workshop is open to colleagues from other universities. Apart from the dissemination of state-of-the-art scientific results, it aims to train advanced students and to provide a platform for young researchers.

Registration: Please send a short mail to **rene.schilling (at) tu-dresden.de**
There will be a small fee for tea & coffee during the breaks.

Organizer: René SCHILLING (TU Dresden), Gerald TRUTNAU (Seoul National University), Kazutoshi YAMAZAKI (Kansai University)

Location: C 207, Willersbau, Zellescher Weg 12-14, Dresden
A detailed description how to find us is given here.

Workshop on Stochastic Analysis and Related Topics 2015

Update

Thursday Nov 5th	
09.00 – 09.35	**Mateusz Kwasnicki** (Wroclaw) Martin kernels for jump-type processes
09.40 – 10.15	**Toshihiro Uemura** (Kansai) A note on the Mosco convergence of symmetric Dirichlet forms
	Coffee
10.45 – 11.20	**Daniel Lenz** (Jena) Short Time Asymptotics of Heat Semigroups
11.25 – 12.00	**Marcel Schmidt** (Jena) Does diffusion determine the geometry?
	Lunch
14.00 – 14.35	**Joanna Tumilewicz** (Wroclaw) Insurance contracts based on drawdown and drawup events of spectrally negative Lévy processes
14.40 – 15.15	**Irmina Czarna** (Wroclaw) Exit problems for spectrally negative Lévy processes with Parisian delay and a lower ultimate bankrupt barrier implementation
	Coffee
15.45 – 16.20	**Niels Jacob** (Swansea) Surprises and gaps in our understanding of transition functions
16.25 – 17.00	**Kazutoshi Yamazaki** (Kansai) Refracted-reflected spectrally negative Lévy processes
17.05 – 17.40	**Stefano De Marco** (Paris) Robust super-hedging of options on the VIX volatility index and Martingale optimal transport
19.00 –	**Dinner**

Friday Nov 6th	
09.00 – 09.35	**Gerald Trutnau** (Seoul) Recurrence criteria for generalized Dirichlet forms
09.40 – 10.15	**Panki Kim** (Seoul) Estimates of Dirichlet heat kernel for symmetric Markov processes
	Coffee
10.45 – 11.20	**Matsuyo Tomisaki** (Nara) From diffusion processes to bi-generalized diffusion processes
11.25 – 12.00	**Victoria Knopova** (Kiev) Chung-type law of the iterated logarithm for Lévy type processes
	Lunch
14.00 – 14.35	**Paolo Di Tella** (Dresden) The Chaotic Representation Property of Compensated-Covariation Stable Families of Martingales
14.40 – 15.15	**Bartolomiej Dyda** (Wroclaw) Fractional calculus for power functions
	Coffee
15.45 – 16.20	**Tomasz Grzywny** (Wroclaw) Kato classes for Levy processes
16.25 – 17.00	**Kamil Kaleta** (Wroclaw) Contractivity and ground state domination properties for non-local Schrödinger operators

WORKSHOP ON STOCHASTIC ANALYSIS AND RELATED TOPICS 2015

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Arrival Information

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Zoom (4,7 MB)

Schedule: List of abstracts

Thursday, November 5th

09:00–09:35	**Mateusz Kwasnicki** (TU Wroclaw, Poland)	Martin kernels for jump-type processes
09:40–10:15	**Toshihiro Uemura** (Kansai U, Osaka, Japan)	A note on the Mosco convergence of symmetric Dirichlet forms
	Coffee break	
10:45–11:20	**Daniel Lenz** (U Jena, Germany)	Short Time Asymptotics of Heat Semigroups
11:25–12:00	**Marcel Schmidt** (U Jena, Germany)	Does diffusion determine the geometry?
	Lunch	
14:00–14:35	**Joanna Tumilewicz** (U Wroclaw, Poland)	Insurance contracts based on drawdown and drawup events of spectrally negative Lévy processes
14:40–15:15	**Irmina Czarna** (U Wroclaw, Poland)	Exit problems for spectrally negative Lévy processes with Parisian delay and a lower ultimate bankrupt barrier implementation
	Coffee break	
15:45–16:20	**Niels Jacob** (Swansea U, UK)	Surprises and gaps in our understanding of transition functions
16:25–17:00	**Kazutoshi Yamazaki** (Kansai U, Osaka, Japan)	Refracted-reflected spectrally negative Lévy processes
17:05–17:40	**Stefano De Marco** (X - Ecole Polytechnique, France)	Robust super-hedging of options on the VIX volatility index and Martingale optimal transport
19.00–	**Dinner**	

Friday, November 6th --- Update ---

09:00–09:35	**Gerald Trutnau** (Seoul National U, S. Korea)	Recurrence criteria for generalized Dirichlet forms
09:40–10:15	**Panki Kim** (Seoul National U, S. Korea)	Estimates of Dirichlet heat kernel for symmetric Markov processes
	Coffee break	
10:45–11:20	**Matsuyo Tomisaki** (Nara Women's U, Japan)	From diffusion processes to bi-generalized diffusion processes
11:25–12:00	**Victoria Knopova** (Academy of Sciences, Kyiv, Ukraine)	Chung-type law of the iterated logarithm for Lévy type processes
	Lunch	
14:00–14:35	**Paolo Di Tella** (TU Dresden, Germany)	The Chaotic Representation Property of Compensated-Covariation Stable Families of Martingales
14:40–15:15	**Bartolomiej Dyda** (TU Wroclaw)	Fractional calculus for power functions
	Coffee break	
15:45-16:20	**Tomasz Grzywny** (TU Wroclaw, Poland)	Kato classes for Levy processes
16:25–17:00	**Kamil Kaleta** (TU Wroclaw, Poland)	Contractivity and ground state domination properties for non-local Schrödinger operators

Further Participants:
Apart from the speakers, the following colleagues will come to TU Dresden
Frank Balke
Krzysztof Bogdan (TU Wroclaw),
Minjung Gim (Seoul National U),
Tomasz Juszczyszyn (TU Wroclaw),
Kyung-youn Kim (Seoul National U & TU Dresden),
Jacek Mucha (TU Wroclaw),
Mariusz Olszewski (TU Wroclaw),
Artur Rutkowski (TU Wroclaw),
Nikola Sandric (Zagreb U & TU Dresden),
Takahiro Shindo (Kansai U),
Kohei Suzuki (Kyoto U, z.Zt. U Bonn),
Zbigniew Palmowski (U Wroclaw),
Melchior Wirth (U Jena)

Organizers:

René SCHILLING (TU Dresden)
Gerald TRUTNAU (Seoul National University)
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Location:

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Abstracts

Irmina Czarna (U Wrocław)

Exit problems for spectrally negative Lévy processes with Parisian delay and a lower ultimate bankrupt barrier implementation

Abstract: We will present ruin problem, where there is a Parisian delay and a lower ultimate bankrupt barrier. In this problem we will say that a risk process get ruined when it stays below zero longer than a fixed amount of time $d > 0$ or goes below a fixed level $-a$ (ultimate bankruptcy level). We focus on a general spectrally negative Lévy insurance risk process. For this class of processes using fluctuation theory we identify the Laplace transforms of the ruin probability, two-sided exit problems in terms of so-called q-scale functions and second-generation q-scale functions.

Stefano de Marco (X - Paris)

Robust super-hedging of options on the VIX volatility index and martingale optimal transport

Abstract: Several recent works have shown how the super-heding problem of a financial derivative can be formulated as an optimal transport problem with an additional martingale constraint, leading to duality results and to super-hedging strategies that are robust, in the sense that they are independent of any parametric modeling assumption. VIX options traded on the CBOE have become popular volatility derivatives. In this work, we bound VIX options from S& P500 vanilla options and VIX futures. This leads us to introduce a new martingale optimal transport problem with additional constraints (that can eventually be solve numerically). Analytical lower and upper bounds are also provided, which highlight some (potential) arbitrage opportunities. We fully characterize the class of marginal distributions for which these explicit bounds are optimal, and illustrate numerically that they seem to be optimal also for the market-implied marginal distributions. Joint work with P. Henry-Labordère

Paolo di Tella (TU Dresden)

The chaotic representation property compensated-covariation stable families of martingales

Abstract: We study the chaotic representation property for certain families $\mathcal{X}$ of square integrable martingales on a finite time interval $[0, T]$. For this purpose, we introduce the notion of compensated-covariation stability of such families. The chaotic representation property will be defined using iterated integrals with respect to a given family $\mathcal{X}$ of square integrable martingales having deterministic mutual predictable covariation. The main result is: If $\mathcal{X}$ is a compensated-covariation stable family of square integrable martingales such that $\langle X, Y \rangle$ is deterministic for all $X, Y \in \mathcal{X}$ and, furthermore, the system of monomials generated by $\mathcal{X}$ is total in $L^2(\Omega, \mathcal{F}_T, \mathbb{P})$, then $\mathcal{X}$ possesses the chaotic representation property. We shall then give concrete examples in the case of Lévy processes. This talk is based on a joint paper with H.-J. Engelbert.

Bartolomiej Dyda(TU Wrocław)

Fractional calculus for power functions

Abstract:

Zorana Grbac (U Paris VII)

Approximate option pricing in the Lévy Libor model

Abstract: In this work we consider pricing of options on interest rates (caplets and swaptions) in the Lévy Libor model developed by Eberlein and Özkan (2005). This model is an extension to Lévy driving processes of the classical Libor market model (LMM) driven by a Brownian motion. Option pricing is significantly less tractable in this model than in the LMM due to the appearance of stochastic terms in the jump part of the driving process when performing the measure changes which are standard in pricing of interest rate derivatives. To this end, we propose to treat a given Lévy Libor model as a suitable perturbation of the LMM and thus obtain closed form approximations for caplet and swaption prices. The method is inspired by recent works by Černy, Denkl and Kallsen (2013) and Tankov and Ménassé (2014). The approximate option prices in the Lévy Libor model are given as the corresponding LMM prices plus correction terms which depend on the characteristics of the underlying Lévy process and some additional terms obtained from the LMM model. This is joint work with D. Krief and P. Tankov.

Tomasz Grzywny (TU Wrocław)

Kato classes for Levy processes

Abstract: Let $\{X_t\}_{t>0}$ be a Lévy process in $\mathbb{R}^d$ and let q be a Borel function $q : \mathbb{R}^d \mapsto \mathbb{R}$. In the talk I will discuss relations, and their consequences, of the following two Kato conditions,

$$\lim_{t\to 0^+} \left[\sup_x \mathbb{E}^x \int_0^t |q(X_s)| ds \right] = 0$$

and

$$\lim_{r\to 0^+} \left[\sup_x \mathbb{E}^x \int_0^\infty e^{-\lambda t} \mathbb{1}_{B(x,r)} |q(X_t) dt| ds \right] = 0$$

for some $\lambda > 0$ (equivalently for every $\lambda > 0$). Through the Khas'minskii Lemma the first condition is crucial in the study of the Schrödinger (Feynman-Kac) semigroup. On the other hand, the second condition usually transforms into an analytic expression which allows to read off the acceptable singularity of q. A fundamental example of the equivalence of the above two conditions is the Brownian motion in $\mathbb{R}^d$, $d \geq 2$. The equivalence essentially fails if $d = 1$. The talk is based on a joint work with Karol Szczypkowski.

Niels Jacob (Swansea U)

Surprises and gaps in our understanding of transition functions

Abstract: Two observations: Every positive power of the negative Laplacian generates an L^2-semigroup with density. For powers between 0 and 1 we deal with positivity preserving semigroups. For integer powers we have densities with exponential decay, the decay is controlled by the Legendre transform of the symbol which turns out to be the symbol of a symmetric stable process. Of course for non-integer powers we cannot have exponential decay.

With a proper normalization the Fourier transforms of certain convolution semigroups define a strongly continuous family of contractions. Some are related to additive processes and give insights about transition functions.

The Fourier transform and the Legendre transform are "duality operations" and the question arises whether we should or even have to not embed our studies of transition functions of Lévy processes into a much wider context.

Kamil Kaleta (TU Wrocław)

Contractivity and ground state domination properties for non-local Schrödinger operators

Abstract: First I will show that the classical hypercontractivity, supercontractivity and ultracontractivity properties of the ground state transformed semigroups corresponding to the fairly general self-adjoint operators can be refined by the concept of the ground state domination. Next I will apply this observation to the semigroups of non-local Schrödinger operators based on generators of symmetric jump-paring Lévy processes with Kato-class confining potentials. This class of processes has the property that the intensity of single large jumps dominates the intensity of all multiple large jumps, and the related operators include pseudo-differential operators of interest in mathematical physics. As a consequence, I will obtain for a large subclass of confining potentials that, on the one hand, supercontractivity and ultracontractivity, on the other hand, hypercontractivity and asymptotic ultracontractivity of the transformed semigroup are equivalent properties. This is in stark contrast to classical Schrödinger operators, for which all these properties are known to be different. My talk is based on joint preprint with M. Kwaśnicki and J. Lőrinczi: arXiv:1509.08319.

Panki Kim (Seoul National U)

Estimates of Dirichlet heat kernel for symmetric Markov processes

Abstract: We consider a large class of symmetric pure jump Markov processes dominated by isotropic unimodal Lévy processes with weak scaling conditions. We first establish sharp two-sided heat kernel estimates for this processes in $C^{1,\rho}$ open sets. As a corollary of our main result, we obtain a sharp two-sided Green function and a scale invariant boundary Harnack inequality with explicit decay rates in $C^{1,\rho}$ open sets. This is a joint work with Tomasz Grzywny and Kyung-Youn Kim.

Victoria Knopova (U Kiev & Ukrainian NAS)

Chung-type law of Tthe iterated logarithm for Lévy type processes

Abstract: The talk is devoted to the Chung-type liminf law of the iterated logarithm at zero for a class of (pure-jump) Lévy-type processes. Our result extends earlier work by Dupuis [C. Dupuis: Mésure de Hausdorff de la trajectoire de certains processus à accroissements indépendants et stationnaires. In: *Séminaire de Prohabilités VIII* (1972/73). Springer, Lecture Notes in Mathematics **381**, Berlin 1974, 40–77] for Lévy processes, but the lack of independence of increments in the Lévy type case makes the problem much

more complicated. The norming function is given in terms of the symbol of the infinitesimal generator of the process.

This talk is based on the following paper:

V. Knopova, R.Schilling . On the small-time behaviour of Lévy-type processes. *Stochastic Processes and their Applications* **124** (2014) 2249–2265.

Mateusz Kwasnicki (TU Wrocław)

Martin kernels for jump-type processes

Abstract:

I will discuss the *boundary Harnack inequality* (BHI) and *Martin representation* for positive functions harmonic with respect to a Markov process with jumps. BHI states that positive harmonic functions in a domain D which converge to zero at some part of the boundary of D have common decay rate. In other words, ratios of such functions are comparable to a constant. Such a result was proved recently for a fairly general class of processes in my paper with Krzysztof Bogdan and Takashi Kumagai.

BHI is a self-improving estimate and, under appropriate conditions, the ratios of harmonic functions are not only bounded, but they extend continuously to the boundary. This allows one to define the *Martin kernel* as the boundary limit of the ratio of Green functions. General theory of Martin boundary implies then a representation theorem for harmonic functions. This part is based on my preprint with Tomasz Juszczyszyn.

Processes covered by our results include (not necessarily symmetric) strictly stable Lévy processes, many symmetric stable-like processes on manifolds and on fractals, and a wide class of isotropic unimodal Lévy processes. A detailed study of the last example in this list is an ongoing work with Tomasz Grzywny.

Daniel Lenz (U Jena)

Short Time Asymptotics of Heat Semigroups

Abstract:

Marcel Schmidt (U Jena)

Does diffusion determine the geometry?

Abstract: Can one hear the shape of a drum? In mathematical terms this famous question of M. Kac asks whether two unitarily equivalent Laplacians live on the same geometric object. It is now known, that in general the answer is negative. Following an idea of Wolfgang Arendt, we replace the unitary transformation intertwining the Laplacians by an order preserving one and then ask how much of the geometry is preserved. As in this situation the associated semigroups are equivalent up to an order isomorphism our question becomes as stated in the title. In this talk we try to give an answer. (this is joint work with Matthias Keller, Daniel Lenz and Melchior Wirth)

Matsuyo Tomisaki (Kansai U & Nara Women's U)

From diffusion processes to bi-generalized diffusion processes

Abstract: The theory of one-dimensional diffusion processes was developed in

the 1950's. A one-dimensional diffusion process is a strong Markov process with continuous sample paths, and it is determined by a strictly increasing continuous scale function s and a positive speed measure dm on an interval in the real line. The positivity of dm was soon relaxed to nonnegativity, and appeared the notion of generalized diffusion processes or gap processes. A generalized diffusion process is a strong Markov process with right continuous sample paths, which may jump only to the nearest neighbours in the support of dm, and it is determined by a strictly increasing continuous scale function s and a nonnegative speed measure dm. The class of one-dimensional diffusion processes or generalized diffusion processes forms an effective one from probabilistic and analytic points of view. However, in the 1980's, there appeared a one-dimensional Markov process corresponding to the scale function with jumps in applications of the theory of one-dimensional diffusion processes or generalized diffusion processes. The one-dimensional Markov process corresponding to a non-decreasing scale function s and a nonnegative speed measure dm, which we call a bi-generalized diffusion process, neither is strong Markov nor has right continuous sample paths in general anymore. In this talk, we survey some applications of bi-generalized diffusion processes.

Gerald Trutnau (Seoul National U)

Recurrence criteria for generalized Dirichlet forms

Abstract: We develop sufficient analytic conditions for recurrence and transience of non-sectorial perturbations of possibly non-symmetric Dirichlet forms on a general state space. These form an important subclass of generalized Dirichlet forms. In case there exists an associated process, we show how the analytic conditions imply recurrence and transience in the classical probabilistic sense. As an application, we consider a generalized Dirichlet form given on a closed or open subset of the Euclidean space which is given as a divergence free first order perturbation of a non-symmetric energy form. Then using volume growth conditions of the sectorial part and the non-sectorial first order part, we derive an explicit criterion for recurrence. Moreover, we present concrete examples with applications to Muckenhoupt weights and counterexamples. The counterexamples show that the non-sectorial case differs qualitatively from the symmetric or non-symmetric sectorial case. Namely, we make the observation that one of the main criteria for recurrence in these cases fails to be true for generalized Dirichlet forms.

Joanna Tumilewicz (U Wrocław)

Insurance contracts based on drawdown and drawup events of spectrally negative Lévy processes

Abstract: We consider insurances polices based on drawdown/drawup event where underlying asset is derived by spectrally negative Lévy process. We analyze four contracts. In the first one the protection buyer will pay constant premium p continuously until the drawdown of size a occurs. In return he receives the insured amount of α at the drawdown time. Another contract provides protection from any specified drawdown with drawup contingency. This contract may expire early if a drawup event occurs prior to drawdown

one. The last two contracts are extension of the previous ones by additional cancellable feature, which allows investor to terminate contract earlier. We focus on two problems: calculate fair premium for basic contracts and find the optimal stopping time for polices with cancellable feature. In analysis we use heavily the fluctuation theory of drawdown Lévy processes and general theory of construction of optimal stopping rules for Markov process. This is a joint work with Zbigniew Palmowski.

Toshihiro Uemura (Kansai U)

A note on the Mosco convergence of symmetric Dirichlet forms

Abstract: In the talk, we will show that a sequence of jump type symmetric Dirichlet forms on $L^2(\mathbb{R}^d)$ converges to the Dirichlet form associated with a symmetric diffusion process in the sense of the Mosco convergence. Furthermore we will also give an example for which the Hausdorff dimension of the paths up to time 1 of the limit process can drastically change those of a sequence of symmetric Lévy processes under the Mosco convergence.

Kazutoshi Yamazaki (Kansai U)

Refracted-reflected spectrally negative Lévy processes

Abstract: We study a combination of the refracted and reflected Lévy processes. Given a spectrally negative Lévy process and two boundaries, it is reflected at the lower boundary while, whenever it is above the upper boundary, a linear drift at a constant rate is subtracted from the increments of the process. Using the scale functions, we compute the resolvent measure, the Laplace transform of the occupation times as well as other fluctuation identities that will be useful in applied probability including insurance, queues, and inventory management. (Joint with Jose Luis Perez, CIMAT)