# Archive:

# Workshop on Jump Processes – JUMPS 2014

## Thursday 6th and Friday 7th February 2014

The aim of this workshop is to bring together a range of researchers who are interested in stochastic processes with jumps to present their work and exchange ideas.

**Organizers:**
Anita BEHME (TU Dresden) and René SCHILLING (TU Dresden)

**Location:**
C 207, Willersbau, Zellescher Weg 12-14, Dresden
A detailed description how to find us is given here.


**Booklet** (page 2-13, with Schedule and Abstracts)
**WebSite** (page 14+15)

# Jumps 2014
# Workshop on Jump Processes

## Schedule and Abstracts

**6th - 7th February 2014**
Room: WIL C 207
Willersbau, Zellescher Weg 12-14
01069 Dresden

# Schedule:

**Thursday February 6th**

| | | |
|---|---|---|
| 09.30-10.15 | **Frank Aurzada** (Darmstadt) | First exits of Lévy processes over a one-sided moving boundary |
| 10.20-11.05 | **Thomas Simon** (Lille) | Hitting densities for stable processes |
| | **Coffee break** | |
| 11.30-12.15 | **Alexander Schnurr** (Siegen) | A Criterion for Invariant Measures of Itô Processes Based on the Symbol |
| | **Lunch** | |
| 14.00-14.25 | **Michael Schwarzenberger** (Dresden) | Affine processes and symbols |
| 14.30-14.55 | **Elke Rosenberger** (Potsdam) | Agmon-type estimates for a class of jump processes |
| 15.00-15.25 | **Chang-Song Deng** (Dresden/Wuhan) | Some analysis on the path space of Lévy processes |
| | **Coffee break** | |
| 16.00-16.45 | **Mátyás Barczy** (Debrecen) | A SDE with jumps for a multi-type CBI process |
| 16.50-17.35 | **Gyula Pap** (Szeged) | Moment estimates and a convergence result for a multi-type CBI process |

**Friday February 7th**

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| 09.30-10.15 | **Kathrin Glau** (Munich) | Feynman-Kac for Lévy processes: from killing rates to the supremum process |
| 10.20-11.05 | **Andreas Basse-O'Connor** (Aarhus) | Uniform convergence of random series in the Skorohod space D[0,1] |
| | **Coffee break** | |
| 11.30-12.15 | **Mihály Kovács** (Otago) | Euler approximation of the stochastic Allen-Cahn equation |
| | **Lunch** | |
| 14.00-14.45 | **Mateusz Kwasnicki** (Wroclaw) | Hitting times of points for symmetric Lévy processes with completely monotone jumps |
| 14.50-15.35 | **Ilya Pavlyukevich** (Jena) | Limiting behaviour of finite and infinite dimensional stochastic systems driven by stable Lévy processes |

# Abstracts

## First exits of Lévy processes over a one-sided moving boundary

Frank Aurzada, TU Darmstadt

We study the probability that a Lévy process at time $T$ has not yet crossed a given deterministic function $f$ (called the moving boundary). If $f$ is constant this question is in the realm of classical fluctuation theory. We determine for what functions $f$ the asymptotic order (as $T$ tends to infinity) of this probability is the same as for a constant function. This is joint work with Tanja Kramm.

**References:**

[1] Aurzada, A. and Kramm, T. The first passage time problem over a moving boundary for asymptotically stable Lévy processes. arXiv:1305.1203.

[2] Aurzada, A. and Kramm, T. First exit of Brownian motion from a one-sided moving boundary. High Dimensional Probability VI: The Banff Volume. Progress in Probability 66 (2013), 215-219. arXiv:1203.4691.

[3] Aurzada, A., Kramm, T., and Savov, M. First passage times of Lévy processes over a one-sided moving boundary. arXiv:1201.1118

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## Hitting densities of stable processes

Thomas Simon (Université Lille 1)

We consider a Lévy stable process $X$ on the line which visits points a.s. We derive an identity in law for the hitting time $\tau$ of a given point, in terms of a size-bias of the positive branch of a drifted Cauchy random variable and of a positive stable random variable. This generalizes an identity by Yano, Yano and Yor [5] in the symmetric case and by myself [4] in the spectrally positive case. Together with an argument relying on self-decomposability and log-concavity, this identity makes it possible to show the unimodality of these hitting times, in the spirit of a classical result by Rösler [3] for linear diffusions. We will compare the identity for hitting times with Zolotarev's identity [6] for the positive branch of $X_1$ and also, if time permits, with the recent computations by Kuznetsov [1] on the first passage times of $X$.

This is joint work with Julien Letemplier [2].

**References:**

[1] A. Kuznetsov. On the density of the supremum of a stable process. *Stoch. Proc. Appl.* **123** (3), 986-1003, 2013.

[2] J. Letemplier and T. Simon. Unimodality of hitting times for stable processes. `http://arxiv.org/abs/1309.5321`.

[3] U. Rösler. Unimodality of passage times for one-dimensional strong Markov processes. *Ann. Probab.* **8** (4), 853-859, 1980.

[4] T. Simon. Hitting densities for spectrally positive stable processes. *Stochastics* **83** (2), 203-214, 2011.

[5] K. Yano, Y. Yano and M. Yor. On the laws of first hitting times of points for one-dimensional symmetric stable Lévy processes. *Sémin. Probab.* **XLII**, 187-227, 2009.

[6] V. M. Zolotarev. *One-dimensional stable distributions.* Nauka, Moskva, 1983.

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## A Criterion for Invariant Measures of Itô Processes Based on the Symbol

Alexander Schnurr, Siegen University

Having introduced the class of Itô processes (in the sense of Cinlar, Jacod, Protter and Sharpe), we give a short overview on the probabilistic symbol and its applications. In the main part of the talk we develop an integral criterion for the existence of an invariant measure of an Itô process. This class of processes is quite general: it contains rich Feller processes, solutions of Lévy driven SDEs and, more general, solutions of Skorokhod-type SDEs. Our criterion in based on the probabilistic symbol. In contrast to the standard integral criterion for invariant measures of Markov processes based on the generator, no test functions and hence no information on the domain of the generator is needed.

This is joint work with Anita Behme (TU Dresden).

**References:**

[1] Behme, A. and Schnurr, A. (2013+) A Criterion for Invariant Measures of Itô Processes Based on the Symbol. *Submitted.*

[2] Cinlar, E., Jacod, J., Protter, P., and Sharpe, M. J. (1980) Semimartingales and Markov processes. *Z. Wahrscheinlichkeitstheorie verw. Gebiete* **54**, 161–219.

## Affine processes and symbols

Michael Schwarzenberger, TU Dresden

The concept of affine processes was introduced in 2003 by Duffie, Filipović and Schachermayer. This new class summarizes many well-known processes, especially from the field of mathematical finance, like the squared Bessel process or the Ornstein-Uhlenbeck process.
For jump processes the characterization via the generator and its symbol has proven to be very useful. Many properties of the process are derived from the symbol.
In this talk we bring these two fields together. Therefore we present some facts about affine processes and give a short introduction to generators and symbols. Then we show the symbol of an affine process and its special features. As an application we deduce the conservativeness of affine processes from the symbol.

**References:**

[1] Böttcher, B., Schilling, R. L., and Wang, J. (2013) Lévy Matters III. *Lecture Notes Math.*

[2] Duffie, D., Filipović, D., and Schachermayer, W. (2003) Affine processes and applications in finance. *Ann. Appl. Probab.* **13**, 984–1053

[3] Keller-Ressel, M., Schachermayer, W., and Teichmann, J. (2011) Affine processes are regular. *Probab. Theory Relat. Fields* **3-4**, 591–611

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## Agmon-type estimates for a class of jump processes

Elke Rosenberger, Universität Potsdam

In the limit $\varepsilon \to 0$ we consider the generators $H_\varepsilon$ of families of reversible jump processes in $\mathbb{R}^d$ associated with a class of symmetric non-local Dirichlet-forms. It turns out, that the eigenfunctions decay exponentially fast with a rate given by a Finsler distance. This distance function is given as solution of a certain eikonal equation. The estimates are analog to the semiclassical Agmon estimates for differential operators of second order.

This is joint work with Christian Leonard and Markus Klein.

**References:**

[1] Fukushima, M., Oshima, Y. and Takeda, M.(1994) Dirichlet Forms and Symmetric Markov Processes, *de Gruyter*

[2] Helffer, B. and Sjöstrand, J. (1984) Multiple wells in the semi-classical limit I. *Comm. in P.D.E.***9**, 337-408

[3] Klein, M. and Rosenberger, E. (2008) Agmon-Type Estimates for a class of Difference Operators . *Ann. Henri Poincaré***9**, 1177-1215

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## Some analysis on the path space of Lévy processes

Chang-Song Deng, Dresden University of Technology, Wuhan University

A Cameron-Martin type theorem (and the integration by parts formula) will be presented for subordinate Brownian motions. Moreover, I shall talk about functional inequalities on the path space of Lévy processes.

# A SDE with jumps for a multi-type CBI process

Mátyás Barczy, University of Debrecen

First, we prove a generalization of some results of Yamada and Watanabe [4] for a stochastic differential equation (SDE) with jumps (given as follows) which may have interest on its own. Let $U$ be a complete, separable metric space equipped with its Borel $\sigma$-algebra $\mathcal{B}(U)$, and let $m$ be a $\sigma$-finite Radon measure on $(U, \mathcal{B}(U))$. Let $U_0, U_1 \in \mathcal{B}(U)$ be disjoint subsets, and $d$ and $r$ be positive integers. Let $b : [0,\infty) \times \mathbb{R}^d \to \mathbb{R}^d$, $\sigma : [0,\infty) \times \mathbb{R}^d \to \mathbb{R}^{d\times r}$, $f : [0,\infty) \times \mathbb{R}^d \times U \to \mathbb{R}^d$ and $g : [0,\infty) \times \mathbb{R}^d \times U \to \mathbb{R}^d$ be Borel measurable functions. Consider a $d$-dimensional SDE

$$\begin{aligned} \boldsymbol{X}_t = \boldsymbol{X}_0 &+ \int_0^t b(s, \boldsymbol{X}_s)\,\mathsf{d}s + \int_0^t \sigma(s, \boldsymbol{X}_s)\,\mathsf{d}\boldsymbol{W}_s \\ &+ \int_0^t \int_{U_0} f(s, \boldsymbol{X}_{s-}, u)\,\widetilde{N}(\mathsf{d}s, \mathsf{d}u) + \int_0^t \int_{U_1} g(s, \boldsymbol{X}_{s-}, u)\,N(\mathsf{d}s, \mathsf{d}u) \end{aligned} \tag{1}$$

for $t \in [0,\infty)$, where $(\boldsymbol{W}_t)_{t\in\mathbb{R}_+}$ is an $r$-dimensional standard Brownian motion, $N(\mathsf{d}s, \mathsf{d}u)$ is a Poisson random measure on $(0,\infty) \times U$ with intensity measure $\mathsf{d}s\, m(\mathsf{d}u)$, $\widetilde{N}(\mathsf{d}s, \mathsf{d}u) := N(\mathsf{d}s, \mathsf{d}u) - \mathsf{d}s\, m(\mathsf{d}u)$, and $(\boldsymbol{X}_t)_{t\in\mathbb{R}_+}$ is a suitable process with values in $\mathbb{R}^d$. We prove that pathwise uniqueness for the SDE (1) implies uniqueness in the sense of probability law; and that weak existence and pathwise uniqueness for the SDE (1) imply strong existence.

In the second part of the talk, a multi-type continuous state continuous time branching process with immigration satisfying some moment conditions is identified as a pathwise unique strong solution of some SDE with jumps of type (1) with some explicitly given functions $b$, $\sigma$, $f$ and $g$ not depending on $s$. This generalizes the corresponding one- and two-dimensional results of Li [2,Theorem 9.31] and Ma [2,Theorem 3.2], respectively.

The full presentation of our generalizations of Yamada–Watanabe results for the SDE (1) can be found in [1].

This is a joint work with Zenghu Li (Beijing Normal University) and Gyula Pap (University of Szeged).

The research of M. Barczy and G. Pap was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP-4.2.4.A/2-11/1-2012-0001 'National Excellence Program'.

**References:**

[1] Barczy, M., Li, Z. and Pap, G. (2013) Yamada–Watanabe results for stochastic differential equations with jumps. ArXiv: `http://arxiv.org/abs/1312.4485`

[2] Li, Z. H. (2011) Measure-Valued Branching Markov Processes. *Springer-Verlag, Heidelberg.*

[3] Ma, R. G. (2013) Stochastic equations for two-type continuous-state branching processes with immigration. *Acta Mathematica Sinica, English Series* **29(2)**, 287–294.

[4] Yamada, T. and Watanabe, S. (1971) On the uniqueness of solutions of stochastic differential equations. *Journal of Mathematics of Kyoto University* **11(1)**, 155–167.

## Moment estimates and a convergence result for a multi-type CBI process

Gyula Pap, University of Szeged

First, a recursion for moments of a multi-type continuous state continuous time branching process with immigration (CBI process) is derived. As a consequence, we obtain several useful moment estimates. An important tool during the proof is the identification of a multi-type CBI process satisfying some moment conditions as the pathwise unique strong solution of certain stochastic differential equation with jumps given in the talk of Mátyás Barczy.

In the second part of the talk, under natural assumptions, a Feller type diffusion approximation is derived for critical, indecomposable multi-type CBI processes. Namely, it is proved that a sequence of appropriately scaled random step functions formed from a critical, indecomposable multi-type CBI process converges weakly towards a squared Bessel process supported by a ray determined by the Perron vector of a matrix related to the branching mechanism of the CBI process.

This is joint work with Mátyás Barczy (University of Debrecen) and Zenghu Li (Beijing Normal University).

The research of M. Barczy and G. Pap was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP-4.2.4.A/2-11/1-2012-0001 'National Excellence Program'.

## Feynman-Kac for Lévy processes: from killing rates to the supremum process

Kathrin Glau, Technische Universität München

We establish assertions of type Feynman-Kac, answering the following question: under which conditions can we represent the weak solution to the Kolmogorov backward equation of a Lévy process with killing rate as a conditional expectation? We do this for the parabolic equation on $\mathcal{R}^d$ and on a subdomain of $\mathcal{R}^d$ with zero boundary conditions.

To this end, we classify Lévy processes according to the solution spaces of the associated parabolic PIDEs: we show that the corresponding evolution problem has a unique weak solution in the Sobolev-Slobodeckii space $H^{\alpha/2}$ if and only if the process has Sobolev index $\alpha$. We relate the Sobolev index to the Blumenthal-Getoor index.

Next, we present Feynman-Kac representations for variational solutions of Kolmogorov backward equations for a class of time-inhomogeneous Lévy processes killed with exponential killing rate. This class comprises processes with Brownian part and a wide class of pure jump processes, such as generalized hyperbolic, multivariate normal inverse Gaussian, tempered stable and semi-stable processes.

The results have applications in probability theory, relativistic quantum mechanics, financial mathematics and insurance. In particular, we characterize the distribution function of the running supremum of a Lévy process. We deduce smoothness results for the distribution of the supremum of a Lévy process and derive new conditions under which it has a density.

**References:**

[1] Glau, K. (2013) Classification of Lévy processes with parabolic Kolmogorov backward equations, submitted

[2] Glau, K. (2014) Feynman-Kac formula for Lévy processes with killing rate, submitted

[3] Glau, K. (2014) Feynman-Kac for Lévy processes – from boundary value problems to the supremum process, manuscript in preparation.

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## Uniform convergence of random series in the Skorohod space D[0,1]

Andreas Basse-O'Connor, Aarhus University

In this talk we will consider convergence of series of independent stochastic processes with cadlag sample paths (cadlag means right-continuous with left-hand limits). For symmetric processes, we show that finite dimensional convergence to a cadlag limit implies uniform convergence almost surely, and thereby extend the Ito-Nisio theorem to the Skorohod space D[0,1]. The main difficulties of dealing with D[0,1] in this context are its non-separability under the supremum norm and the discontinuity of addition under Skorohod's topology. We apply this result to

prove the uniform convergence of series representations of stable processes in a general non-Markovian setting. As a consequence, we obtain explicit representations of the jump process, and of related path functionals.

The talk is based on joint work with Jan Rosinski, The University of Tennessee.

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## Euler approximation of the stochastic Allen-Cahn equation

Mihály Kovács, University of Otago

We consider the stochastic Allen-Cahn equation perturbed by smooth additive Gaussian noise in a spatial domain with smooth boundary in dimension $d \leq 3$. We study the semidiscretization in time of the equation by an implicit Euler method. We show that the scheme converges pathwise with a rate of almost $1/2$ and that it also converges strongly but with no rate given.

## Hitting times of points for symmetric Lévy processes with completely monotone jumps

Mateusz Kwaśnicki, Wrocław University of Technology

Let $X_t$ be a symmetric Lévy process with completely monotone jumps (which means that the Lévy measure of $X_t$ has a completely monotone density function on $(0,\infty)$), and let $\tau_x$ be the first hitting time of $\{x\}$ for $X_t$. In [1], a semi-explicit expression for $\mathbf{P}(t < \tau_x < \infty)$ is proved, as a result of the development of the spectral theory of the process $X_t$ killed upon hitting 0. This expression is used in [2] to derive estimates and asymptotic expansions of $\mathbf{P}(t < \tau_x < \infty)$ and $\mathbf{P}(\tau_x \in dt)$.

More precisely, let $\Psi$ be the Lévy–Khintchine exponent of $X_t$ and $\Psi^{-1}$ the inverse function of the restriction of $\Psi$ to $(0,\infty)$. If

$$\inf_{\xi>0} \frac{\xi\Psi''(\xi)}{\Psi'(\xi)} > 0,$$

then there are positive constants $C_1$, $C_2$, $C_3$ such that

$$\frac{C_1}{t|x|\Psi(1/|x|)\Psi^{-1}(1/t)} \le \mathbf{P}(t < \tau_x < \infty) \le \frac{C_2}{t|x|\Psi(1/|x|)\Psi^{-1}(1/t)}$$

$$\frac{C_1}{t^2|x|\Psi(1/|x|)\Psi^{-1}(1/t)} \le \frac{d}{dt}\mathbf{P}(\tau_x \in dt) \le \frac{C_2}{t^2|x|\Psi(1/|x|)\Psi^{-1}(1/t)}$$

for all $t > 0$ and $x \in \mathbf{R} \setminus \{0\}$ such that $t\Psi(1/|x|) \ge C_3$. If in addition $\Psi$ is regularly varying at $\infty$ and at 0, then the leading terms of $\mathbf{P}(t < \tau_x < \infty)$ and $\mathbf{P}(\tau_x \in dt)$ are identified as $x \to 0$ and as $t \to \infty$.

I will present the above results in more detail and discuss possible extensions.

This is joint work with Tomasz Juszczyszyn.

**References:**

[1] Kwaśnicki, M. (2012), Spectral theory for one-dimensional symmetric Lévy processes killed upon hitting the origin. *Electron. J. Probab.* **17**, 83:1–29.

[2] Juszczyszyn, T. and Kwaśnicki, M., Hitting times of points for symmetric Lévy processes with completely monotone jumps. *In preparation.*

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## Limiting behaviour of finite and infinite dimensional stochastic systems driven by stable Lévy processes

Ilya Pavlyukevich, FSU Jena

We consider one dimensional Langevin equations with linear and non-linear friction as well as multi-(infinitely-) dimensional Langevin equations driven by stable Lévy processes. We study their asymptotic behaviour in the limit of large friction with the focus on the convergence in the Skorokhod $M_1$-topology.

This is joint work with M. Riedle (King's College London) and M. Gradinaru (U Rennes 1).

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The abstract-booklet can be downloaded here .

**Organizers:**
Anita Behme and René Schilling

**Location:**
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A detailed description how to find us is also given here.

Stand: 17.02.2021 14:47
Autor: René Schilling

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