

# Hans-Jürgen Hoehnke (1925 - 2007)

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Hans-Jürgen Hoehnke passed away on June 10, 2007. In Hans-Jürgen Hoehnke the international community of algebraists lost one of the leading researchers in the field of general algebra. As a student of Heinrich Brandt in Halle he began his career as a researcher during the years of a stormy development within algebra as well as of the discovery of several links with neighbouring disciplines. These links created generalizations of groups, rings and other “classical” algebraic structures. “Modern Algebra ” in the sense of van der Waerden’s book with the same title changed to “General Algebra”. Beginning with the thirties of the last century, the old branches of General Algebra - the theory of fields, of associative and associative - commutative rings to which van der Waerden’s book was mainly devoted - have undergone far-reaching changes. Within the framework of the classical parts of general algebra, independent trends arose: homological algebra leading to numerous results in topology and algebraic geometry, projective algebra, including the elements of projective geometry and differential algebra. The theory of semigroups, of quasigroups and of  $n$ -groups ceased to be simple theories of “generalized groups” and found their own paths of development and their own areas of application. Eventually, the general theory of universal algebra came into being, as well as model theory which is interwoven with mathematical logic and later with theoretical computer science. Category theory provides a wide overview on algebraic thinking. This part of the history of mathematics finds its reflection in H.-J. Hoehnke’s papers.

Born on October, 1925, in the Free City of Danzig (Gdansk) Hans-Jürgen Hoehnke was not only a witness of an era in mathematics, but also a contemporary witness of political developments in Europe. During this time Danzig was a so-called Free City under the protection of the League of Nations. However in 1933 the Nazi party gained control of the government of the city and in September 1939 German troops occupied it and incorporated it into the Reichsgau Danzig-West Prussia. Hans-Jürgen was fourteen years old at this time and as a student of the secondary school “Konradinum” he had become interested in constructing short-wave radios. This was a hobby which was viewed with suspicion by the authorities and Hoehnke chose another more harmless subject of interest: mathematics and physics. Since his parents wanted to offer him the “best possible” education they decided to send him to a half-military school, the “Ordensburg” Sonthofen. He hated the military training, became home sick and escaped several times, but the parents took him back. In 1943 he graduated from this school and had to attend the Germain Air Force school. Only after the end of the World War II he was able to continue his education.

In 1946 he began the study mathematics and physics at the Martin-Luther Universität Halle/ Wittenberg. After obtaining his diploma in mathematics he moved with his wife to the small town of Vacha to work here as a mathematician in a company. His first publication entitled “Die Konstanten der Wellenleitungen - eine

Ausdehnung der Abrahamschen Leitungstheorie" dates back to this period. But he came back to the city of Halle and began research under the algebraist Heinrich Brandt. He was awarded his doctorate by Halle university in 1952. His thesis is entitled "Über eine Transformationseigenschaft der Nonionen" and was published in 1978 in "Mathematische Nachrichten". His main interests during this period were constructive questions in the theories of rings and algebras. In his paper "Über die definierenden Gleichungen für Matrizeneinheiten in primären Ringen" (1956) he looks at multiplicative semigroups inside matrix rings over completely primary rings. Constructive problems were also attacked in papers such as "Lösung eines Problems von Ch. Hopkins" (1957), "Über komponierbare Formen und konkordante hyperkomplexe Größen" (1958) and "Konstruktive Methoden in der Theorie der Algebren" (1960). Other topics in H.-J. Hoehnke's early papers are annihilators and nilpotence. In "Nilpotenzkriterien" (1957) he looks for conditions on a ring which force certain radicals to be nilpotent. Twenty years later he came back to constructive methods. In a letter to M. A. Knus (Zurich) he wrote "Wie Sie wissen, interessiere ich mich sehr für konstruktive Fragen der Theorie der Algebren und beabsichtige, nach Abschluß meiner Arbeiten auf dem Grenzgebiet zwischen allgemeiner Algebra und Informatik in etwa einem Jahr, in dieser Richtung, die ich inzwischen nur informativ weiter verfolgt habe, wieder neu zu beginnen." A joint paper with K. W. Johnson from 1998 entitled " $K$ -characters and group invariants" shows that he was fascinated by constructive questions from which he began his research work. While K. W. Johnson later on made the step from group characters to quasigroup characters, H.-J. Hoehnke contributed to General Algebra with a series of nine papers entitled "Zur Theorie der Gruppoide" (see also [3]). Around this time, motivated by his investigations of groupoids, he embarked on an ambitious programme to examine the structure theory of semigroups. In [3] L. Marki explains this programme.

In 1956 Hoehnke left the university of Halle and continued his academic career at the Mathematical Institute of the Academy of Sciences in Berlin. He got his habilitation in 1965 from Humboldt University with a thesis entitled "Zur Theorie der Gruppoide I, II". Working on the structure theory of semigroups he defined various concepts of radicals in analogy to radicals of rings. He began also to develop an axiomatic theory of radicals, not only for semigroups but for arbitrary algebraic structures. His notion of a radical of an algebra (named Hoehnke radical) is the congruence which yields the greatest homomorphic image of the algebra in the semisimple class (classes closed under subdirect products) generated by this algebra.

In the next period of H.-J. Hoehnke's scientific work his main interest turned more and more towards non-classical parts of general algebra. Unfortunately, the fundamental ideas and the most important results of modern general algebra are not a part of the scientific equipment of every well-educated mathematician. A wide

circle of mathematicians at the Academy of Sciences in the German Democratic Republic had an acquaintance with the achievements of general algebra that remained rather on the level of the early thirties of the last century. Based on the political system they tried to eliminate all branches of mathematical research which were far from their own scientific interests. Hoehnke resisted vehemently. He was a person which tried to live uprightly without making compromises, answering only to his own conscience. They misunderstood him and considered him a rebel against socialism. Only people which are familiar with the conditions of the political system existing at that time can understand what this means and can appreciate the personal problems and disadvantages connected with his upright attitude. When he once came back from a research stay abroad he had to learn that he was not longer the head of the institute for pure mathematics. Despite being a highly qualified researcher, he never was given the title of a professor. Many of his actions and reactions have to be considered in the light of this background. It is not surprising that in this situation he was often unhappy. Once he said to me: “Only if you feel really unhappy, you will become creative”. In the contribution “Vorstellungen über die Entwicklung der abstrakten Algebra am ZIMM (Zentralinstitut für Mathematik und Mechanik der Akademie der Wissenschaften) he wrote: “Sogenannte Schwerpunktbildungen tragen nicht nur theoretisch sondern faktisch dazu bei, daß die abstrakte Algebra in der DDR von Jahr zu Jahr immer weiter eliminiert wird, und dieser steile Abstieg wird nirgends so deutlich, als beim Besuch ausländischer Tagungen. Eine solche Beeinflussung, die vergißt, daß jedes Mehr auf einem Teilgebiet der Mathematik durch ein Minder auf anderen Teilgebieten erkaufte werden muß und jeder künstliche Eingriff in wissenschaftliche Entwicklungen stets irreparabel ist, beruht darauf, daß eine Minderheit von Wissenschaftsstrategen für sich in Anspruch nimmt, alle Erkenntnisse in sich zu konzentrieren und stört die ausgewogene kontinuierliche Entwicklung der Mathematik in der DDR und damit alle Anstrengungen und Erwartungen, die man im Hinblick auf die große Vergangenheit der Mathematik in Deutschland mit Recht in uns setzt.” A second answer was provided by his deep contributions to the research in general algebra. In “Superposition partieller Funktionen” (1972) he began to consider the concept of a clone, nowadays one of the fundamental concepts of general algebra. In particular, he looked for an abstract characterization of clones of partial operations. This concept played an important role in the discussions during several conferences organized by him.

He initiated the series of conferences entitled “Arbeitstagungen Allgemeine Algebra und Grenzgebiete”. We appreciated these activities very much since these conferences offered one of the rare opportunities to come into contact with algebraists from “western” countries. Another series of conferences, the “Conferences for Young Algebraists” organized mostly in Potsdam, was also strongly influenced by his ideas and by his presence. On the occasion of the 4-th conference for young algebraists in Potsdam 1988 he posed the question of finding

a Cayley-type-characterization for partial clones. In his thesis ([1]) F. Börner gave an answer to this question on the level of universal algebra. Hoehnke's own answer used enriched cartesian-closed categories, the so-called *dht*-symmetric categories (Hoehnke-categories). In a series of papers entitled "On certain classes of categories and monoids constructed from abstract Mal'cev clones", I - V, he developed these ideas and in the book "Partial algebras and their theories" (2007, with J. Schrekenberger), he summarized all results in this area. Another motivation for the use of category theory as a language to express mathematical ideas came from theoretical computer science. In the monograph "Automaten und Funktoren" (with L. Budach) the authors generalized the concept of an algebraic theory in the sense of F. W. Lawvere from the one-based to the many-based case. They extended the approach of G. Hotz on the synthesis of switching circuits to arbitrary networks of automata in certain Kronecker categories and developed the concept of a Kronecker-double category as a generalization of double categories in the sense of Ehresmann. In the papers "Struktursätze der Algebra und Kompliziertheit logischer Schemata" I, II, he turned to complexity theory. While clones of varieties of total algebras, or dually Lawvere theories, are well-studied, there were many problems for quasivarieties. Here also Hoehnke's contributions brought "light into the darkness". Quasivarieties are also considered in "Fully-invariant closure systems and congruences on quasivarieties of algebras" where he generalized the concept of a fully invariant congruence. Very naturally he moved from universal algebra and theoretical computer science to mathematical logic. From the late 1960's on he was thinking about classes of algebras algebraizing a logical system. Under his guidance I contributed with my Ph. D. thesis "Algebraische Fragen nichtklassischer Aussagenkalküle" to this problem which was solved later by W. Blok and D. Pigozzi ([2]).

Dr. Hans-Jürgen Hoehnke had many scientific contacts, all over the world. He knew A. I. Mal'cev, A. Tarski, L. V. Skornjakov, S. McLane, B. Neumann and many other leading mathematicians personally. His students draw much profit from his far-reaching contacts.

As one of his students I am always very grateful for his ideas, his help and encouragement. He never forced me to follow his advices. There was much freedom to follow my own ideas. When I began my research work with him I was a teacher at the small town of Thale in the Harz mountains, far from universities and cities. But he visited us quite often and became a "honorary member" of my family. Sometimes he brought along guests, mathematicians from all over the world.

Dr. Hans-Jürgen Hoehnke was a member of the editorial board of several journals, for instance of "Mathematica Japonica (Scientiae Mathematicae Japonicae)" and "Semigroup Forum". He edited and co-edited series of books published by the Akademie-Verlag in Berlin and by Kluwer. He had a broad knowledge and many

interests in mathematics.

His interests apart from mathematics included music. He liked the music of Bach, Beethoven, Mahler and other composers. He himself played the transversal flute. He was interested in literature, philosophy and history. We discussed together the books of H. Hesse, J. Joyce and W. Faulkner, about I. Kant and the historian L. v. Ranke.

Dr. H.-J. Hoehnke was a talented mathematician and an interesting presence. There was always something to learn from him. We will miss his person. There remain his ideas, his papers and books. I am convinced that his work will influence the development of several parts of algebra also in the future.

## References

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