

INVERSE SYSTEM REALIZATION WITH OPERATIONAL AMPLIFIERS: STABILITY VS. NONIDEAL OP-AMP. CHARACTERISTICS

Ute Feldmann* and Martin Hasler**

*Faculty of Electrical Engineering, Technical University Dresden, Germany

**Department of Electrical Engineering, Swiss Federal Institute of Technology, Lausanne

Abstract

We present a criterion useful for inverse system design which establishes the correct functioning of op-amps. in terms of network structure.

1 Introduction

One of the promising methods to transmit information on a chaotic carrier signal is the inverse system approach [1]. The key idea is that the roles of input and output of the transmitter, the *original system*, are exchanged at the receiver, which is therefore called the *inverse system*.

The receiver has to recover the information signal. If the output of the receiver asymptotically copies the original input signal, we say that the receiver synchronizes. In order to do this the inverse system has to have unique asymptotic behaviour. These things are discussed in-depth in [2] as well as the crucial result that a synchronizing inverse can be achieved by appropriate choice of the output of the original system.

Usually, the circuit realization of this method uses both for the transmitter and the receiver a nonlinear dynamic one-port [1], the voltage and current of which are treated alternatively as input and output (Fig. 1a). We have shown [2] that the restriction to one-ports is not necessary. Indeed, the information signal can be injected into the circuit by, say, a voltage source and any other voltage or current, not just the source current, can serve as chaotic signal to be transmitted. This is to treat the circuit as two-port. The inversion of this input-output-situation can be achieved, using an operational amplifier. One of the four possible situations is shown in Fig. 1b. We replaced the op-amp. by a nullator-norator pair.

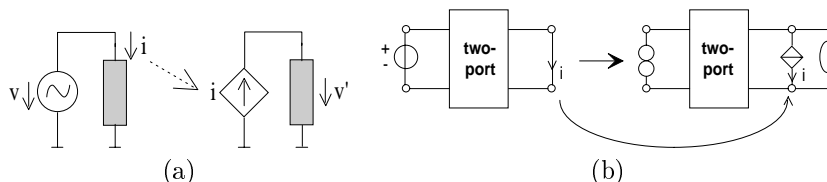


Figure 1: Inverse system realization: the $V \rightarrow I \rightarrow V$ method (a) with a 1-port (b) with a 2-port

In the sequel we take it for granted that the inverse system synchronizes with the original system in case of an ideal op-amp. But we still have to check that the non ideal characteristics of the operational amplifier do not destroy the correct functioning of the inverse system. (The question of the correct functioning of circuits containing op-amps. is a general one and not restricted to inverse system design. Although our results are applicable to all these circuits we will stick to inverse systems here) The purpose of this paper is to give conditions which ensure that the output of the inverse system deviates only slightly from the original information signal when the operational amplifier saturates and has a single pole frequency characteristic.

2 State Equations

If too large an input signal is applied to the inverse system, it will be driven into saturation and the retrieved signal will be heavily distorted. Therefore, we suppose that the dynamic range of the input signal is limited in such a way that the output of the operational amplifier, when its ideal model is taken, stays well below the limits imposed by the saturation effect of its non ideal model. This is necessary for correct functioning, but not sufficient. It could

still happen that the dynamic effects of the operational amplifier drive it into saturation. We use the following model of the nonideal op-amp.:

$$\tau v_{out} \dot{v}_{out} + v_{out} = f(v_{in}) \quad (1)$$

where $f(\cdot)$ indicates a saturation characteristic depicted in Fig. 2a, i.e. the op-amp. is now regarded as an open circuit at the input port and an *inertial* voltage controlled voltage source at the output port.

It is understood that the circuit structure can change crucially when a nullator-norator pair is replaced by a pair of an open circuit and an inertial vc. voltage source. Namely, the dimension of the state space (i.e. the number of independent capacitor charges and inductor fluxes) may change. Furthermore, the appearance of input derivatives in the system description (as is the case for inverse systems with reduced state space, cf. [2]) makes our slow-fast approach (section 3) difficult to apply. Therefore, we require for the application of our approach that all capacitors and inductors represent states in the ideal system as well as in the nonideal system.

The inverse system with ideal op-amp. is described by:

$$\begin{aligned} \dot{\mathbf{x}} &= h(v_{out}, \mathbf{x}, s) \\ v_{in} &= g(v_{out}, \mathbf{x}, s) = 0 \end{aligned} \quad (2)$$

whereas the system with model (1) of the op-amp. is described by:

$$\begin{aligned} \dot{\mathbf{x}} &= h(v_{out}, \mathbf{x}, s) \\ \tau v_{out} \dot{v}_{out} + v_{out} &= f(g(v_{out}, \mathbf{x}, s)) \end{aligned} \quad (3)$$

Here $\mathbf{x} \in \mathbb{R}^N$ is the state vector, $s(t)$ is the input signal and $v_{in} = g(v_{out}, \mathbf{x}, s)$ represents the *two port characteristics of the rest-network* cf. section (3).

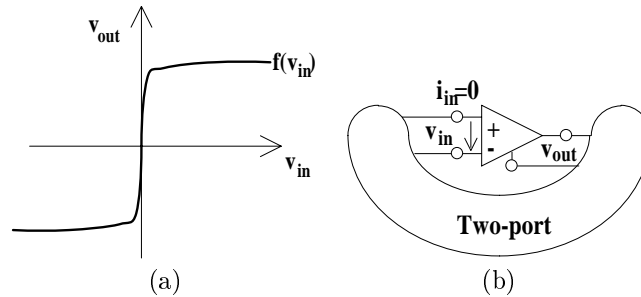


Figure 2: (a) Saturation characteristic of the op-amp. $f(v_{in})$ (b) Inverse system as a connection of the op-amp. two-port with the rest-network

3 Slow-Fast Approach

We have to show that the system (3) produces nearly the same signals as the ideal system (2) which moves on a hypersurface \mathcal{H}_{id} in $\mathbb{R}^{N+2} : v_{in} = g(v_{out}, \mathbf{x}, s) = 0$.

We take the following simplifying point of view: The dynamics of the ideal circuit are supposed to be much slower than the additional dynamics caused by the frequency roll-off of the operational amplifier. This is due to the very large slope A of $f(\cdot)$ at 0 at least if v_{in} is not very very close to zero or large. Therefore, we analyse the fast dynamics in the limit of constant \mathbf{x} and constant s . This is that we consider the inverse system as the connection of the operational amplifier, with its dynamic model, to the *static rest-network*, Fig. 2b.

$$\tau v_{out} \dot{v}_{out} = f(g(v_{out}, \mathbf{x}, s)) - v_{out} \quad (4)$$

The set of dc-operating points of Equ. (4) $\bar{v}_{out}(\mathbf{x}, s)$ for all \mathbf{x} and s represents a hypersurface \mathcal{H}_{cd} in \mathbb{R}^{N+2} too, which is described by:

$$f^{-1}(v_{out}) = g(v_{out}, \mathbf{x}, s) \approx \frac{v_{out}}{A} \quad (5)$$

Next we show that this hypersurface is close to the hypersurface \mathcal{H}_{id} the system with ideal op-amp. is moving on. Then we derive conditions to ensure that the hypersurface \mathcal{H}_{cd} is globally asymptotically stable under the dynamics (4).

The dc-operating point can for specific circuits be shown to be unique and to depend continuously on the the inverse of the slope A by applying criteria of [3]. Note, the dc-operating point of (4) is a solution of a resistive circuit which consists of a nonlinear vc. voltage source: $v_{out} = f(v_{in})$ and the static rest-network where capacitors and inductors are replaced by voltage resp. current sources. Due to the continuity of the solution in $\frac{1}{A}$ the hypersurface \mathcal{H}_{cd} , i.e. the solution of (5) is close to \mathcal{H}_{id} . This result can also be obtained by inspection of the intersection of the two-port characteristics under the constraint $i_{in} = 0$, Fig. 3b.

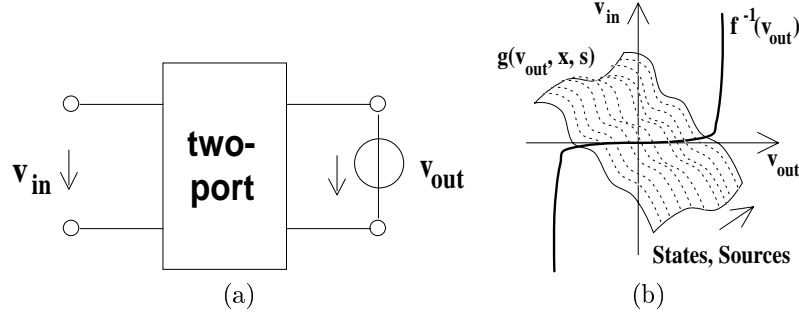


Figure 3: (a) Circuit to detect the static two-port characteristics under the constraint $i_{in} = 0$: $v_{in} = g(v_{out}, \mathbf{x}, s)$ (b) Inverse saturation characteristic of the op-amp. $f^{-1}(v_{in})$ and two-port characteristic of the rest-network $v_{in} = g(v_{out}, \mathbf{x}, s)$, the intersection of both determines the dc-operation point

From our slow-fast point of view, this dc-operating point varies slowly and the solution of the inverse system will track it, provided it is asymptotically stable. In order to show this one can use a Lyapunov function $w(v_{out}, \mathbf{x}, s) = \frac{(v_{out} - \bar{v}_{out}(\mathbf{x}, s))^2}{2}$.

$$\dot{w} = (v_{out} - \bar{v}_{out}(\mathbf{x}, s)) \cdot (f(g(v_{out}, \mathbf{x}, s)) - v_{out}) \leq 0 \quad (6)$$

If $g(v_{out}, \mathbf{x}, s)$ is a strictly decreasing function then, because $f(\cdot)$ is strictly increasing, $f(g(v_{out}, \mathbf{x}, s)) - v_{out}$ is strictly decreasing as well. Both multipliers of (6) are zero at $v_{out} = \bar{v}_{out}(\mathbf{x}, s)$. This and the fact that they are strictly increasing, resp. decreasing functions of v_{out} proves the validity of (6).

It remains to show that $g(v_{out}, \mathbf{x}, s)$ is a strictly decreasing function. In case $v_{in} = g(v_{out}, \mathbf{x}, s)$ is strictly increasing one can change the op-amp polarity to obtain correct functioning.

Proposition 1 Consider resistive networks which:

1. consist of resistors with strictly monotonic characteristics, independent sources and ideal op-amps
2. has for all source values exactly one solution.

If all admissible partial orientations, where exactly one source is oriented, lead to the same orientation of a branch voltage (current) then all resistive networks with this structure have a strictly monotonic dependence of the branch voltage (current) with respect to the source value.

An admissible orientation is an orientation of network branches which does not contradict Kirchhoffs Laws. The branch orientations represent voltage and current differences of the solutions with different source values. This proposition is derived from [3] where a proof is given.

In our slow-fast approach we are interested in the dependence $v_{in}(v_{out})$. Therefore, we have to treat the two-port as a resistive network according to Fig. 3a and to check the mentioned conditions. For this all slowly varying states are replaced by sources, namely, capacitors (inductors) as voltage (current) sources.

If now all possible partial orientations, where the only oriented source is v_{out} , lead to the same orientation of v_{in} then the circuit has the monotonicity property necessary to stabilize the slow manifold.

The result of this orientation check does not depend on specific source values as it corresponds to a circuit structure. Therefore all voltage (current) sources can be replaced by short (open) circuits as will be demonstrated for an example in the next section.

4 Example: Colpitts Oscillator

We designed a transmission system using the Colpitts oscillator. For this we manipulated its basis-emitter voltage with an information voltage signal. The original and inverse circuit are depicted in Fig. 4. Since the linear part of

the circuit is stable the inverse system synchronizes and recovers the information signal at least if an ideal op-amp. is used for the two-port inversion. This has been confirmed by simulation experiments.

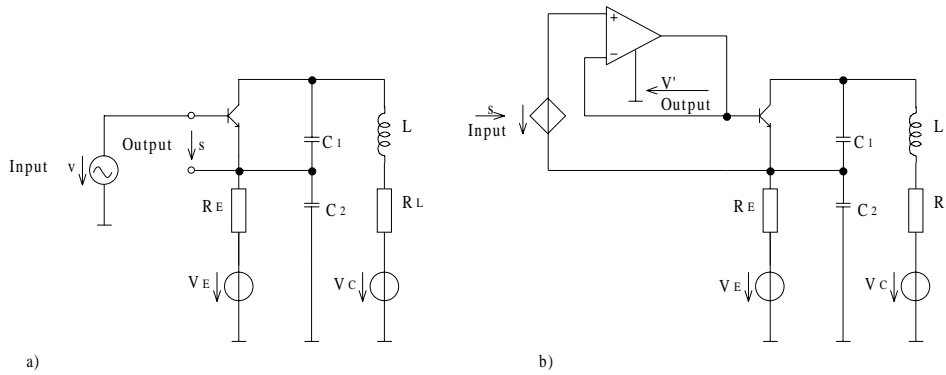


Figure 4: Transmission system using the Colpitts oscillator (a) Transmitter (b) Receiver

In Fig. 5a is the circuit structure shown, which corresponds to the inverse Colpitts system with an ideal op-amp. i.e. a nullator-norator pair. In Fig. 5b the only possible partial orientation starting with v_{out} is shown. This indicates that the inverse system has the necessary monotonicity property. And it is possible to choose the op-amp. polarity so that the slowly varying dc-point near $v_{in} = 0$ is stabilized. In this case the retrieved signal will be close to the desired information signal.

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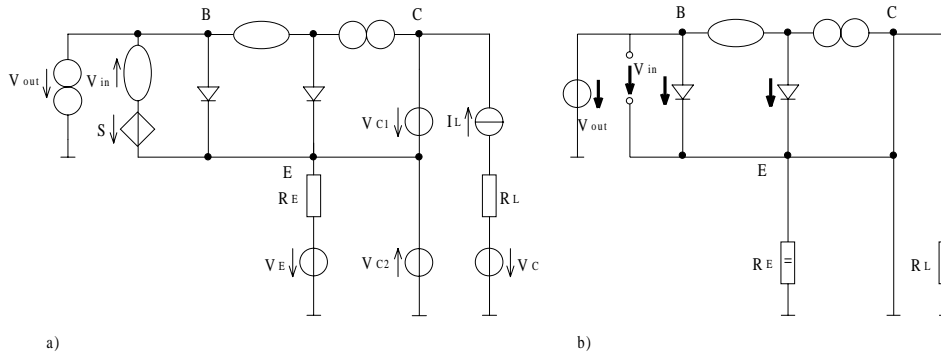


Figure 5: (a) Circuit structure corresponding to the inverse Colpitts system of Fig. 4b) the controlled current source of the transistor is modeled by an additional nonlinear resistor and a nullator-norator pair; (b) Network structure for consideration of monotonicity with the only possible voltage orientation when v_{out} is oriented, '=' indicates that the branch is not oriented

5 Conclusion

Under reasonable assumptions on the system and its inverse, the circuit theoretic property of monotonic dependence as discussed in [3] enables to assure the correct functioning of the inverse system and of op-amp. circuits in general when saturation and dynamic behaviour of the operational amplifier is taken into account. We have verified this property for a transmission system using the Colpitts oscillator.

References

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