Machine Learning
AdaBoost

Dmitrij Schlesinger

WS2013/2014, 31.01.2014

COMPUTER VISION LAB DRESDEN

TECHNISCHE UNIVERSITÄT DRESDEN

Idea

Compose a **"strong"** classifier from **"weak"** ones
Compare with SVM – *complex* feature spaces, *one* classifier.
Given:

- a set of weak classifiers $\mathcal{H}$. Example: linear classifiers for two classes $h \in \mathcal{H} : \mathcal{X} \to \{-1, +1\}$

$$h(x) = \operatorname{sign}\Big(\langle x, w\rangle + b\Big)$$

- labeled training data $\Big((x_1, k_1), (x_2, k_2) \ldots (x_m, k_m)\Big)$, $x_i \in \mathcal{X}$, $k_i \in \{-1, +1\}$

Find a strong classifier

$$f(x) = \operatorname{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

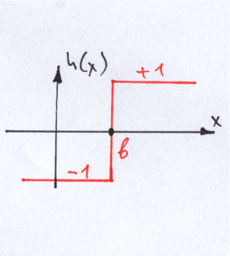
with $h_t \in \mathcal{H}$, $\alpha_t \in \mathbb{R}$ that separates the training data as good as possible.

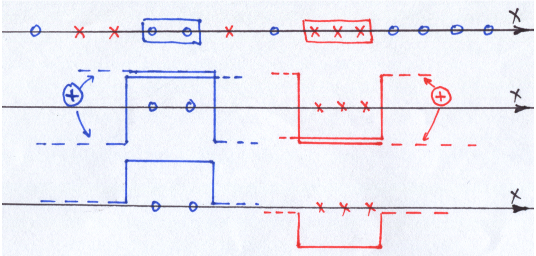
Power of the set of decision strategies

Is it possible to classify any training data without errors?
If not any, which ones?

Yes, it is (if the number of used classifiers is not restricted).

Example for $x \in \mathbb{R}$, $h(x) = \text{sign}(\langle x, w \rangle + b) = \pm\text{sign}(x - b)$

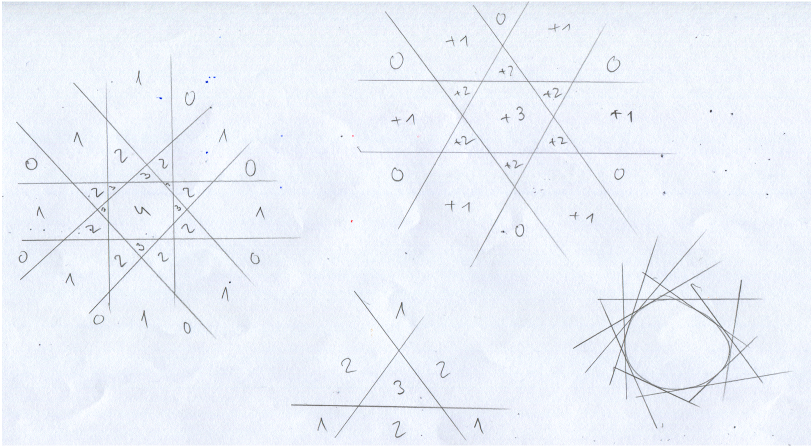




Key idea: it is possible to build an "elementary" classifier for each particular data point that is "neutral" for all others.

Power of the set of decision strategies

Examples in $x \in \mathbb{R}^2$



Algorithm

Given: $\Big((x_1, k_1), (x_2, k_2) \dots (x_m, k_m)\Big)$, $x_i \in \mathcal{X}$, $k_i \in \{-1, +1\}$

Initialize **Weights** for all samples as $D^{(1)}(i) = 1/m$

For $t = 1, \dots, T$

(1) Choose (learn) a weak classifier $h_t \in \mathcal{H}$ taking into account the actual weights $D^{(t)}$

(2) Choose α_t

(3) Update weights:

$$D^{(t+1)}(i) = \frac{D^{(t)}(i) \cdot \exp\Big(-\alpha_t y_i h_t(x_i)\Big)}{Z_t}$$

with a normalizing factor Z_t so that $\sum_i D^{(t+1)}(i) = 1$.

The final strong classifier is:

$$f(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

Algorithm (1)

Choose (learn) a weak classifier $h_t \in \mathcal{H}$.

$$h_t = \arg\min_{h \in \mathcal{H}} \epsilon(D^{(t)}, h) = \arg\min_{h \in \mathcal{H}} \sum_i D^{(t)}(i) \cdot \delta\Big(y_i, h(x_i)\Big)$$

i.e. choose the best one from a pre-defined faimy $\mathcal{H}$
– it can be SVM, Hinge Loss, whatever ...

Note: AdaBoost is a meta-algorithm, it can work practically with any classifier family $\mathcal{H}$

The specialty here – the data poins are weighted with $D(i)$

Pre-requirement: the error $\epsilon(D^{(t)}, h) < 1/2$
– the best h_t should be not worse than a random choice.

Otherwise – Stop.

Algorithm (2)

Choose α_t.

The goal is to build $f(x)$ so that its error

$$\epsilon(f) = \sum_i \delta\Big(y_i, f(x_i)\Big)$$

is minimal.

The upper bound for the overall error is $\epsilon(f) \leq \prod_{t=1}^{T} Z_t$.
$\rightarrow$ choose α_t (greedy) so that the actual Z_t is minimal

$$Z_t = \sum_i D^{(t)}(i) \cdot \exp\Big(-\alpha_t y_i h_t(x_i)\Big) \rightarrow \min_{\alpha_t}$$

The task is convex and differentiable $\rightarrow$ analytical solution

$$\alpha_t = 1/2 \ln\left(\frac{1-\epsilon(D^{(t)}, h_t)}{\epsilon(D^{(t)}, h_t)}\right)$$

Algorithm (3)

Update weights.

$$D^{(t+1)}(i) \sim D^{(t)}(i) \cdot \exp\Big(-\alpha_t y_i h_t(x_i)\Big)$$

Note: $\alpha_t > 0$

$$\begin{aligned} y_i h_t(x_i) > 0 \text{ (correct)} \quad &\Rightarrow \quad \exp\Big(-\alpha_t y_i h_t(x_i)\Big) < 1 \\ y_i h_t(x_i) < 0 \text{ (error)} \quad &\Rightarrow \quad \exp\Big(-\alpha_t y_i h_t(x_i)\Big) > 1 \end{aligned}$$

The samples that are actually missclassified are supported

$\Rightarrow$ the classifier h_{t+1} in the next round will attempt to classify properly just these.

Examples by Sochman, Tippetts, Freund
`http://cseweb.ucsd.edu/~yfreund/adaboost/`

Sumary

History:

1990 – Boost-by-majority algorithm (Freund)

1995 – AdaBoost (Freund & Schapire)

1997 – Generalized version of AdaBoost (Schapire & Singer) (**today**)

2001 – AdaBoost in Face Detection (Viola & Jones)

Some interesting properties:

- AB is a simple linear combination of (linear) classifiers
- AB converges to the logarithm of the likelihood ratio
- AB has good generalization capabilities (?)
- AB is a feature selector

See also: `http://www.boosting.org/`

Viola & Jones's Face Detection

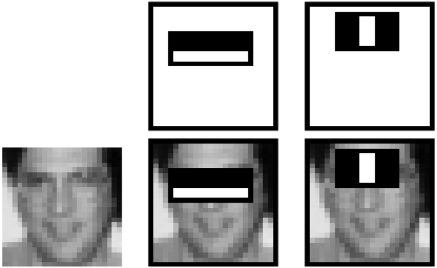
Rapid Object Detection using a Boosted Cascade of Simple Features (CVPR 2001)

Haar features can be computed very fast

24×24 window $\times \ldots \rightarrow 180.000$ feature values per position!!!

Weak classifier – convolution with Haar-kernel $\lessgtr$ threshold

AdaBoost for learning – feature selector

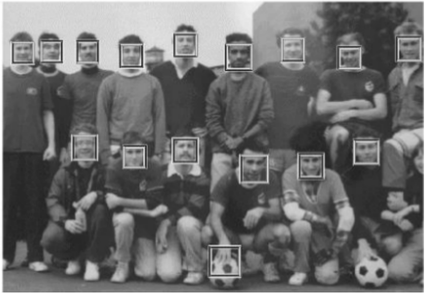


The two best features

Best features – 0.1 to 0.3 error, the other ones – 0.4 to 0.5

Viola & Jones's Face Detection

Database: 130 images, 507 faces



Overall error rate – about 7%