

$$(*) \quad \text{rot } E = F, \quad \text{div } E = g$$

$$\begin{array}{c} \uparrow \\ \text{rot } E = 0 \end{array}$$

$$E = E_F + E_g$$

$$\text{rot } E_F = F, \quad \text{div } E_F = 0$$

$$\text{rot } E_g = 0, \quad \text{div } E_g = g$$

$$① \quad \text{rot } E_F = F$$

?

$$② \quad \text{div } E_g = g$$

$$\mathcal{H}_g := \{ E : \text{rot } E = 0 \wedge \text{div } E = 0 \}$$

$$F \in \mathcal{R}(\text{rot}) \quad \wedge \quad g \in \mathcal{R}(\text{div})$$

$$E_F := \widetilde{\text{rot}}^{-1} F, \quad \widetilde{\text{rot}} \text{ u.i.j.}$$

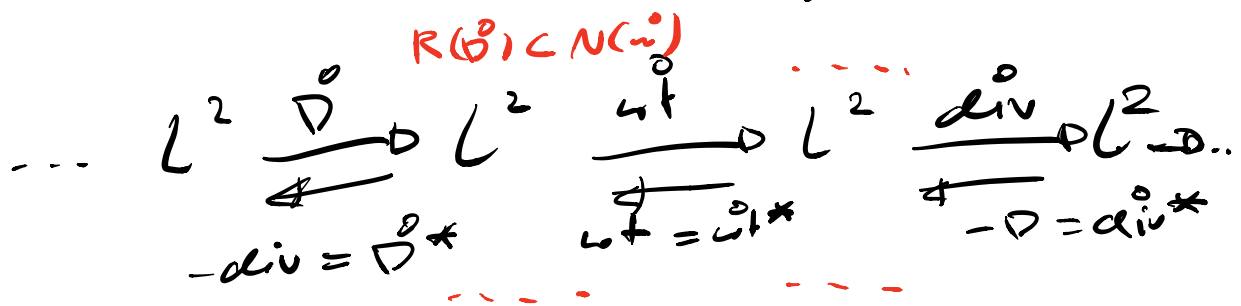
$$E_g := \widetilde{\text{div}}^{-1} g, \quad \widetilde{\text{div}} \text{ u.i.j.}$$

$$\underline{\text{Note}} \quad E_F \in \mathcal{N}(\text{rot})^\perp = \overline{\mathcal{R}(\text{rot}^*)} = \overline{\mathcal{R}(\text{rot})}$$

$$E_g \in \mathcal{N}(\text{div})^\perp = \overline{\mathcal{R}(\text{div}^*)} = \overline{\mathcal{R}(\mathcal{D})}$$

$R(\mathcal{O}), R(\text{div}), R(\mathcal{O}(1)), \dots$  closed

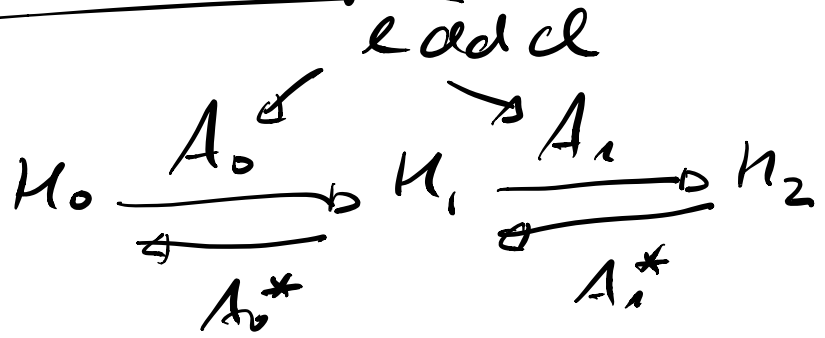
$E_F \in \overline{R(\mathcal{O}(1))} \subset N(\text{div})$  } complex  
 $E_G \in \overline{R(\mathcal{O})} \subset N(\mathcal{O}(1))$  } purity



de Rham complex

Wiesner complex

$$\boxed{R(A_0) \subset N(A_1)}$$



$$A_1 x = f, \quad \begin{matrix} \mathcal{O} \\ \mathcal{O}(1) \end{matrix}$$

$$A_0^* x = g, \quad \begin{matrix} \mathcal{O} \\ \text{div} \end{matrix}$$

$$\mathbb{D}(A_1) \cap \mathbb{D}(A_0^*) \hookrightarrow H_1$$

$$\begin{aligned} & \mathbb{D}(\text{curl}) \cap \mathbb{D}(\text{div}) \hookrightarrow L^2 \\ & = H_0(\text{curl}) \cap H(\text{div}) \end{aligned}$$