$(*)$ $\quad \mathring{\operatorname{rot}}\, E = F \quad , \quad \operatorname{div} E = g$

$$\cancel{n \times E\big|_{\partial\Omega} = 0}$$

$$E = E_F + E_g$$

$$\mathring{\operatorname{rot}}\, E_F = F \quad , \quad \operatorname{div} E_F = 0$$

$$\mathring{\operatorname{rot}}\, E_g = 0 \quad , \quad \operatorname{div} E_g = g$$

① $\mathring{\operatorname{rot}}\, E_F = F$

?

② $\operatorname{div} E_g = g$

$$\mathcal{H}_D := \{ E : \mathring{\operatorname{rot}}\, E = 0 \wedge \operatorname{div} E = 0 \}$$

$$F \in R(\mathring{\operatorname{rot}}) \quad \wedge \quad g \in R(\operatorname{div})$$

$$E_F := \widetilde{\mathring{\operatorname{rot}}}^{-1} F \quad , \quad \widetilde{\mathring{\operatorname{rot}}} \text{ inj}$$

$$E_g := \widetilde{\operatorname{div}}^{-1} g \quad , \quad \widetilde{\operatorname{div}} \text{ inj.}$$

<u>note</u> $\quad E_F \in N(\mathring{\operatorname{rot}})^{\perp} = \overline{R(\mathring{\operatorname{rot}}^*)} = \overline{R(\operatorname{rot})}$

$\qquad\quad E_g \in N(\operatorname{div})^{\perp} = \overline{R(\operatorname{div}^*)} = \overline{R(\mathring{\nabla})}$

$R(\overset{\circ}{\nabla})$, $R(\mathrm{div})$, $R(\overset{\circ}{\mathrm{curl}})$, ... closed

$$\left.\begin{aligned} E_F &\in \overline{R(\mathrm{curl})} \subset N(\mathrm{div}) \\ E_g &\in \overline{R(\overset{\circ}{\nabla})} \subset N(\overset{\circ}{\mathrm{curl}}) \end{aligned}\right\} \text{complex property}$$

$R(\overset{\circ}{\nabla}) \subset N(\overset{\circ}{\mathrm{curl}})$

$$\cdots \quad L^2 \underset{-\mathrm{div} = \overset{\circ}{\nabla}^*}{\overset{\overset{\circ}{\nabla}}{\rightleftarrows}} L^2 \underset{\mathrm{curl} = \overset{\circ}{\mathrm{curl}}^*}{\overset{\overset{\circ}{\mathrm{curl}}}{\rightleftarrows}} L^2 \underset{-\nabla = \overset{\circ}{\mathrm{div}}^*}{\overset{\overset{\circ}{\mathrm{div}}}{\rightleftarrows}} L^2 \rightarrow \cdots$$

de Rham complex

Hilbert complex

$$\boxed{R(A_0) \subset N(A_1)}$$

closed

$$H_0 \underset{A_0^*}{\overset{A_0}{\rightleftarrows}} H_1 \underset{A_1^*}{\overset{A_1}{\rightleftarrows}} H_2$$

$$\underset{\overset{\circ}{\mathrm{curl}}}{\overset{\text{"}}{A_1}} x = f, \qquad \underset{\mathrm{div}}{\overset{\text{"}}{A_0^*}} x = g$$

$$\boxed{D(A_1) \cap D(A_0^*) \hookrightarrow H_1}$$

$$D(\mathring{\mathrm{curl}}) \cap D(\mathrm{div}) \hookrightarrow L^2$$

$$= H_0(\mathrm{curl}) \cap H(\mathrm{div})$$