Poincaré meets Korn via Maxwell: Extending Korn's First Inequality to Incompatible Tensor Fields

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Tiny Motivation

gradient plasticity theory, micromorphic models, dislocation theory

new variational formulation by Patrizio Neff for the plastic variable p:
Find plastic tensor field $p : \Omega \subset \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ with vanishing row-wise tangential components on some part Γ_t of the boundary $\Gamma = \partial\Omega$ s.t. for all tensor field q (like p)

$$b(p,q) := \lambda \langle \operatorname{Curl} p, \operatorname{Curl} q \rangle_{L^2(\Omega)} + \langle \operatorname{sym} p, \operatorname{sym} q \rangle_{L^2(\Omega)}$$
$$= \langle f, q \rangle_{L^2(\Omega)}, \quad f = -\operatorname{sym} \nabla u, \quad \lambda > 0$$

$\Leftrightarrow$ p solves problem with mixed boundary conditions

$$\begin{aligned} \lambda \operatorname{Curl} \operatorname{Curl} p + \operatorname{sym} p &= f && \Omega \\ \tau p &= 0 && \Gamma_t && \text{(tangential Dirichlet bc)} \\ \tau \operatorname{Curl} p &= 0 && \Gamma_n && \text{(tangential Neumann bc)} \end{aligned}$$

Here: u, ∇u, $\operatorname{sym} \nabla u$ classical displacement, deformation, strain
open problems: well defined?, Hilbert space (Curl and tangential trace)?, coercive?, unique solution?
answer: new estimate $\Rightarrow$ unique solution by Lax-Milgram and ...

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- $\Omega \subset \mathbb{R}^N$ bounded domain with Lipschitz boundary $\Gamma := \partial\Omega$, first $N = 3$
- $\emptyset \neq \Gamma_t \subset \Gamma$ relatively open, separated from $\Gamma_n := \partial\Omega \setminus \overline{\Gamma_t}$ by Lipschitz curve
- Ω sliceable ('any domain is sliceable')
- semi-norm $|\!|\!| \cdot |\!|\!|$ for tensor fields $T \in \mathrm{H}(\mathrm{Curl};\Omega)$ ($\mathrm{H}(\mathrm{curl};\Omega)$ row-wise!)

$$|\!|\!| T |\!|\!|^2 := \|\mathrm{sym}\, T\|^2_{\mathrm{L}^2(\Omega)} + \|\mathrm{Curl}\, T\|^2_{\mathrm{L}^2(\Omega)}$$

Theorem

$$\exists\, c > 0 \quad \forall\, T \in \overset{\circ}{\mathrm{H}}(\mathrm{Curl};\Gamma_t,\Omega) \qquad \|T\|_{\mathrm{L}^2(\Omega)} \leq c\, |\!|\!| T |\!|\!|$$

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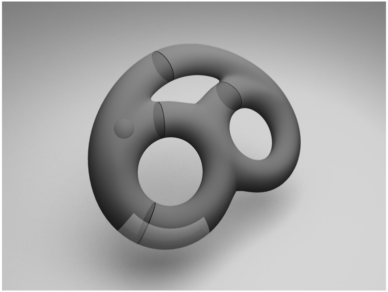
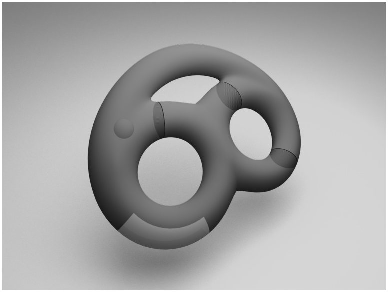
Sliceable Domains

Two ways to cut a sliceable domain:

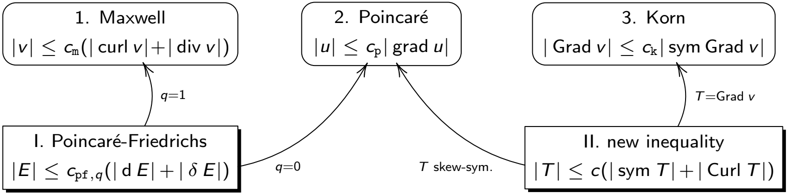




(Many Thanks to Kostas Pamfilos for the pictures.)

Interesting Consequences

The three fundamental inequalities are implied by two!



$$c_{\mathrm{p}} = c_{\mathrm{pf},0}, \quad c_{\mathrm{m}} = c_{\mathrm{pf},1}, \quad c_{\mathrm{k}}, c_{\mathrm{p}} \leq c$$

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δ	0	div	$-$ curl	grad
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$\iota^*_{\Gamma_t} E$	$E\vert_{\Gamma_t}$	$\nu \times E\vert_{\Gamma_t}$	$\nu \cdot E\vert_{\Gamma_t}$	0
$\circledast\, \iota^*_{\Gamma_n} * E$	0	$\nu \cdot E\vert_{\Gamma_n}$	$-\nu \times (\nu \times E)\vert_{\Gamma_n}$	$E\vert_{\Gamma_n}$

identification table for q-forms and vector proxies in $\mathbb{R}^3$

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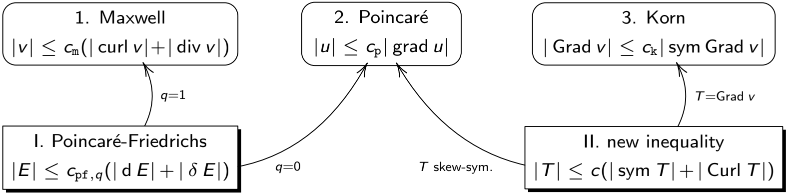

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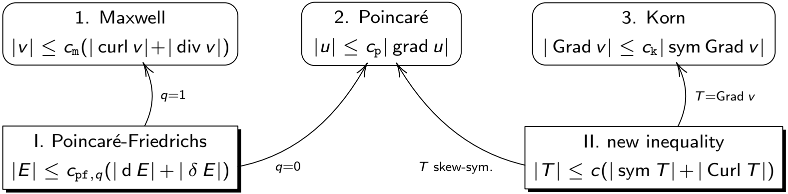

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Proof of Main Inequality: Tools

combination of techniques from

- electro-magnetic (static Maxwell equations with mixed boundary conditions)
- elastic theory

three crucial tools:

(HD) Helmholtz' decomposition for tensor fields, i.e.,

$$\mathsf{L}^2(\Omega) = \overset{\circ}{\mathsf{H}}(\mathrm{Curl}_0;\Gamma_t,\Omega) \oplus \mathrm{Curl}\,\overset{\circ}{\mathsf{H}}(\mathrm{Curl};\Gamma_n,\Omega)$$

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(SD) sliceable domains to get KI

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combination of techniques from

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$T \in \overset{\circ}{\mathrm{H}}(\mathrm{Curl}; \Gamma_t, \Omega)$

- HD $\Rightarrow$ $T = R + S \in \overset{\circ}{\mathrm{H}}(\mathrm{Curl}_0; \Gamma_t, \Omega) \oplus \mathrm{Curl}\, \overset{\circ}{\mathrm{H}}(\mathrm{Curl}; \Gamma_n, \Omega)$
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