$$\overset{\circ}{\mathrm{rot}}\, E = F, \qquad \operatorname{div} E = g$$

$$\cancel{n \times E\big|_{\partial\Omega} = 0}$$

$$E = E_F + E_g$$

$$\overset{\circ}{\mathrm{rot}}\, E_F = F, \qquad \operatorname{div} E_F = 0$$

$$\overset{\circ}{\mathrm{rot}}\, E_g = 0, \qquad \operatorname{div} E_g = g$$

(1) $\overset{\circ}{\mathrm{rot}}\, E_F = F$?

(2) $\operatorname{div} E_g = g$?

$$\mathcal{H}_D := \{ E : \overset{\circ}{\mathrm{rot}}\, E = 0 \wedge \operatorname{div} E = 0 \}$$

$$F \in R(\overset{\circ}{\mathrm{rot}}), \qquad g \in R(\operatorname{div})$$

$$E_F := \widetilde{\overset{\circ}{\mathrm{rot}}}^{-1} F, \qquad \widetilde{\overset{\circ}{\mathrm{rot}}} \text{ inj.}$$

$$E_g := \widetilde{\operatorname{div}}^{-1} g, \qquad \widetilde{\operatorname{div}} \text{ inj.}$$

$$E_F \in N(\mathring{\mathrm{curl}})^{\perp} = \overline{R(\mathring{\mathrm{curl}}^{*})} = \overline{R(\mathrm{curl})}$$

$$E_g \in N(\mathrm{div})^{\perp} = \overline{R(\mathrm{div}^{*})} = \overline{R(\mathring{\nabla})}$$

$$\left.\begin{array}{l} E_F \in \overline{R(\mathrm{curl})} \subset N(\mathrm{div}) \\ E_g \in \overline{R(\mathring{\nabla})} \subset N(\mathring{\mathrm{curl}}) \end{array}\right\} \text{complex prop.}$$

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de Rham complex

$$\cdots \to L^2 \underset{\xleftarrow[-\mathrm{div}]{}}{\xrightarrow{\mathring{\nabla}}} L^2 \underset{\xleftarrow[\mathrm{curl}]{}}{\xrightarrow{\mathring{\mathrm{curl}}}} L^2 \underset{\xleftarrow[-\nabla]{}}{\xrightarrow{\mathring{\mathrm{div}}}} L^2 \to \cdots$$

Hilbert complex

$$\cdots \underset{\leftarrow}{\to} H_0 \underset{\xleftarrow[A_0^{*}]{}}{\xrightarrow{A_0}} H_1 \underset{\xleftarrow[A_1^{*}]{}}{\xrightarrow{A_1}} H_2 \underset{\leftarrow}{\to} \cdots$$

$$A_1 x = f, \qquad A_0^{*} x = g$$

$$\boxed{D(A_1) \cap D(A_1^*) \hookrightarrow H_1}$$

$$D(\mathring{\mathrm{curl}}) \cap D(\mathrm{div}) \hookrightarrow L^2$$

$$= H_0(\mathrm{curl}) \cap H(\mathrm{div})$$