Poincaré meets Korn via Maxwell: Extending Korn's First Inequality to Incompatible Tensor Fields

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(joint work with **Patrizio Neff**, **Karl-Josef Witsch**, Universität Essen)

RG75: Optimization and PDEs with Applications
Dedicated to the 75th anniversary of **Roland Glowinski**

Jyväskylän Yliopisto, Suomi

June 18, 2012

Tiny Motivation: Energy Principle

NEW MODEL (gradient plasticity for finite deformations)
Patrizio Neff ('06) for (non-sym.!) plastic deformation (distortion) tensor (PDT) P:

Let u class. displacement, $G := \nabla u$ class. deformation, $\emptyset \neq \Gamma_t(\text{open}) \subset \Gamma := \partial\Omega$:

MINIMIZATION PROBLEM ❶ Find PDT-field $\hat{P} : \Omega \subset \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ with $\tau\hat{P} = \tau G$ on Γ_t minimizing

$$\min_P \tilde{\mathcal{E}}(P) = \tilde{\mathcal{E}}(\hat{P}),$$

where $\hat{P}, P \in \mathsf{H}(\mathrm{Curl};\Omega)$ and (energy functional)

$$\tilde{\mathcal{E}}(P) := \|\mathrm{sym}(P - G)\|^2_{\mathsf{L}^2(\Omega)} + \lambda\,\|\mathrm{Curl}\,P\|^2_{\mathsf{L}^2(\Omega)} + \kappa\,\|\mathrm{tr}\,P\|^2_{\mathsf{L}^2(\Omega)}\,, \quad \lambda > 0, \quad \kappa \geq 0.$$

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for all $T \in \overset{\circ}{\mathrm{H}}(\mathrm{Curl};\Gamma_t,\Omega)$.

OPEN PROBLEMS ('06-'11) well defined problem?,
right Hilbert space (tangential trace)?, b coercive?, unique solution?

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Main Results

- $\Omega \subset \mathbb{R}^N$ bounded domain with Lipschitz boundary $\Gamma := \partial\Omega$ (think of $N = 2, 3$)
- $\emptyset \neq \Gamma_t \subset \Gamma$ relatively open, separated from $\Gamma_n := \Gamma \setminus \overline{\Gamma_t}$ by Lipschitz curve
- Ω sliceable ('Any domain is sliceable!')
- semi-norm $|\!|\!| \cdot |\!|\!|$ for tensor fields $T \in \mathsf{H}(\mathrm{Curl};\Omega)$ ($\mathsf{H}(\mathrm{curl};\Omega)$ row-wise!)

$$|\!|\!| T |\!|\!|^2 := \|\mathrm{sym}\, T\|^2_{\mathsf{L}^2(\Omega)} + \|\mathrm{Curl}\, T\|^2_{\mathsf{L}^2(\Omega)}$$

Theorem

$$\exists\, c > 0 \quad \forall\, T \in \overset{\circ}{\mathsf{H}}(\mathrm{Curl};\Gamma_t,\Omega) \qquad \|T\|_{\mathsf{L}^2(\Omega)} \leq c\, |\!|\!| T |\!|\!|$$

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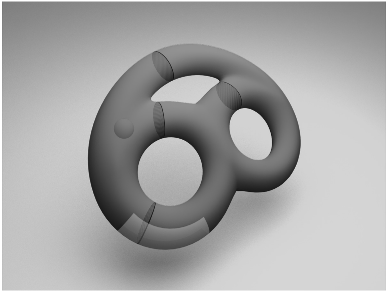
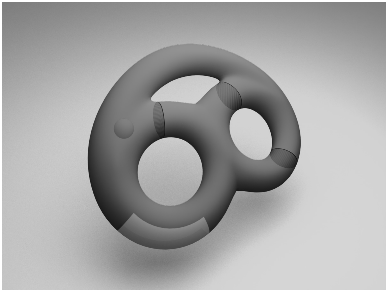
Sliceable Domains

Two ways to cut a sliceable domain

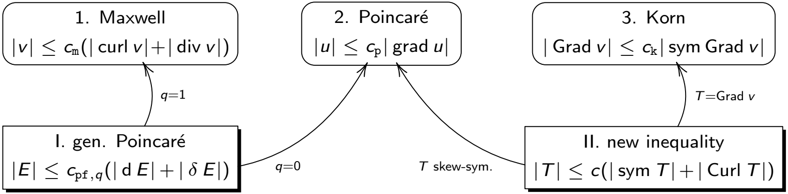




(Many Thanks to Kostas Pamfilos for the pictures)

Interesting Mathematical Consequences

The three fundamental inequalities are implied by two!



$$c_{\mathtt{p}} = c_{\mathtt{pf},0}, \quad c_{\mathtt{m}} = c_{\mathtt{pf},1}, \quad c_{\mathtt{k}}, c_{\mathtt{p}} \leq c$$

q	0	1	2	3
d	grad	curl	div	0
δ	0	div	$-$ curl	grad
$\overset{\circ}{\mathrm{D}}{}^q(\Gamma_t,\Omega)$	$\overset{\circ}{\mathrm{H}}(\mathrm{grad};\Gamma_t,\Omega)$	$\overset{\circ}{\mathrm{H}}(\mathrm{curl};\Gamma_t,\Omega)$	$\overset{\circ}{\mathrm{H}}(\mathrm{div};\Gamma_t,\Omega)$	$\mathrm{L}^2(\Omega)$
$\overset{\circ}{\Delta}{}^q(\Gamma_n,\Omega)$	$\mathrm{L}^2(\Omega)$	$\overset{\circ}{\mathrm{H}}(\mathrm{div};\Gamma_n,\Omega)$	$\overset{\circ}{\mathrm{H}}(\mathrm{curl};\Gamma_n,\Omega)$	$\overset{\circ}{\mathrm{H}}(\mathrm{grad};\Gamma_n,\Omega)$
$\iota^*_{\Gamma_t} E$	$E\|_{\Gamma_t}$	$\nu \times E\|_{\Gamma_t}$	$\nu \cdot E\|_{\Gamma_t}$	0
$\circledast \iota^*_{\Gamma_n} * E$	0	$\nu \cdot E\|_{\Gamma_n}$	$-\nu \times (\nu \times E)\|_{\Gamma_n}$	$E\|_{\Gamma_n}$

identification table for q-forms and vector proxies in $\mathbb{R}^3$

Interesting Mathematical Consequences

The three fundamental inequalities are implied by two!

1. Maxwell
$|v| \leq c_{\mathrm{m}}(|\operatorname{curl} v| + |\operatorname{div} v|)$

2. Poincaré
$|u| \leq c_{\mathrm{p}}|\operatorname{grad} u|$

3. Korn
$|\operatorname{Grad} v| \leq c_{\mathrm{k}}|\operatorname{sym}\operatorname{Grad} v|$

I. gen. Poincaré
$|E| \leq c_{\mathrm{pf},q}(|\mathrm{d}\, E| + |\delta\, E|)$

II. new inequality
$|T| \leq c(|\operatorname{sym} T| + |\operatorname{Curl} T|)$

I. → 1.: $q=1$
I. → 2.: $q=0$
II. → 2.: T skew-sym.
II. → 3.: $T=\operatorname{Grad} v$

$$c_{\mathrm{p}} = c_{\mathrm{pf},0}, \quad c_{\mathrm{m}} = c_{\mathrm{pf},1}, \quad c_{\mathrm{k}}, c_{\mathrm{p}} \leq c$$

q	0	1	2	3
d	grad	curl	div	0
δ	0	div	− curl	grad
$\overset{\circ}{\mathrm{D}}{}^q(\Gamma_t,\Omega)$	$\overset{\circ}{\mathrm{H}}(\operatorname{grad};\Gamma_t,\Omega)$	$\overset{\circ}{\mathrm{H}}(\operatorname{curl};\Gamma_t,\Omega)$	$\overset{\circ}{\mathrm{H}}(\operatorname{div};\Gamma_t,\Omega)$	$\mathrm{L}^2(\Omega)$
$\overset{\circ}{\Delta}{}^q(\Gamma_n,\Omega)$	$\mathrm{L}^2(\Omega)$	$\overset{\circ}{\mathrm{H}}(\operatorname{div};\Gamma_n,\Omega)$	$\overset{\circ}{\mathrm{H}}(\operatorname{curl};\Gamma_n,\Omega)$	$\overset{\circ}{\mathrm{H}}(\operatorname{grad};\Gamma_n,\Omega)$
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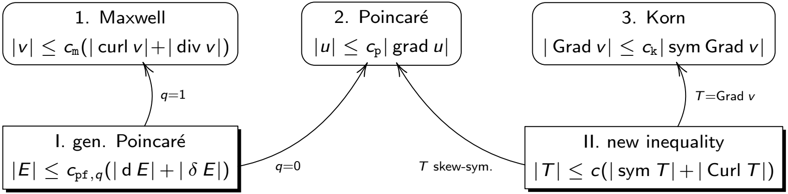

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$\mathring{\mathrm{D}}^q(\Gamma_t, \Omega)$	$\mathring{\mathrm{H}}(\operatorname{grad}; \Gamma_t, \Omega)$	$\mathring{\mathrm{H}}(\operatorname{curl}; \Gamma_t, \Omega)$	$\mathring{\mathrm{H}}(\operatorname{div}; \Gamma_t, \Omega)$	$\mathrm{L}^2(\Omega)$
$\mathring{\Delta}^q(\Gamma_n, \Omega)$	$\mathrm{L}^2(\Omega)$	$\mathring{\mathrm{H}}(\operatorname{div}; \Gamma_n, \Omega)$	$\mathring{\mathrm{H}}(\operatorname{curl}; \Gamma_n, \Omega)$	$\mathring{\mathrm{H}}(\operatorname{grad}; \Gamma_n, \Omega)$
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identification table for q-forms and vector proxies in $\mathbb{R}^3$

Proof of Main Inequality: Tools

combination of techniques from

- electro-magnetic theory (static Maxwell equations with mixed bc)
- linear elasticity theory

three crucial tools:

(HD) Helmholtz' decomposition for tensor fields, i.e.,

$$\mathsf{L}^2(\Omega) = \overset{\circ}{\mathsf{H}}(\mathrm{Curl}_0; \Gamma_t, \Omega) \oplus \mathrm{Curl}\,\overset{\circ}{\mathsf{H}}(\mathrm{Curl}; \Gamma_n, \Omega)$$

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$$\|T\|_{\mathsf{L}^2(\Omega)} \leq c_{\mathrm{m}} \big(\|\mathrm{Curl}\, T\|^2_{\mathsf{L}^2(\Omega)} + \|\mathrm{Div}\, T\|^2_{\mathsf{L}^2(\Omega)} \big)^{1/2}$$

$$\text{for all } T \in \overset{\circ}{\mathsf{H}}(\mathrm{Curl}; \Gamma_t, \Omega) \cap \overset{\circ}{\mathsf{H}}(\mathrm{Div}; \Gamma_n, \Omega) \cap (\mathcal{H}(\Omega)^3)^\perp$$

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(SD) sliceable domains to get KI

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Proof of Main Inequality (almost trivial)

$T \in \overset{\circ}{\mathsf{H}}(\mathrm{Curl};\Gamma_t,\Omega)$

- HD $\Rightarrow$ $T = R + S \in \overset{\circ}{\mathsf{H}}(\mathrm{Curl}_0;\Gamma_t,\Omega) \oplus \mathrm{Curl}\,\overset{\circ}{\mathsf{H}}(\mathrm{Curl};\Gamma_n,\Omega)$
- $\Rightarrow$ $\mathrm{Curl}\,S = \mathrm{Curl}\,T$ and

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- MI $\Rightarrow$ $\|S\|_{\mathsf{L}^2(\Omega)} \leq c_{\mathrm{m}} \|\mathrm{Curl}\,T\|_{\mathsf{L}^2(\Omega)}$ $(*)$
- KI and $(*)$ $\Rightarrow$

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$$\leq c_{\mathrm{k}}^2 \|\mathrm{sym}\,R\|^2_{\mathsf{L}^2(\Omega)} + \|S\|^2_{\mathsf{L}^2(\Omega)} \leq 2c_{\mathrm{k}}^2 \|\mathrm{sym}\,T\|^2_{\mathsf{L}^2(\Omega)} + (1 + 2c_{\mathrm{k}}^2)\|S\|^2_{\mathsf{L}^2(\Omega)}$$

- $\Rightarrow$ $\|T\|^2_{\mathsf{L}^2(\Omega)} \leq c^2 \,\|T\|^2$ $\square$

note: $c = \max\{\sqrt{2}c_{\mathrm{k}}, c_{\mathrm{m}}\sqrt{1 + 2c_{\mathrm{k}}^2}\}$

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Almost Last Slide!

FIRST PAPERS

[1, 2] $\Omega \subset \mathbb{R}^3$, $\Gamma_t = \Gamma$ connected
[3] $\Omega \subset \mathbb{R}^N$, $\Gamma_t = \Gamma$ connected (differential forms, curl := d ...)
[4] $\Omega \subset \mathbb{R}^3$, $\Gamma_t \subset \Gamma$ (this talk!)
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NUMERICS/COMPUTATIONS/SIMULATIONS first promising results in cooperation with Jan Valdman (Ostrava), new tensor-H(Curl)-FE with hom. restricted tan. traces

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FIRST PAPERS

[1, 2] $\Omega \subset \mathbb{R}^3$, $\Gamma_t = \Gamma$ connected
[3] $\Omega \subset \mathbb{R}^N$, $\Gamma_t = \Gamma$ connected (differential forms, curl := d ...)
[4] $\Omega \subset \mathbb{R}^3$, $\Gamma_t \subset \Gamma$ (this talk!)
[5] $\Omega \subset \mathbb{R}^N$, $\Gamma_t \subset \Gamma$ (differential forms)

ONGOING WORK exterior domains, non-homogeneous tangential traces, L^p, inhomogeneous media ... (already done, needs to be LaTexed ...)

NUMERICS/COMPUTATIONS/SIMULATIONS first promising results in cooperation with Jan Valdman (Ostrava), new tensor-H(Curl)-FE with hom. restricted tan. traces

References

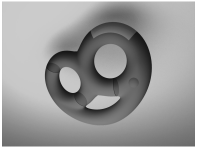
Last Slide!

P. Neff, D. Pauly, and K.-J. Witsch.
A canonical extension of Korn's first inequality to H(Curl) motivated by gradient plasticity with plastic spin.
(CRAS) C. R. Acad. Sci. Paris, Ser. I, 349:1251–1254, 2011.

P. Neff, D. Pauly, and K.-J. Witsch.
On a canonical extension of Korn's first and Poincaré's inequality to H(Curl).
(POMI) Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov., 397:115125, 2011.

P. Neff, D. Pauly, and K.-J. Witsch.
Maxwell meets Korn: A new coercive inequality for tensor fields in $\mathbb{R}^{N\times N}$ with square-integrable exterior derivative.
(M2AS) Math. Methods Appl. Sci., 35:65–71, 2012.

P. Neff, D. Pauly, and K.-J. Witsch.
Poincaré meets Korn via Maxwell: Extending Korn's first inequality to incompatible tensor fields.
(Crelle's Journal) J. Reine Angew. Math., submitted, 2012.



Thank You and **HAPPY BIRTHDAY**

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