

ISem 23, June 24, 2020

Dirichlet

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$$\operatorname{curl}_0 E = \bar{F}, \quad \operatorname{div} E = g \quad \Omega \subset \mathbb{R}^3$$
$$\uparrow \quad \uparrow$$
$$\cancel{\nu \times E|_{\partial\Omega} = 0} \quad \cancel{\nu \cdot E|_{\partial\Omega} = 0}$$

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$$E = E_{\bar{F}} + E_g$$

$$\operatorname{curl}_0 E_{\bar{F}} = \bar{F}, \quad \operatorname{div} E_{\bar{F}} = 0$$

$$\operatorname{curl}_0 E_g = 0, \quad \operatorname{div} E_g = g$$

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$$\textcircled{1} \quad \operatorname{curl}_0 E_{\bar{F}} = \bar{F}$$

$$\textcircled{2} \quad \operatorname{div} E_g = g$$

$$\mathcal{H}_D := \{ E : \operatorname{curl}_0 E = 0 \wedge \operatorname{div} E = 0 \}$$

$$F \in R(\text{curl}_0) \quad \wedge \quad g \in R(\text{div})$$

$$\begin{aligned} E_F &:= \widetilde{\text{curl}_0}^{-1} F & \text{curl}_0 & \text{inj.} \\ E_g &:= \widetilde{\text{div}}^{-1} g & \widetilde{\text{div}} & \text{inj.} \end{aligned}$$

$$\begin{aligned} \text{notes } E_F &\in N(\text{curl}_0) \stackrel{!}{=} \overline{R(\text{curl}_0^*)} = \overline{R(\text{curl})} \\ E_g &\in N(\text{div}) \stackrel{!}{=} \overline{R(\text{div}^*)} = \overline{R(\mathcal{D}_0)} \end{aligned}$$

$$\boxed{R(\text{curl}) \text{ closed, } R(\mathcal{D}_0) \text{ closed}}$$

$$\begin{aligned} E_F &\in \overline{R(\text{curl})} \subset N(\text{div}) \\ E_g &\in \overline{R(\mathcal{D}_0)} \subset N(\text{curl}_0) \end{aligned} \left. \begin{array}{l} \text{complex} \\ \text{property} \end{array} \right\}$$

de Rham complex

$$\begin{array}{ccccc} L^2 \mathcal{D}_0 & \xrightarrow{2,3} & L^{2,3} \text{curl}_0 & \xrightarrow{2,3} & L^2 \\ \uparrow & & \uparrow & & \uparrow \\ \mathcal{D}_0^* = -\text{div} & & \text{curl}^* = \text{curl} & & \text{div}_0^* = -\mathcal{D} \end{array}$$

Hilbert complex

$$\begin{array}{c}
 H_0 \xrightarrow{A_0(1 \text{ ddc})} H_1 \xrightarrow{A_1} H_2 \\
 \swarrow A_0^* \quad \searrow A_1^*
 \end{array}$$

A diagram of a Hilbert complex. It shows a sequence of Hilbert spaces  $H_0$ ,  $H_1$ , and  $H_2$  connected by operators  $A_0(1 \text{ ddc})$  and  $A_1$ . Below the spaces are adjoint operators  $A_0^*$  and  $A_1^*$ . A large oval encircles the operators  $A_0(1 \text{ ddc})$ ,  $A_1$ ,  $A_0^*$ , and  $A_1^*$ .

$$\boxed{A_1 x = f, \quad A_0^* x = g}$$

Co complex prop. :  $\boxed{A_1 A_0 = 0}$

$$\boxed{D(A_1) \cap D(A_0^*) \overset{\text{cpt}}{\hookrightarrow} H_1}$$

$$\begin{aligned}
 & D(\text{curl}) \cap D(\text{div}) \hookrightarrow L^{2,3} \\
 & = H_0(\text{curl}) \cap H(\text{div})
 \end{aligned}$$