Hodge-Helmholtz Decompositions of Weighted Sobolev Spaces

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Introduction: Helmholtz Decomposition

- $\Omega \subset \mathbb{R}^3$ exterior domain
- $\varepsilon : \Omega \to \mathbb{R}^{3\times3}$ medium property: bd., sym., unif. pos. def. matrix

$\Rightarrow$ Helmholtz decomposition

$$\mathrm{L}^2(\Omega) = \overline{\operatorname{grad} \overset{\circ}{\mathrm{H}}{}^1(\Omega)} \oplus_\varepsilon \mathcal{H}_{D,\varepsilon}(\Omega) \oplus_\varepsilon \varepsilon^{-1}\overline{\operatorname{curl} \mathrm{H}(\operatorname{curl}, \Omega)}$$

$2\times$ proj. theo. in Hilbert space, $R(A)^\perp = \ker(A^*)$, $H = \overline{R(A)} \oplus \ker(A^*)$,
Dirichlet fields:

$$\mathcal{H}_{D,\varepsilon}(\Omega) = \{E \in \mathrm{L}^2(\Omega) \mid \operatorname{curl} E = 0\,,\ \operatorname{div} \varepsilon E = 0\,,\ \nu \times E|_{\partial\Omega} = 0\}$$

$\partial\Omega$ Lipschitz (even weaker LMCP) $\Rightarrow$

$$\mathrm{L}^2(\Omega) = \operatorname{grad} \overset{\circ}{\mathrm{H}}{}^1_{-1}(\Omega) \oplus_\varepsilon \mathcal{H}_{D,\varepsilon}(\Omega) \oplus_\varepsilon \varepsilon^{-1} \operatorname{curl} \mathrm{H}_{-1}(\operatorname{curl}, \Omega)$$

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$$\operatorname{curl} \mathrm{H}_{-1}(\operatorname{curl}, \Omega) = \operatorname{curl} \{E \in \mathrm{L}^2_{-1}(\Omega) \mid \operatorname{curl} E \in \mathrm{L}^2(\Omega)\,,\ \operatorname{div} E = 0\,,\ \nu \cdot E|_{\partial\Omega} = 0\}$$

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- $\Omega \subset \mathbb{R}^N$ exterior domain, $q \in \{0, \ldots, N\}$
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question: Hodge-Helmholtz decomposition

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d: exterior derivative, $\delta = \pm * \mathrm{d} *$: co-derivative, formally skew adjoint

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- P. ('08)

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methods not applicable for non smooth boundaries and $\varepsilon \neq \mathrm{id}$

- P. ('08)

Lipschitz $\Rightarrow$ LMCP: $\overset{\circ}{\mathrm{D}}{}^q_s(\Omega) \cap \Delta^q_s(\Omega) \hookrightarrow \mathrm{L}^{2,q}_t(\Omega)$ compact $\forall\, s > t$

Decomposition Results (P. '08)

- $\Omega \subset \mathbb{R}^N$ (LMCP/Lipschitz exterior domain)
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Decomposition Results (P. '08)

- $\Omega \subset \mathbb{R}^N$ (LMCP/Lipschitz exterior domain)
- weight $s \in \mathbb{R} \setminus \mathbb{I}$ ($\mathbb{I}$ discrete set)
- $\varepsilon = \mathrm{id} + \hat{\varepsilon} \in \mathrm{L}^\infty(\Omega)$, i.e., L^∞-perturbation of id
- near infinity $\hat{\varepsilon} \in \mathrm{C}^1$
- decaying $\hat{\varepsilon} = \mathcal{O}(r^{-\tau})$, $\partial_n \hat{\varepsilon} = \mathcal{O}(r^{-\tau-1})$ with $\tau > 0$

then: Hodge-Helmholtz 'decompositions'

$$\mathrm{L}^{2,q}_s(\Omega) = \begin{cases} \mathrm{d}\,\overset{\circ}{\mathrm{D}}{}^{q-1}_{s-1}(\Omega) + \varepsilon^{-1}\,\delta\,\Delta^{q+1}_{s-1}(\Omega) & , s \in (-\infty, -N/2) \\ \mathrm{d}\,\overset{\circ}{\mathrm{D}}{}^{q-1}_{s-1}(\Omega) \dot{+} \varepsilon^{-1}\,\delta\,\Delta^{q+1}_{s-1}(\Omega) \dot{+} \overset{\circ}{\mathrm{B}}{}^q(\Omega) & , s \in (-N/2, N/2) \\ \mathrm{d}\,\overset{\circ}{\mathrm{D}}{}^{q-1}_{s-1}(\Omega) \dot{+} \varepsilon^{-1}\,\delta\,\Delta^{q+1}_{s-1}(\Omega) \dot{+} \overset{\circ}{\mathrm{B}}{}^q(\Omega) \dot{+} \Delta_\varepsilon \eta \mathcal{P}^q_{s-2} & , s \in (N/2, \infty) \end{cases}$$

here:

- $\Delta_\varepsilon = \mathrm{d}\,\delta + \varepsilon^{-1}\,\delta\,\mathrm{d} = \Delta + \check{\varepsilon}\,\delta\,\mathrm{d}$
- $\mathcal{P}^q_{s-2} \ni P_\ell = r^{-\ell} P$, where $\ell \in \mathbb{N}$ and P hom. harm. polyn. q-form in $\mathbb{R}^N$ $\mathrm{d}\,P = 0$, $\delta\,P = 0$ and $\Delta P_\ell = 0$
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Decomposition and Representation Results for Large Weights (P. '08)

remember: for $s > N/2$

$$\mathrm{L}^{2,q}_s(\Omega) = \mathrm{d}\,\overset{\circ}{\mathrm{D}}{}^{q-1}_{s-1}(\Omega) \dotplus \varepsilon^{-1}\,\delta\,\Delta^{q+1}_{s-1}(\Omega) \dotplus \overset{\circ}{\mathrm{B}}{}^q(\Omega) \dotplus \Delta_\varepsilon \eta \mathcal{P}^q_{s-2}$$

for $s > N/2 + 1$

- no repres. by co-closed (δ-free) or co-exact resp. closed (d-free) or exact forms from $\mathrm{L}^{2,q}_{s-1}(\Omega)$
- either representation in $\mathrm{L}^{2,q}_{s-1}(\Omega)$
- or co-closed resp. closed representation in $\mathrm{L}^{2,q}_{<N/2}(\Omega)$

$$\mathrm{d}\,\overset{\circ}{\mathrm{D}}{}^{q-1}_{s-1}(\Omega) = \mathrm{d}\left(\left(\overset{\circ}{\mathrm{D}}{}^{q-1}_{s-1}(\Omega) \boxplus \eta\mathcal{H}^{q-1}_{s-1}\right) \cap {}_0\Delta^{q-1}_{<N/2}(\Omega)\right)$$

$$\delta\,\Delta^{q+1}_{s-1}(\Omega) = \delta\left(\left(\Delta^{q+1}_{s-1}(\Omega) \boxplus \eta\mathcal{H}^{q+1}_{s-1}\right) \cap {}_0\overset{\circ}{\mathrm{D}}{}^{q+1}_{<N/2}(\Omega)\right)$$

here:

- $\mathcal{H}^{q\pm1}_{s-1} \ni H_\ell = r^{-\ell}H$, where $\ell \in \mathbb{N}$ and H hom. polyn. $(q\pm1)$-form in $\mathbb{R}^N$
  $\mathrm{d}\,H_\ell = 0$, $\delta\,H_\ell = 0$ and $\Delta H = 0$
- $\eta\mathcal{H}^q_{s-2}$ finite dim. subspace of $\mathrm{L}^{q\pm1}_{<N/2}(\Omega)$ of smooth forms

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for $s \geq 0$

$$\mathrm{L}^{2,q}_{s}(\Omega) \cap \mathcal{H}^{q}_{\varepsilon,-s}(\Omega)^{\perp_{\varepsilon}} = \mathrm{d}\,\overset{\circ}{\mathrm{D}}{}^{q-1}_{s-1}(\Omega) \oplus_{\varepsilon} \varepsilon^{-1}\,\delta\,\Delta^{q+1}_{s-1}(\Omega)$$

for $-N/2 \leq s < N/2 - 1$

$$\mathcal{H}^{q}_{\varepsilon,s}(\Omega) = \mathcal{H}^{q}_{\varepsilon,0}(\Omega) = \mathcal{H}^{q}_{\varepsilon}(\Omega)$$

for $s < -N/2$

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$\circledast$ 'contains' finitely many growing hom. harm. polyn.

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$$\mathcal{H}_{\varepsilon,s}^q(\Omega) = \mathcal{H}_{\varepsilon,0}^q(\Omega) = \mathcal{H}_\varepsilon^q(\Omega)$$

for $s < -N/2$

$$\mathcal{H}_{\varepsilon,s}^q(\Omega) = \underbrace{\mathcal{H}_{\varepsilon,s}^q(\Omega) \cap \mathring{\mathrm{B}}^q(\Omega)^{\perp_\varepsilon}}_{=:\circledast} \dotplus \mathcal{H}_\varepsilon^q(\Omega)$$

$\circledast$ 'contains' finitely many growing hom. harm. polyn.

Decomposition Results for Large Weights (P. '08)

remember: for $s > N/2$

$$\mathrm{L}^{2,q}_{s}(\Omega) = \mathrm{d}\,\mathring{\mathrm{D}}^{q-1}_{s-1}(\Omega) \dotplus \varepsilon^{-1}\,\delta\,\Delta^{q+1}_{s-1}(\Omega) \dotplus \mathring{\mathrm{B}}^{q}(\Omega) \dotplus \Delta_{\varepsilon}\eta\mathcal{P}^{q}_{s-2}$$

for $s \geq 0$

$$\mathrm{L}^{2,q}_{s}(\Omega) \cap \mathcal{H}^{q}_{\varepsilon,-s}(\Omega)^{\perp_{\varepsilon}} = \mathrm{d}\,\mathring{\mathrm{D}}^{q-1}_{s-1}(\Omega) \oplus_{\varepsilon} \varepsilon^{-1}\,\delta\,\Delta^{q+1}_{s-1}(\Omega)$$

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- pick $F \in \mathrm{L}^{2,q}_s(\Omega)$
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- non smooth boundaries
- inhomogeneities $\varepsilon \neq \mathrm{id}$

basic tools:

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applications:

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