The div-curl-Lemma by the FA-Toolbox

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Open-Minded ;-)

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Bergische Universität Wuppertal

classical div-curl-lemma

Let $\Omega \subset \mathbb{R}^3$ be open.

Lemma (classical div-curl-lemma)

Assumptions:

(i) $(E_n), (H_n)$ *bounded in* $\mathrm{L}^2(\Omega)$

(ii) $(\operatorname{rot} E_n)$ *bounded in* $\mathrm{L}^2(\Omega)$

(ii') $(\operatorname{div} H_n)$ *bounded in* $\mathrm{L}^2(\Omega)$

$\Rightarrow \exists$ E, H *and subsequences s.t.* $E_n \rightharpoonup E$, $\operatorname{rot} E_n \rightharpoonup \operatorname{rot} E$ *and* $H_n \rightharpoonup H$, $\operatorname{div} H_n \rightharpoonup \operatorname{div} H$ *and*

$$\forall\, \varphi \in \mathring{\mathrm{C}}^\infty(\Omega) \qquad \int_\Omega \varphi(E_n \cdot H_n) \to \int_\Omega \varphi(E \cdot H)$$

classical div-curl-lemma is local!

div-curl-lemma

Let $\Omega \subset \mathbb{R}^3$ be a bounded weak Lipschitz domain with boundary Γ and weak Lipschitz boundary parts Γ_t and $\Gamma_n = \Gamma \setminus \overline{\Gamma_t}$.

Lemma (div-curl-lemma (global version))

Assumptions:

- **(i)** $(E_n), (H_n)$ *bounded in* $\mathsf{L}^2(\Omega)$
- **(ii)** $(\operatorname{rot} E_n)$ *bounded in* $\mathsf{L}^2(\Omega)$
- **(ii')** $(\operatorname{div} H_n)$ *bounded in* $\mathsf{L}^2(\Omega)$
- **(iii)** $\nu \times E_n = 0$ *on* Γ_t
- **(iii')** $\nu \cdot H_n = 0$ *on* Γ_n

$\Rightarrow\ \exists\ E,\ H$ *and subsequences s.t.* $E_n \rightharpoonup E$, $\operatorname{rot} E_n \rightharpoonup \operatorname{rot} E$ *and* $H_n \rightharpoonup H$, $\operatorname{div} H_n \rightharpoonup \operatorname{div} H$ *and*

$$\int_\Omega E_n \cdot H_n \to \int_\Omega E \cdot H$$

Proof.

- generalize and fa-toolbox
- crucial points: complex property and compact embedding □

literature

original papers (local div-curl-lemma):

- Murat, F.: *Compacité par compensation*,
  Annali della Scuola Normale Superiore di Pisa-Classe di Scienze, 1978
- Tartar, L.: *Compensated compactness and applications to partial differential equations*,
  Nonlinear analysis and mechanics, Heriot-Watt symposium, 1979

recent papers (global div-curl-lemma):

- Gloria, A., Neukamm, S., Otto, F.: *Quantification of ergodicity in stochastic homogenization: optimal bounds via spectral gap on Glauber dynamics*,
  (IM) Invent. Math., 2015
- Kozono, H., Yanagisawa, T.: *Global compensated compactness theorem for general differential operators of first order*,
  (ARMA) Arch. Ration. Mech. Anal., 2013
- Schweizer, B.: *On Friedrichs inequality, Helmholtz decomposition, vector potentials, and the div-curl lemma*,
  accepted preprint, 2018
- Waurick, M.: *A Functional Analytic Perspective to the div-curl Lemma*,
  (JOP) J. Operator Theory, 2018

fa-toolbox for linear problems/systems

idea: solve problem with general and simple linear functional analysis ($\Rightarrow$ fa-toolbox) ...

literature: probably very well known for ages, but hard to find ...

Friedrichs, Weyl, Hörmander, Fredholm, von Neumann, Riesz, Banach, ... ?

Why not rediscover?

A_0^*-A_1-lemma (generalized global div-curl-lemma)

Let $A_0 : D(A_0) \subset H_0 \to H_1$, $A_1 : D(A_1) \subset H_1 \to H_2$ (possibly and generally unbounded) be two densely defined and closed linear operators on three Hilbert spaces H_0, H_1, H_2 with Hilbert space adjoints $A_0^* : D(A_0^*) \subset H_1 \to H_0$, $A_1^* : D(A_1^*) \subset H_2 \to H_1$. Moreover, let $A_1 A_0 = 0$, i.e. $R(A_0) \subset N(A_1)$. (complex property)

Lemma (A_0^*-A_1-lemma)

Let $D(A_1) \cap D(A_0^*) \hookrightarrow H_1$ be compact, and

(i) *(x_n) bounded in $D(A_1)$,*

(ii) *(y_n) bounded in $D(A_0^*)$.*

$\Rightarrow \exists\ x \in D(A_1)$, $y \in D(A_0^*)$ and subsequences s.t. $x_n \rightharpoonup x$ in $D(A_1)$ and $y_n \rightharpoonup y$ in $D(A_0^*)$ as well as

$$\langle x_n, y_n \rangle_{H_1} \to \langle x, y \rangle_{H_1}.$$

Proof.

... blackboard ... □

A_0^*-A_1-lemma (generalized global div-curl-lemma)

compact embedding

$$D(A_1) \cap D(A_0^*) \hookrightarrow H_1$$

in global div-curl-lemma reads:

$$\{E \in L^2(\Omega) : \operatorname{rot} E \in L^2(\Omega),\ \operatorname{div} E \in L^2(\Omega),\ \nu \times E = 0 \text{ on } \Gamma_t,\ \nu \cdot E = 0 \text{ on } \Gamma_n\} \hookrightarrow L^2(\Omega)$$

is compact

(Weck's selection theorem, '74,
also Bauer, Costabel, Kuhn, Jochmann, Osterbrink, P, Picard, Schomburg, Weber, Witsch)

classical de Rham complex in 3D (∇-rot-div-complex)

$\Omega \subset \mathbb{R}^3$ bounded weak Lipschitz domain, $\partial\Omega = \Gamma = \overline{\Gamma_t \dot{\cup} \Gamma_n}$

(electro-magneto dynamics, Maxwell's equations)

$$\{0\} \;\underset{\pi_{\{0\}}}{\overset{\iota_{\{0\}}}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{-\operatorname{div}}{\overset{\mathring{\nabla}}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{\operatorname{rot}}{\overset{\mathring{\operatorname{rot}}}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{-\nabla}{\overset{\mathring{\operatorname{div}}}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{\iota_{\mathbb{R}}}{\overset{\pi_{\mathbb{R}}}{\rightleftarrows}}\; \mathbb{R}$$

mixed boundary conditions and inhomogeneous and anisotropic media

$$\{0\} \text{ or } \mathbb{R} \;\underset{\pi}{\overset{\iota}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{-\operatorname{div}_{\Gamma_n}\varepsilon}{\overset{\nabla_{\Gamma_t}}{\rightleftarrows}}\; \mathsf{L}^2_{\varepsilon} \;\underset{\varepsilon^{-1}\operatorname{rot}_{\Gamma_n}}{\overset{\operatorname{rot}_{\Gamma_t}}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{-\nabla_{\Gamma_n}}{\overset{\operatorname{div}_{\Gamma_t}}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{\iota}{\overset{\pi}{\rightleftarrows}}\; \mathbb{R} \text{ or } \{0\}$$

classical de Rham complex in 3D (∇-rot-div-complex)

$\Omega \subset \mathbb{R}^3$ bounded weak Lipschitz domain, $\partial\Omega = \Gamma = \overline{\Gamma_t \dot\cup \Gamma_n}$

(electro-magneto dynamics, Maxwell's equations with mixed boundary conditions)

$$\{0\} \text{ or } \mathbb{R} \underset{\pi}{\overset{\iota}{\rightleftarrows}} \mathsf{L}^2 \underset{-\operatorname{div}_{\Gamma_n}\varepsilon}{\overset{\nabla_{\Gamma_t}}{\rightleftarrows}} \mathsf{L}^2_\varepsilon \underset{\varepsilon^{-1}\operatorname{rot}_{\Gamma_n}}{\overset{\operatorname{rot}_{\Gamma_t}}{\rightleftarrows}} \mathsf{L}^2 \underset{-\nabla_{\Gamma_n}}{\overset{\operatorname{div}_{\Gamma_t}}{\rightleftarrows}} \mathsf{L}^2 \underset{\iota}{\overset{\pi}{\rightleftarrows}} \mathbb{R} \text{ or } \{0\}$$

related fos

$$\begin{array}{llllllllllll} \nabla_{\Gamma_t} u = A & \text{in } \Omega & | & \operatorname{rot}_{\Gamma_t} E = J & \text{in } \Omega & | & \operatorname{div}_{\Gamma_t} H = k & \text{in } \Omega & | & \pi v = b & \text{in } \Omega \\ \pi u = a & \text{in } \Omega & | & -\operatorname{div}_{\Gamma_n} \varepsilon E = j & \text{in } \Omega & | & \varepsilon^{-1} \operatorname{rot}_{\Gamma_n} H = K & \text{in } \Omega & | & -\nabla_{\Gamma_n} v = B & \text{in } \Omega \end{array}$$

related sos

$$\begin{array}{lllllllll} -\operatorname{div}_{\Gamma_n} \varepsilon \nabla_{\Gamma_t} u = j & \text{in } \Omega & | & \varepsilon^{-1} \operatorname{rot}_{\Gamma_n} \operatorname{rot}_{\Gamma_t} E = K & \text{in } \Omega & | & -\nabla_{\Gamma_n} \operatorname{div}_{\Gamma_t} H = B & \text{in } \Omega \\ \pi u = a & \text{in } \Omega & | & -\operatorname{div}_{\Gamma_n} \varepsilon E = j & \text{in } \Omega & | & \varepsilon^{-1} \operatorname{rot}_{\Gamma_n} H = K & \text{in } \Omega \end{array}$$

corresponding compact embeddings:

$$D(\nabla_{\Gamma_t}) \cap D(\pi) = D(\nabla_{\Gamma_t}) = \mathsf{H}^1_{\Gamma_t} \hookrightarrow \mathsf{L}^2 \qquad \text{(Rellich's selection theorem)}$$

$$D(\operatorname{rot}_{\Gamma_t}) \cap D(-\operatorname{div}_{\Gamma_n} \varepsilon) = \mathsf{R}_{\Gamma_t} \cap \varepsilon^{-1} \mathsf{D}_{\Gamma_n} \hookrightarrow \mathsf{L}^2_\varepsilon \qquad \text{(Weck's selection theorem, '74)}$$

$$D(\operatorname{div}_{\Gamma_t}) \cap D(\varepsilon^{-1} \operatorname{rot}_{\Gamma_n}) = \mathsf{D}_{\Gamma_t} \cap \mathsf{R}_{\Gamma_n} \hookrightarrow \mathsf{L}^2 \qquad \text{(Weck's selection theorem, '74)}$$

$$D(\nabla_{\Gamma_n}) \cap D(\pi) = D(\nabla_{\Gamma_n}) = \mathsf{H}^1_{\Gamma_n} \hookrightarrow \mathsf{L}^2 \qquad \text{(Rellich's selection theorem)}$$

Weck's selection theorem for weak Lip. dom. and mixed bc: Bauer/P/Schomburg ('16)

de Rham complex in ND or on Riemannian manifolds (d-complex)

$\Omega \subset \mathbb{R}^N$ bd w. Lip. dom. or Ω Riemannian manifold with cpt cl. and Lip. boundary Γ

(generalized Maxwell equations)

$$\{0\} \underset{\pi_{\{0\}}}{\overset{\iota_{\{0\}}}{\rightleftarrows}} \mathsf{L}^{2,0} \underset{-\,\delta}{\overset{\mathring{\mathrm{d}}}{\rightleftarrows}} \mathsf{L}^{2,1} \underset{-\,\delta}{\overset{\mathring{\mathrm{d}}}{\rightleftarrows}} \ldots \mathsf{L}^{2,q} \underset{-\,\delta}{\overset{\mathring{\mathrm{d}}}{\rightleftarrows}} \mathsf{L}^{2,q+1} \ldots \mathsf{L}^{2,N-1} \underset{-\,\delta}{\overset{\mathring{\mathrm{d}}}{\rightleftarrows}} \mathsf{L}^{2,N} \underset{\iota_{\mathbb{R}}}{\overset{\pi_{\mathbb{R}}}{\rightleftarrows}} \mathbb{R}$$

de Rham complex in ND or on Riemannian manifolds (d-complex)

$\Omega \subset \mathbb{R}^N$ bd w. Lip. dom. or Ω Riemannian manifold with cpt cl. and Lip. boundary Γ

(generalized Maxwell equations)

$$\{0\} \text{ or } \mathbb{R} \underset{\pi}{\overset{\iota}{\rightleftarrows}} \mathsf{L}^{2,0} \underset{-\delta^1_{\Gamma_n}}{\overset{\mathrm{d}^0_{\Gamma_t}}{\rightleftarrows}} \mathsf{L}^{2,1} \underset{-\delta^2_{\Gamma_n}}{\overset{\mathrm{d}^1_{\Gamma_t}}{\rightleftarrows}} \dots \mathsf{L}^{2,q} \underset{-\delta^{q+1}_{\Gamma_n}}{\overset{\mathrm{d}^q_{\Gamma_t}}{\rightleftarrows}} \mathsf{L}^{2,q+1} \dots \mathsf{L}^{2,N-1} \underset{-\delta^N_{\Gamma_n}}{\overset{\mathrm{d}^{N-1}_{\Gamma_t}}{\rightleftarrows}} \mathsf{L}^{2,N} \underset{\iota}{\overset{\pi}{\rightleftarrows}} \mathbb{R} \text{ or } \{0\}$$

related fos

$$\begin{aligned} \mathrm{d}^q_{\Gamma_t} E &= F && \text{in } \Omega \\ -\delta^q_{\Gamma_n} E &= G && \text{in } \Omega \end{aligned}$$

related sos

$$\begin{aligned} -\delta^{q+1}_{\Gamma_n} \mathrm{d}^q_{\Gamma_t} E &= F && \text{in } \Omega \\ -\delta^q_{\Gamma_n} E &= G && \text{in } \Omega \end{aligned}$$

includes: EMS rot / div, Laplacian, rot rot, and more...
corresponding compact embeddings:

$$D(\mathrm{d}^q_{\Gamma_t}) \cap D(\delta^q_{\Gamma_n}) \hookrightarrow \mathsf{L}^{2,q} \qquad \text{(Weck's selection theorems, '74)}$$

Weck's selection theorem for Lip. manifolds and mixed bc: Bauer/P/Schomburg ('17)

elasticity complex in 3D (sym ∇-Rot $\text{Rot}_{\mathbb{S}}^{\top}$-$\text{Div}_{\mathbb{S}}$-complex)

$\Omega \subset \mathbb{R}^3$ bounded strong Lipschitz domain

$$\{0\} \;\underset{\pi_{\{0\}}}{\overset{\iota_{\{0\}}}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{-\operatorname{Div}_{\mathbb{S}}}{\overset{\mathring{\operatorname{sym}}\,\nabla}{\rightleftarrows}}\; \mathsf{L}^2_{\mathbb{S}} \;\underset{\operatorname{Rot}\operatorname{Rot}_{\mathbb{S}}^{\top}}{\overset{\mathring{\operatorname{Rot}\operatorname{Rot}}_{\mathbb{S}}^{\top}}{\rightleftarrows}}\; \mathsf{L}^2_{\mathbb{S}} \;\underset{-\operatorname{sym}\nabla}{\overset{\mathring{\operatorname{Div}}_{\mathbb{S}}}{\rightleftarrows}}\; \mathsf{L}^2 \;\underset{\iota_{\mathsf{RM}}}{\overset{\pi_{\mathsf{RM}}}{\rightleftarrows}}\; \mathsf{RM}$$

elasticity complex in 3D (sym ∇-Rot $\mathrm{Rot}_{\mathbb{S}}^{\top}$-$\mathrm{Div}_{\mathbb{S}}$-complex)

$\Omega \subset \mathbb{R}^3$ bounded strong Lipschitz domain

$$\{0\} \underset{\pi_{\{0\}}}{\overset{\iota_{\{0\}}}{\rightleftarrows}} \mathsf{L}^2 \underset{-\mathrm{Div}_{\mathbb{S}}}{\overset{\mathrm{sym}\,\mathring{\nabla}}{\rightleftarrows}} \mathsf{L}^2_{\mathbb{S}} \underset{\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S}}^{\top}}{\overset{\mathrm{Rot}\,\mathring{\mathrm{Rot}}_{\mathbb{S}}^{\top}}{\rightleftarrows}} \mathsf{L}^2_{\mathbb{S}} \underset{-\mathrm{sym}\,\nabla}{\overset{\mathring{\mathrm{Div}}_{\mathbb{S}}}{\rightleftarrows}} \mathsf{L}^2 \underset{\iota_{\mathsf{RM}}}{\overset{\pi_{\mathsf{RM}}}{\rightleftarrows}} \mathsf{RM}$$

related fos ($\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S},\Gamma}^{\top}$, $\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S}}^{\top}$ first order operators!)

$\mathrm{sym}\,\nabla_\Gamma v = M$ in Ω	$\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S},\Gamma}^{\top} M = F$ in Ω	$\mathrm{Div}_{\mathbb{S},\Gamma} N = g$ in Ω	$\pi v = r$ in Ω
$\pi v = 0$ in Ω	$-\mathrm{Div}_{\mathbb{S}} M = f$ in Ω	$\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S}}^{\top} N = G$ in Ω	$-\mathrm{sym}\,\nabla v = M$ in Ω

related sos ($\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S}}^{\top}\,\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S},\Gamma}^{\top}$ second order operator!)

$-\mathrm{Div}_{\mathbb{S}}\,\mathrm{sym}\,\nabla_\Gamma v = f$ in Ω	$\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S}}^{\top}\,\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S},\Gamma}^{\top} M = G$ in Ω	$-\mathrm{sym}\,\nabla\,\mathrm{Div}_{\mathbb{S},\Gamma} N = M$ in Ω
$\pi v = 0$ in Ω	$-\mathrm{Div}_{\mathbb{S}} M = f$ in Ω	$\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S}}^{\top} N = G$ in Ω

corresponding compact embeddings:

$$D(\mathrm{sym}\,\nabla_\Gamma) \cap D(\pi) = D(\nabla_\Gamma) = \mathsf{H}^1_\Gamma \hookrightarrow \mathsf{L}^2 \qquad \text{(Rellich's selection theorem and Korn ineq.)}$$

$$D(\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S},\Gamma}^{\top}) \cap D(\mathrm{Div}_{\mathbb{S}}) \hookrightarrow \mathsf{L}^2_{\mathbb{S}} \qquad \text{(new selection theorem)}$$

$$D(\mathrm{Div}_{\mathbb{S},\Gamma}) \cap D(\mathrm{Rot}\,\mathrm{Rot}_{\mathbb{S}}^{\top}) \hookrightarrow \mathsf{L}^2_{\mathbb{S}} \qquad \text{(new selection theorem)}$$

$$D(\pi) \cap D(\mathrm{sym}\,\nabla) = D(\nabla) = \mathsf{H}^1 \hookrightarrow \mathsf{L}^2 \qquad \text{(Rellich's selection theorem and Korn ineq.)}$$

two new selection theorems for strong Lip. dom.: P/Schomburg/Zulehner ('18)

biharmonic / general relativity complex in 3D ($\nabla\nabla$-$\mathrm{Rot}_{\mathbb{S}}$-$\mathrm{Div}_{\mathbb{T}}$-complex)

$\Omega \subset \mathbb{R}^3$ bounded strong Lipschitz domain

$$\{0\} \underset{\pi_{\{0\}}}{\overset{\iota_{\{0\}}}{\rightleftarrows}} \mathsf{L}^2 \underset{\operatorname{div}\operatorname{Div}_{\mathbb{S}}}{\overset{\mathring{\nabla\nabla}}{\rightleftarrows}} \mathsf{L}^2_{\mathbb{S}} \underset{\operatorname{sym}\operatorname{Rot}_{\mathbb{T}}}{\overset{\mathring{\operatorname{Rot}}_{\mathbb{S}}}{\rightleftarrows}} \mathsf{L}^2_{\mathbb{T}} \underset{-\operatorname{dev}\nabla}{\overset{\mathring{\operatorname{Div}}_{\mathbb{T}}}{\rightleftarrows}} \mathsf{L}^2 \underset{\iota_{\mathsf{RT}}}{\overset{\pi_{\mathsf{RT}}}{\rightleftarrows}} \mathsf{RT}$$

biharmonic / general relativity complex in 3D ($\nabla\nabla$-$\mathrm{Rot}_{\mathbb{S}}$-$\mathrm{Div}_{\mathbb{T}}$-complex)

$\Omega \subset \mathbb{R}^3$ bounded strong Lipschitz domain

$$\{0\} \underset{\pi_{\{0\}}}{\overset{\iota_{\{0\}}}{\rightleftarrows}} \mathsf{L}^2 \underset{\operatorname{div}\operatorname{Div}_{\mathbb{S}}}{\overset{\mathring{\nabla\nabla}}{\rightleftarrows}} \mathsf{L}^2_{\mathbb{S}} \underset{\operatorname{sym}\operatorname{Rot}_{\mathbb{T}}}{\overset{\mathring{\operatorname{Rot}}_{\mathbb{S}}}{\rightleftarrows}} \mathsf{L}^2_{\mathbb{T}} \underset{-\operatorname{dev}\nabla}{\overset{\mathring{\operatorname{Div}}_{\mathbb{T}}}{\rightleftarrows}} \mathsf{L}^2 \underset{\iota_{\mathsf{RT}}}{\overset{\pi_{\mathsf{RT}}}{\rightleftarrows}} \mathsf{RT}$$

related fos ($\nabla\nabla_\Gamma$, $\operatorname{div}\operatorname{Div}_{\mathbb{S}}$ first order operators!)

$\nabla\nabla_\Gamma u = M$ in Ω	$\operatorname{Rot}_{\mathbb{S},\Gamma} M = F$ in Ω	$\operatorname{Div}_{\mathbb{T},\Gamma} N = g$ in Ω	$\pi v = r$ in Ω
$\pi u = 0$ in Ω	$\operatorname{div}\operatorname{Div}_{\mathbb{S}} M = f$ in Ω	$\operatorname{sym}\operatorname{Rot}_{\mathbb{T}} N = G$ in Ω	$-\operatorname{dev}\nabla v = T$ in Ω

related sos ($\operatorname{div}\operatorname{Div}_{\mathbb{S}} \nabla\nabla_\Gamma = \Delta^2_\Gamma$ second order operator!)

$\operatorname{div}\operatorname{Div}_{\mathbb{S}} \nabla\nabla_\Gamma u = \Delta^2_\Gamma u = f$ in Ω	$\operatorname{sym}\operatorname{Rot}_{\mathbb{T}} \operatorname{Rot}_{\mathbb{S},\Gamma} M = G$ in Ω	$-\operatorname{dev}\nabla\operatorname{Div}_{\mathbb{T},\Gamma} N = T$ in Ω
$\pi u = 0$ in Ω	$\operatorname{div}\operatorname{Div}_{\mathbb{S}} M = f$ in Ω	$\operatorname{sym}\operatorname{Rot}_{\mathbb{T}} N = G$ in Ω

corresponding compact embeddings:

$$D(\nabla\nabla_\Gamma) \cap D(\pi) = D(\nabla\nabla_\Gamma) = \mathsf{H}^2_\Gamma \hookrightarrow \mathsf{L}^2 \qquad \text{(Rellich's selection theorem)}$$

$$D(\operatorname{Rot}_{\mathbb{S},\Gamma}) \cap D(\operatorname{div}\operatorname{Div}_{\mathbb{S}}) \hookrightarrow \mathsf{L}^2_{\mathbb{S}} \qquad \text{(new selection theorem)}$$

$$D(\operatorname{Div}_{\mathbb{T},\Gamma}) \cap D(\operatorname{sym}\operatorname{Rot}_{\mathbb{T}}) \hookrightarrow \mathsf{L}^2_{\mathbb{T}} \qquad \text{(new selection theorem)}$$

$$D(\pi) \cap D(\operatorname{dev}\nabla) = D(\operatorname{dev}\nabla) = D(\nabla) = \mathsf{H}^1 \hookrightarrow \mathsf{L}^2 \qquad \text{(Rellich's selection theorem and Korn type ineq.)}$$

two new selection theorems for strong Lip. dom. and Korn Type ineq.: P/Zulehner ('16)

. . . the world is full of complexes . . . ;)

⇒ relaxing at

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