

LOW-FREQUENCY ASYMPTOTICS FOR TIME-HARMONIC MAXWELL EQUATIONS IN EXTERIOR DOMAINS AND COMPARISON TO EDDY-CURRENT APPROXIMATIONS

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ABSTRACT. In this talk we will give a detailed description of the complete low-frequency asymptotics for time-harmonic Maxwell equations in exterior domains.

RESULTS

We will prove the complete low-frequency asymptotics for time-harmonic Maxwell equations in exterior domains. Starting with introducing the solution theory for time-harmonic electro-magnetic scattering problems via a generalized Fredholm alternative using the limiting absorption principle, we continue with proving an adequate corresponding electro-magneto static solution theory providing also special so-called towers of static solutions. In both cases we will work in polynomially weighted Sobolev spaces. Then a comparison with the whole space solution shows that a generalized asymptotic Neumann series gives the desired asymptotics for low frequencies up to a finite sum of degenerate operators, which can be described explicitly by strongly growing towers. Finally we compare these time-harmonic Maxwell radiation solutions with the corresponding solutions provided by the eddy-current model for low frequencies.

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