Existence of flows for linear Fokker-Planck-Kolmogorov equations and its connection to well-posedness

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Let the coefficients $a_{ij} : [0,T] \times \mathbb{R}^d \to \mathbb{R}$ and $b_i : [0,T] \times \mathbb{R}^d \to \mathbb{R}$, $i,j \leq d$, of the linear Fokker-Planck-Kolmogorov equation (FPK-eq.)

$$\partial_t \mu_t = \partial_i \partial_j (a_{ij} \mu_t) - \partial_i (b_i \mu_t)$$

be Borel measurable, bounded and continuous in space. Assume that for every $s \in [0,T]$ and every Borel probability measure ν on $\mathbb{R}^d$ there is at least one solution $\mu = (\mu_t)_{t \in [s,T]}$ to the FPK-eq. such that $\mu_s = \nu$ and $t \mapsto \mu_t$ is continuous w.r.t. the topology of weak convergence of measures. We prove that in this situation, one can always select one solution $\mu^{s,\nu}$ for each pair (s,ν) such that this family of solutions fulfills

$$\mu_t^{s,\nu} = \mu_t^{r,\mu_r^{s,\nu}} \ \forall\ 0 \leq s \leq r \leq t \leq T,$$

which one interprets as a *flow property* of this solution family. Moreover, we prove that such a flow of solutions is unqiue if and only if the FPK-eq. is well-posed.