On completeness of the space of bounded Dirichlet series

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A general Dirichlet series is a (formal) sum $\sum a_n e^{-\lambda_n s}$, where (a_n) is called the Dirichlet coefficients of D, s is a complex variable and $\lambda = (\lambda_n)$ is a frequency, that is a strictly increasing non negative real sequence which tends to $+\infty$. A natural space of Dirichlet series is given by the space $\mathcal{D}_\infty(\lambda)$ of all λ-Dirichlet series $D = \sum a_n e^{-\lambda_n s}$ which converge on the open right half plane $[Re > 0]$ and define a bounded function there. Endowed with the sup norm, $\mathcal{D}_\infty(\lambda)$ for arbitrary λ forms a normed space . The question about completeness appears to be delicate, since $\mathcal{D}_\infty(\lambda)$ may fail to be complete. In this talk we present sufficient conditions on λ for completeness and especially we focus on the so-called Landau condition (LC), whose sufficiency was recently proven. We give details of the proof, which requires to recall the framework of Hardy spaces on so-called λ-Dirichlet groups and includes a Hardy-Littlewood maximal operator.