A Priori Error Analysis for an Optimal Control Problem Governed by a Variational Inequality of the Second Kind

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We consider the following optimal control problem governed by an elliptic variational inequality of the second kind:

$$\left.\begin{aligned} &\min_{y\in H_0^1(\Omega),\ u\in L^2(\Omega)} J(y,u) := \tfrac{1}{2}\|y-y_d\|^2_{L^2(\Omega)} + \tfrac{\nu}{2}\|u\|^2_{L^2(\Omega)} \\ &\text{s.t.} \quad \int_\Omega \nabla y\cdot\nabla(v-y)\,\mathrm{d}x + \|v\|_{L^1(\Omega)} - \|y\|_{L^1(\Omega)} \geq \langle u, v-y\rangle \quad \forall v\in H_0^1(\Omega), \end{aligned}\right\} \quad \text{(P)}$$

where $\Omega\subset\mathbb{R}^d$ $(d=1,2,3)$ is a bounded domain with $C^{1,1}$-boundary, $y_d\in L^2(\Omega)$ and $\nu>0$ is a fixed number. The optimal control problem is discretized by linear finite elements for the state and a variational discrete approach for the control. We present nearly optimal a priori error estimates for the numerical approximation of (P), i.e. we prove second order convergence (up to logarithmic terms) for the states and first order convergence (up to logarithmic terms) for the controls. The key tools for the proof are strong stationarity conditions and a quadratic growth condition. The derivation of these strong stationarity conditions is based on differentiability properties of the control-to-state operator and needs only mild assumptions on the active set.