Fast Directional Matrix-Vector Multiplications - Analysis and Numerical Experiments

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The solution of boundary value problems for the Helmholtz equation by means of boundary element methods requires the solution of a system of linear equations for matrices whose entries are of the form

$$A[j,k] = f(x_j, y_k) = \frac{\exp(i\kappa|x_j - y_k|)}{4\pi|x_j - y_k|}, \quad j,k \in \{1,...,N\},$$

where $\{x_j\}_{j=1}^N, \{y_k\}_{k=1}^N \subset \mathbb{R}^3$, f is the 3D Helmholtz kernel and $\kappa > 0$ the wave number. Corresponding matrix-vector multiplications have a complexity of order $\mathcal{O}(N^2)$ and are therefore prohibitive for large N. Standard matrix approximation schemes can be used to overcome this problem in low frequency regimes, but are inefficient in high frequency regimes.

We consider and analyze a directional approximation of the Helmholtz kernel. Together with a suitable clustering strategy this allows for an approximation of the matrix A and, correspondingly, an algorithm for fast matrix-vector multiplications, which has a complexity of order $\mathcal{O}(N\log(N))$ in all frequency regimes under suitable assumptions on N, κ and the distribution of points $\{x_j\}_{j=1}^N$ and $\{y_k\}_{k=1}^N$. The effective runtime and accuracy of the algorithm is influenced by the choice of two parameters. We conduct a parameter study to investigate this influence and summarize our observations in a parameter selection strategy.

References

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