Can functional-type a posteriori error estimates unite adaptivity and error control in BEM?

joint work:
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Mathematical model → Representation formula → Boundary integral equation → Boundary elements → Discrete equations → Solution of linear system → Interpretation

Mathematical model

- $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, polygonal boundary $\Gamma := \partial\Omega$,
- **homogeneous** Poisson problem

$$\Delta u = 0 \quad \text{in } \Omega,$$
$$u = g \quad \text{on } \Gamma.$$

Representation formula

Let $G(z) = -\frac{1}{2\pi} \log |z| \quad (d = 2)$ the **fundamental solution** of the Laplacian. Ansatz (**make a choice!**):

- $u(x) = \int_\Gamma G(x - y)\, \phi(y)\, \mathrm{d}y =: [\tilde{V}\phi](x)$ (indir.)
- $u(x) = [\tilde{V}\phi](x) - \underbrace{\int_\Gamma \partial_{n(y)} G(x - y)\, g(y)\, \mathrm{d}y}_{=:[\tilde{K}g](x)}$ (dir.)
- ...

For any $\phi \in \mathsf{H}^{-1/2}(\Gamma)$ it holds $\Delta u = 0$.

- Taking the trace of u (indir.) leads to boundary integral equation

$$V\phi := \gamma_0 \circ \tilde{V}\phi = g \quad \text{in } \mathsf{H}^{1/2}(\Gamma).$$

- variational formulation: Find $\phi \in \mathsf{H}^{-1/2}(\Gamma)$ s.t.

$$\langle\langle \phi\,,\, \psi \rangle\rangle := \langle \psi\,, V\phi \rangle_\Gamma = \langle \psi\,, g \rangle_\Gamma$$

for all $\psi \in \mathsf{H}^{-1/2}(\Gamma)$.

Mathematical model → Representation formula → Boundary integral equation → Boundary elements → Discrete equations → Solution of linear system → Interpretation

Boundary elements

Let $\mathcal{E}_h = \{E_1, \dots, E_N\}$ be a regular triangulation of Γ.

- $\mathcal{P}^0(\mathcal{E}_h) := \{\psi \in \mathrm{L}^\infty(\Gamma) :$
  $\psi|_E$ is const. $\forall E \in \mathcal{E}_h\} \subset \mathrm{H}^{-1/2}(\Gamma)$
- $\mathcal{S}^1(\mathcal{E}_h) := \{\zeta \in \mathrm{C}(\Gamma) :$
  $\zeta|_E$ is affine $\forall E \in \mathcal{E}_h\} \subset \mathrm{H}^{1/2}(\Gamma)$

Discrete equations

Find $\phi_h \in \mathcal{P}^0(\mathcal{E}_h)$ s.t.

$$\langle \psi_h , V\phi_h \rangle_\Gamma = \langle \psi_h , g_h \rangle_\Gamma \quad \text{for all } \psi_h \in \mathcal{P}^0(\mathcal{E}_h).$$

L^2-projection of g

Find $g_h \in \mathcal{S}^1(\mathcal{E}_h) \subset \mathrm{H}^{1/2}(\Gamma)$ s.t.

$$\langle g_h , \zeta_k \rangle_\Gamma = \langle g , \zeta_k \rangle_\Gamma \quad \text{for all } \zeta_k \in \mathcal{S}^1(\mathcal{E}_h).$$

- oscillation error
  $\|g - g_h\|_{\mathrm{H}^{1/2}(\Gamma)} \leq C\, \|h^{1/2}(g - g_h)'\|_{\mathrm{L}^2(\Gamma)}$

Solution of linear system

Interpretation

By means of chosen ansatz (representation formula)

$$u_h = \tilde{V}\phi_h$$

What to remember...

- compute boundary density $\phi_h \approx \phi \in \mathrm{H}^{-1/2}(\Gamma)$
  - in a direct ansatz, we obtain the full Cauchy data $u|_\Gamma = g$ and $\partial_n u|_\Gamma \approx \phi_h$
  - in an indirect ansatz, the computed density has no physical relevance
- $\Delta u_h = 0$, in particular $\Delta(u - u_h) = 0$, in $\mathrm{L}^2(\Omega)$.
- boundary datum g is discretized, e.g. by L^2-projection
- lowest order discretization: $\mathcal{P}^0(\mathcal{E}_h) \subset \mathrm{H}^{-1/2}(\Gamma)$, $\mathcal{S}^1(\mathcal{E}_h) \subset \mathrm{H}^{1/2}(\Gamma)$
- energy error in $\mathrm{H}^{-1/2}(\Gamma)$:

$$\|\!|\phi - \phi_h|\!\| := \sqrt{\langle \phi - \phi_h \, , V(\phi - \phi_h) \rangle_\Gamma} \simeq \|\phi - \phi_h\|_{\mathrm{H}^{-1/2}(\Gamma)}$$

Motivation

Q: Can functional-type a posteriori error estimates unite adaptivity and error control in BEM?

- adaptivity? $\rightsquigarrow \varepsilon = |||\phi - \phi_h||| \simeq \|\phi - \phi_h\|_{H^{-1/2}(\Gamma)}$
- error control? $\rightsquigarrow \varepsilon = \|\nabla(u - u_h)\|_{L^2(\Omega)}, \quad u_h = \tilde{V}\phi_h$
- unity? (Which ε might enable both?)

Motivation

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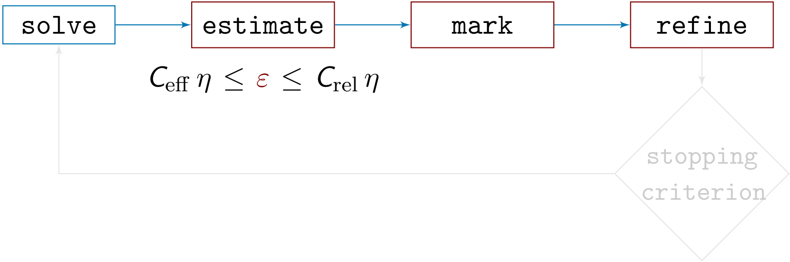
The standard adaptive algorithm:



$$C_{\text{eff}}\,\eta \le \varepsilon \le C_{\text{rel}}\,\eta$$

Motivation

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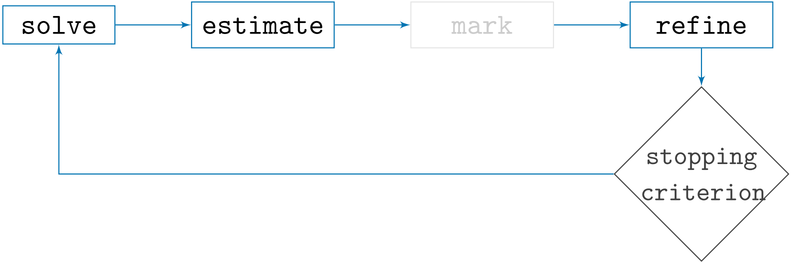
- error control? $\rightsquigarrow \varepsilon = \|\nabla(u - u_h)\|_{L^2(\Omega)}, \quad u_h = \tilde{V}\phi_h$

The error-controlling algorithm (based on uniform refinement):

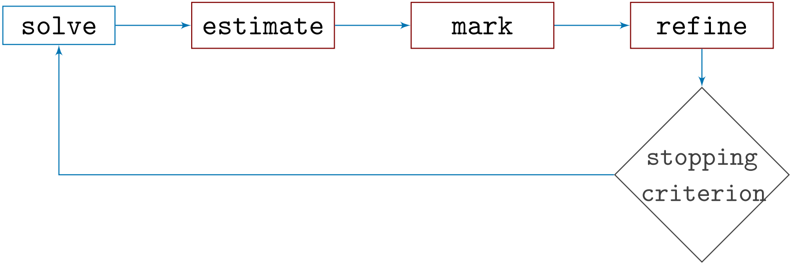


Motivation

Q: `Can functional-type a posteriori error estimates unite adaptivity and error control in BEM?`

- `unity?` (`Which` ε `might enable both?`)

The practical (ideal) adaptive algorithm:



What to remember...

- In BEM, intuitive error functionals for adaptivity and error control do generally not coincide
  - $\varepsilon = |\!|\!|\phi - \phi_h|\!|\!| \simeq \|\phi - \phi_h\|_{H^{-1/2}(\Gamma)}$
  - $\varepsilon = \|\nabla(u - u_h)\|_{L^2(\Omega)}$
- $|\!|\!|\phi - \phi_h|\!|\!| \simeq \|\phi - \phi_h\|_{H^{-1/2}(\Gamma)}$ is non-local, i.e. cannot be written as sum of local contributions
  - localization of $|\!|\!|\cdot|\!|\!|$ will involve incomputable constants
- $\|\nabla(u - u_h)\|_{L^2(\Omega)}$ is local, but (commonly demanded) properties like Galerkin orthogonality are not available on Ω

Key Message of this talk.

- The "natural" error functional is not the only option when aiming at adaptivity.
- **Functional-type** a posteriori error estimates are rather suitable for BEM. (does not require a priori knowledge on u_h, which we do not have, e.g. Galerkin orth.)

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Error identity

For any approximation $v \in \mathrm{H}^1(\Omega)$ with $\nabla v \in \mathrm{H}(\mathrm{div} = 0, \Omega)$, it holds that

$$\max_{\substack{\boldsymbol{\tau} \in \mathrm{H}(\mathrm{div}, \Omega) \\ \mathrm{div} \boldsymbol{\tau} = 0}} \underline{\mathfrak{M}}(\boldsymbol{\tau}) = \|\nabla(u - v)\|^2_{\mathrm{L}^2(\Omega)} = \min_{\substack{w \in \mathrm{H}^1(\Omega) \\ w|_\Gamma = g - v|_\Gamma}} \overline{\mathfrak{M}}(w), \tag{1}$$

where

$$\underline{\mathfrak{M}}(\boldsymbol{\tau}) := \left[2 \int_\Gamma (g - v|_\Gamma) \boldsymbol{\tau} \cdot \boldsymbol{n} - \|\boldsymbol{\tau}\|^2_{\mathrm{L}^2(\Omega)} \right] \tag{2}$$

and

$$\overline{\mathfrak{M}}(w) := \|\nabla w\|^2_{\mathrm{L}^2(\Omega)}. \tag{3}$$

The unique maximizer is $\boldsymbol{\tau} = \nabla(u - v)$, the unique minimizer is $w = u - v$.

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Proof.

The proof is split into two steps.

Step 1 (Upper bound). Let $w \in \mathrm{H}^1(\Omega)$ with $w|_\Gamma = u|_\Gamma = g$. Since $\nabla(u - v) \in \mathrm{H}(\mathrm{div} = 0, \Omega)$, integration by parts shows that

$$\begin{aligned}\|\nabla(u-v)\|^2_{\mathrm{L}^2(\Omega)} &= \langle \nabla(u-w), \nabla(u-v)\rangle_\Omega + \langle \nabla(w-v), \nabla(u-v)\rangle_\Omega \\ &= \langle \nabla(w-v), \nabla(u-v)\rangle_\Omega.\end{aligned}$$

With the Cauchy-Schwarz inequality, we are led to

$$\|\nabla(u-v)\|_{\mathrm{L}^2(\Omega)} \leq \|\nabla(w-v)\|_{\mathrm{L}^2(\Omega)}.$$

By substitution, this proves that

$$\|\nabla(u-v)\|_{\mathrm{L}^2(\Omega)} \leq \inf_{\substack{w \in \mathrm{H}^1(\Omega) \\ w|_\Gamma = g - v|_\Gamma}} \|\nabla w\|_{\mathrm{L}^2(\Omega)},$$

and the infimum is clearly attained for $w = u - v$. □

Proof.

Step 2 (Lower bound). In any Hilbert space H, it holds that

$$\|a\|_{\mathrm{H}}^2 = \max_{b\in\mathrm{H}} \big[2\langle a\,,b\rangle_{\mathrm{H}} - \|b\|_{\mathrm{H}}^2\big] \quad \text{for all } a \in \mathrm{H},$$

where the maximum is attained for $b = a$. With $\nabla(u-v) \in \mathrm{H}(\mathrm{div} = 0\Omega) =: \mathrm{H}$, integration by parts shows that

$$\begin{aligned}\|\nabla(u-v)\|_{\mathrm{L}^2(\Omega)}^2 &= \|\nabla(u-v)\|_{\mathrm{H}(\mathrm{div},\Omega)}^2\\ &= \max_{\substack{\boldsymbol{\tau}\in\mathrm{H}(\mathrm{div},\Omega)\\ div\,\boldsymbol{\tau}=0}} \Big[2\int_\Omega \nabla(u-v)\cdot\boldsymbol{\tau} - \|\boldsymbol{\tau}\|_{\mathrm{L}^2(\Omega)}^2\Big]\\ &= \max_{\substack{\boldsymbol{\tau}\in\mathrm{H}(\mathrm{div},\Omega)\\ div\,\boldsymbol{\tau}=0}} \Big[2\int_\Gamma (g - v|_\Gamma)\,\boldsymbol{\tau}\cdot\boldsymbol{n} - \|\boldsymbol{\tau}\|_{\mathrm{L}^2(\Omega)}^2\Big].\end{aligned}$$

In particular, the maximum is attained for $\boldsymbol{\tau} = \nabla(u-v)$. This concludes the proof. □

Boundary residual

Inside the error identity ($\|\cdot\| := \|\cdot\|_{L^2(\Omega)}$)

$$\max_{\substack{\boldsymbol{\tau}\in H(\mathrm{div},\Omega)\\ \mathrm{div}\boldsymbol{\tau}=0}} \left[2\int_\Gamma \boldsymbol{\tau}\cdot\boldsymbol{n}\,(g - v|_\Gamma) - \|\boldsymbol{\tau}\|^2\right] = \|\nabla(u-v)\|^2 = \min_{\substack{w\in H^1(\Omega)\\ w|_\Gamma = g - v|_\Gamma}} \|\nabla w\|^2,$$

the boundary residual is essential and contains all relevant information.

- evaluations of a BEM approximation v might be expensive at the boundary!
- numerical approximations of $w_h \approx w$ w.r.t. to the majorant will never satisfy the boundary condition exactly, i.e.

$$\underline{\mathfrak{M}}(\boldsymbol{\tau}_h) \leq \max_{\substack{\boldsymbol{\tau}\in H(\mathrm{div},\Omega)\\ \mathrm{div}\boldsymbol{\tau}=0}} \underline{\mathfrak{M}}(\boldsymbol{\tau}) = \|\nabla(u-v)\| = \min_{\substack{w\in H^1(\Omega)\\ w|_\Gamma = g - v|_\Gamma}} \overline{\mathfrak{M}}(w) \leq \overline{\mathfrak{M}}(w_h)$$

will, in general, not hold true.

Majorant

Practical Majorant

Let $J_h : \mathrm{H}^{1/2}(\Gamma) \to \mathrm{H}^{1/2}(\Gamma)$. For any $v \in \mathrm{H}^1(\Omega)$ with $\nabla v \in \mathrm{H}(\mathrm{div} = 0, \Omega)$, it then follows that

$$\|\nabla(u - v)\|_{\mathrm{L}^2(\Omega)} \leq \min_{\substack{\widehat{w} \in \mathrm{H}^1(\Omega) \\ \widehat{w}|_\Gamma = J_h(g - v|_\Gamma)}} \|\nabla \widehat{w}\|_{\mathrm{L}^2(\Omega)} + 2\,\|(1 - J_h)(g - v|_\Gamma)\|_{\mathrm{H}^{1/2}(\Gamma)}. \quad (4)$$

Proof.

Idea: Let $w = u - v$ the exact solution to

$$\min_{\substack{w \in \mathrm{H}^1(\Omega) \\ w|_\Gamma = g - v|_\Gamma}} \overline{\mathfrak{M}}(w)$$

and $\widehat{w}$ the solution to

$$\min_{\substack{w \in \mathrm{H}^1(\Omega) \\ w|_\Gamma = J_h(g - v|_\Gamma)}} \overline{\mathfrak{M}}(w).$$

Then,

$$\|\nabla(u - v)\|_{\mathrm{L}^2(\Omega)} = \|\nabla w\|_{\mathrm{L}^2(\Omega)} \leq \|\nabla \widehat{w}\|_{\mathrm{L}^2(\Omega)} + \underbrace{\|\nabla(w - \widehat{w})\|_{\mathrm{L}^2(\Omega)}}_{\leq \|(1 - J_h)\,(g - v|_\Gamma)\|_{\mathrm{H}^{1/2}(\Gamma)}}.$$

□

Corollary

Let $J_h : \mathrm{H}^{1/2}(\Gamma) \to \mathrm{H}^{1/2}(\Gamma)$. Let $v \in \mathrm{H}^1(\Omega)$ with $\nabla v \in \mathrm{H}(\mathrm{div}=0,\Omega)$. Let $S \subseteq \Omega$ be a Lipschitz domain with $\Gamma \subseteq \partial S$ and $\overline{w} \in \mathrm{H}^1(S)$ the solution of the inhomogeneous Dirichlet problem

$$\langle \nabla \overline{w}, \nabla \varphi \rangle_S = 0 \text{ for all } \varphi \in \mathring{\mathrm{H}}^1(S) \text{ subject to } \overline{w}|_{\partial S} = \begin{cases} J_h(g - v|_\Gamma) & \text{on } \Gamma \subseteq \partial S, \\ 0 & \text{on } \partial S \setminus \Gamma. \end{cases} \tag{5}$$

Then, $$\|\nabla(u-v)\|_{\mathrm{L}^2(\Omega)} \le \|\nabla \overline{w}\|_{\mathrm{L}^2(S)} + 2\,\|(1-J_h)(g - v|_\Gamma)\|_{\mathrm{H}^{1/2}(\Gamma)}. \tag{6}$$

Moreover, let $\mathcal{T}_h$ be a conforming triangulation of S and suppose that $\mathrm{range}(J_h) \subseteq \{v_h|_\Gamma \,:\, v_h \in \mathcal{S}^1(\mathcal{T}_h)\}$. Consider the FEM approximation of $\overline{w}$, i.e., $\overline{w}_h \in \mathcal{S}^1(\mathcal{T}_h)$ satisfies that

$$\langle \nabla \overline{w}_h, \nabla \varphi_h \rangle_S = 0 \text{ for all } \varphi_h \in \mathcal{S}^1_0(\mathcal{T}_h) \text{ with } \overline{w}_h|_{\partial S} = \begin{cases} J_h(g - v|_\Gamma) & \text{on } \Gamma \subseteq \partial S, \\ 0 & \text{on } \partial S \setminus \Gamma. \end{cases} \tag{7}$$

Then, (6) also holds with $\overline{w}$ being replaced by $\overline{w}_h$.

Minorant

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Proposition

For any approximation $v \in \mathsf{H}^1(\Omega)$ with $\nabla v \in \mathsf{H}(\mathsf{div} = 0, \Omega)$, let $(\boldsymbol{\tau}, p) \in \mathsf{H}(\mathsf{div}, \Omega) \times \mathsf{L}^2(\Omega)$ be the solution to the mixed problem

$$\langle \boldsymbol{\tau}, \boldsymbol{\sigma} \rangle_\Omega + \langle \mathsf{div}\, \boldsymbol{\sigma}, p \rangle_\Omega = \langle \boldsymbol{\sigma} \cdot \boldsymbol{n}, g - v|_\Gamma \rangle_\Gamma$$
$$\langle \mathsf{div}\, \boldsymbol{\tau}, q \rangle_\Omega = 0$$

for all $(\boldsymbol{\sigma}, q) \in \mathsf{H}(\mathsf{div}, \Omega) \times \mathsf{L}^2(\Omega)$.
Then, it holds that

$$2 \langle \boldsymbol{\tau} \cdot \boldsymbol{n}, g - v|_\Gamma \rangle_\Gamma - \|\boldsymbol{\tau}\|^2_{\mathsf{L}^2(\Omega)} = \|\nabla(u - v)\|^2_{\mathsf{L}^2(\Omega)}.$$

$\rightsquigarrow$ Implementation available from C. Bahriawati, C. Carstensen in [6].

Proposition (yet implemented version) only $d = 2$

For topologically trivial domains $\Omega \subset \mathbb{R}^2$, we have $\mathsf{H}(\mathsf{div} = 0, \Omega) = \mathsf{rot_z}\, \mathsf{H}(\mathsf{rot_z}, \Omega)$. Hence by $\boldsymbol{\tau} = \mathsf{rot_z}\, \omega = \begin{pmatrix} -\partial_y \omega \\ \partial_x \omega \end{pmatrix}$, we are led to

$$\min_{\omega \in \mathsf{H}^1(\Omega)} \left[2 \langle \mathsf{rot_z}\, \omega \cdot \boldsymbol{n}, g - v|_\Gamma \rangle_\Gamma - \|\nabla \omega\|^2_{\mathsf{L}^2(\Omega)} \right] = \|\nabla(u - u_h)\|^2_{\mathsf{L}^2(\Omega)}.$$

Proposition

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$$\langle \boldsymbol{\tau}\,,\boldsymbol{\sigma}\rangle_\Omega + \langle \mathrm{div}\,\boldsymbol{\sigma}\,,p\rangle_\Omega = \langle \boldsymbol{\sigma}\cdot\boldsymbol{n}\,,g - v|_\Gamma\rangle_\Gamma$$
$$\langle \mathrm{div}\,\boldsymbol{\tau}\,,q\rangle_\Omega = 0$$

for all $(\boldsymbol{\sigma}\,,\, q) \in \mathsf{H}(\mathrm{div}, \Omega) \times \mathsf{L}^2(\Omega)$.
Then, it holds that

$$\mathfrak{M}(\boldsymbol{\tau}_0) \leq 2\,\langle \boldsymbol{\tau}\cdot\boldsymbol{n}\,,g - v|_\Gamma\rangle_\Gamma - \|\boldsymbol{\tau}\|^2_{\mathsf{L}^2(\Omega)} = \|\nabla(u - v)\|^2_{\mathsf{L}^2(\Omega)}.$$

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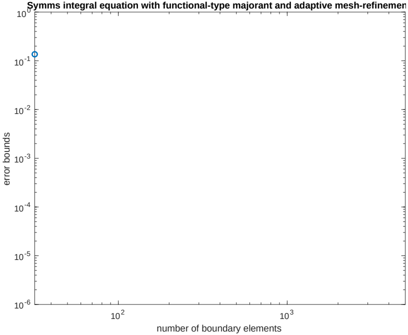
$$\min_{\omega\in\mathsf{H}^1(\Omega)}\Big[2\langle \mathrm{rot}_z\,\omega\cdot\boldsymbol{n}\,,g - v|_\Gamma\rangle_\Gamma - \|\nabla\omega\|^2_{\mathsf{L}^2(\Omega)}\Big] = \|\nabla(u - u_h)\|^2_{\mathsf{L}^2(\Omega)}.$$

Algorithm

Set $\ell = 1$.

1. Extract BEM-mesh $\mathcal{E}_\ell$ from given FEM-mesh $\mathcal{T}_\ell$
2. Compute L^2-projection $g_h \in \mathcal{S}^1(\mathcal{E}_\ell)$ of $g \in \mathsf{H}^1(\Gamma)$
3. Solve boundary integral equation in $\mathcal{P}^0(\mathcal{E}_\ell)$
4. Solve Majorant / Minorant problems via P1-FEM on second order patch Σ, i.e. with $\mathcal{S}^1(\mathcal{T}_\ell^\Sigma) = \mathcal{S}^1(\mathcal{T}_\ell)|_\Sigma$
5. compute error estimator on $\mathcal{T}_\ell^\Sigma$:
$$\eta_\ell(T) = \|\nabla \overline{w}_h\|_{\mathsf{L}^2(T)}$$
for $T \in \mathcal{T}_\ell^\Sigma$, and 0 in $\mathcal{T}_\ell \setminus \mathcal{T}_\ell^\Sigma$
6. Determine set $\mathcal{M}_\ell \subset \mathcal{T}_\ell$ of minimal cardinality such that
$$\theta \sum_{T \in \mathcal{T}_\ell} \eta_\ell^2(T) \leq \sum_{T \in \mathcal{M}_\ell} \eta_\ell^2(T).$$
7. Refine marked elements $\mathcal{M}_\ell$ by newest vertex bisection to obtain $\mathcal{T}_{\ell+1}$

Unit Square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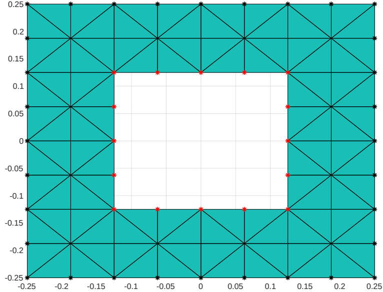
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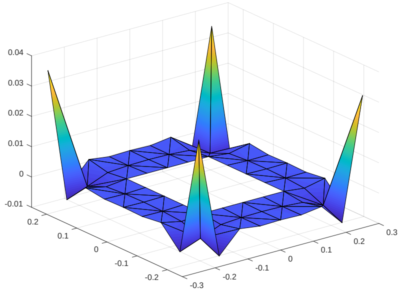
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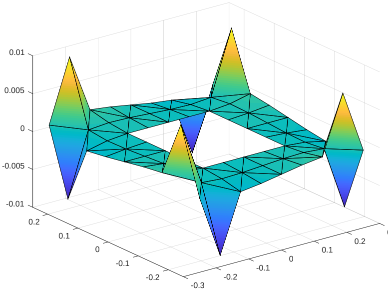
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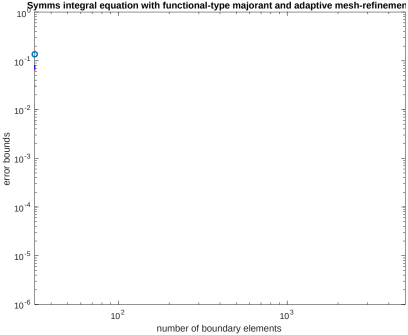
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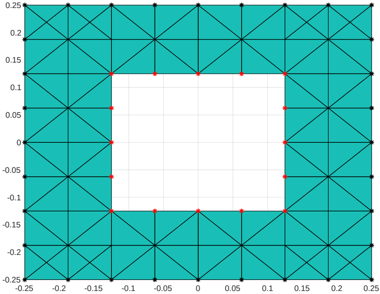

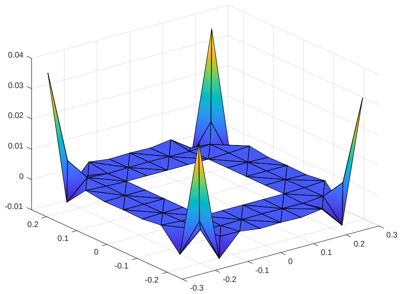

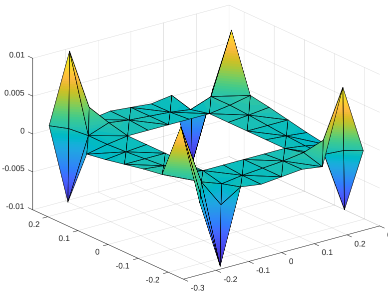

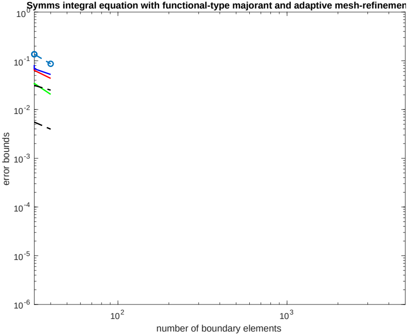
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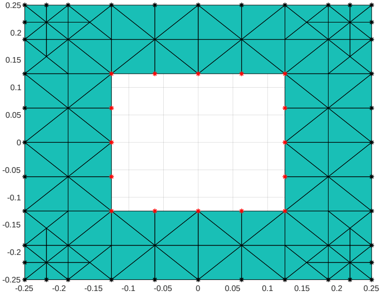
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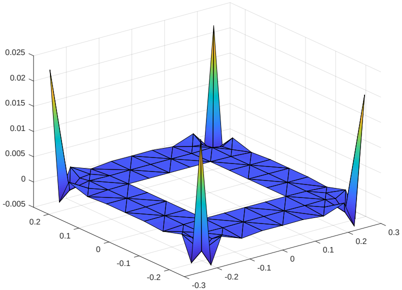
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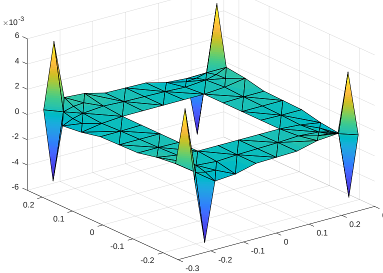
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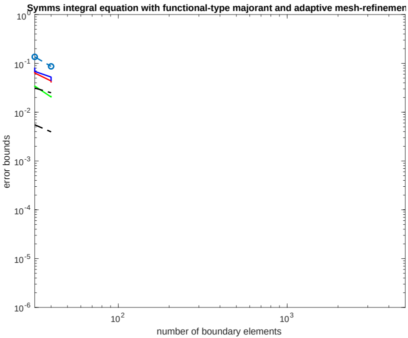
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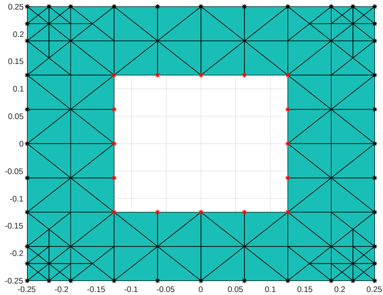

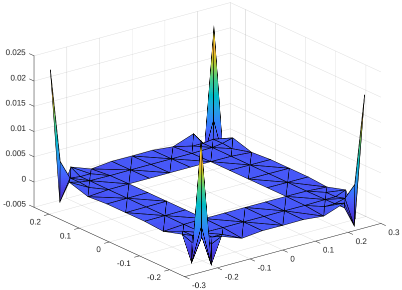

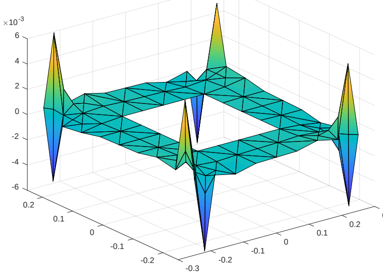
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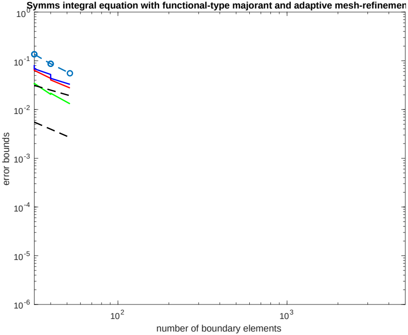
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
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10^{-3}
10^{-4}
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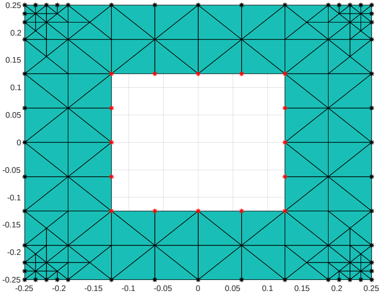
10^{-6}
10^2
10^3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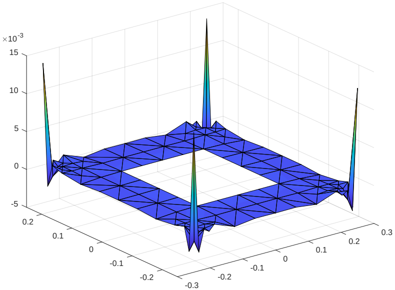
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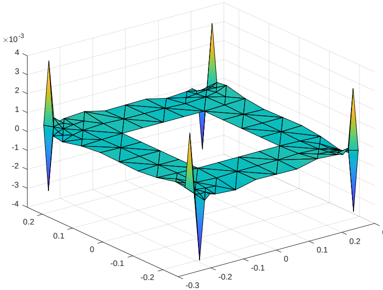
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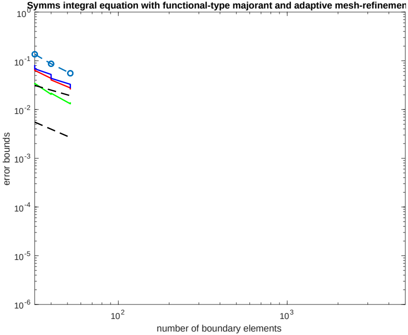

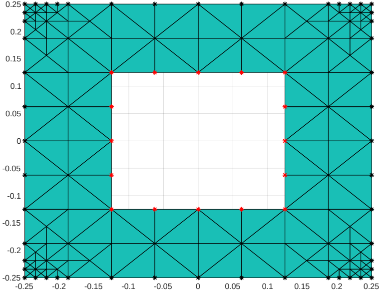

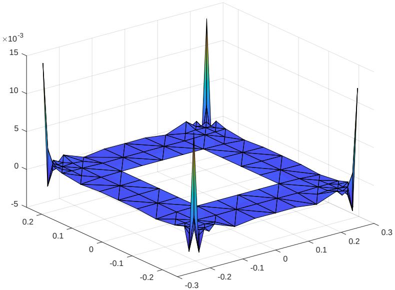

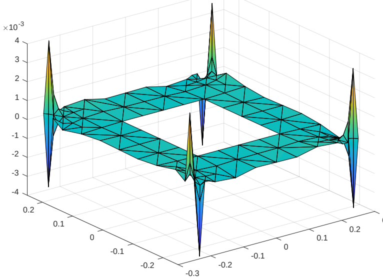

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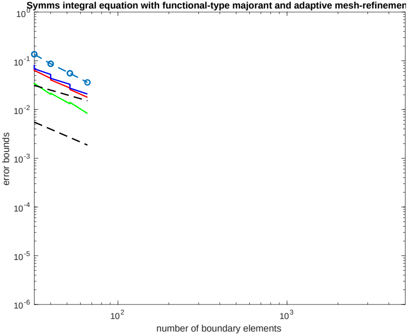

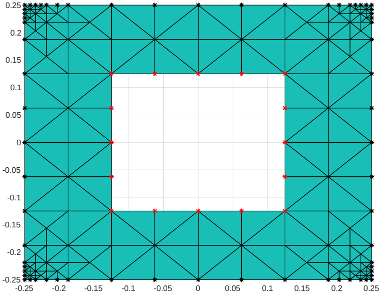

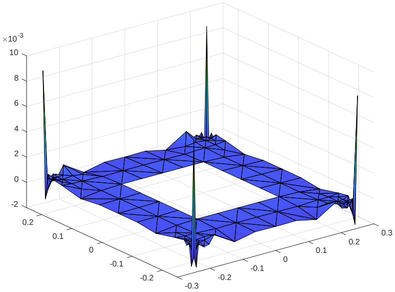

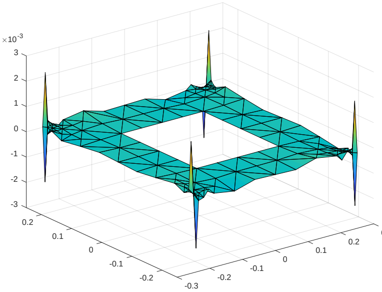

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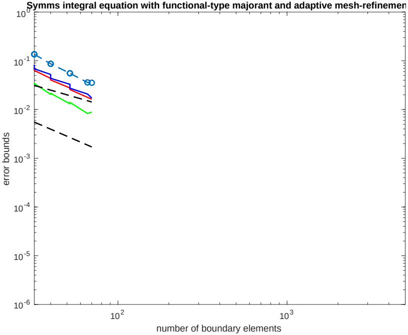
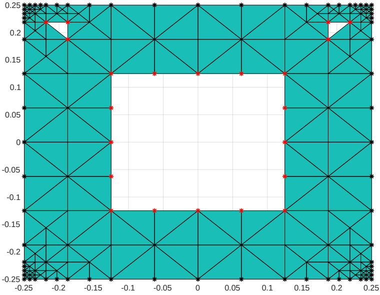
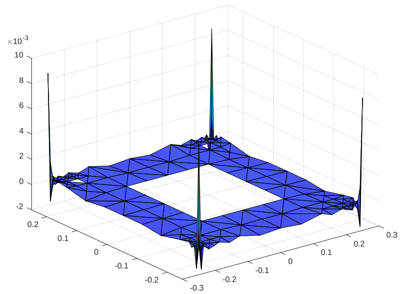
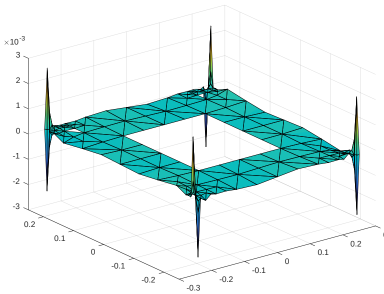
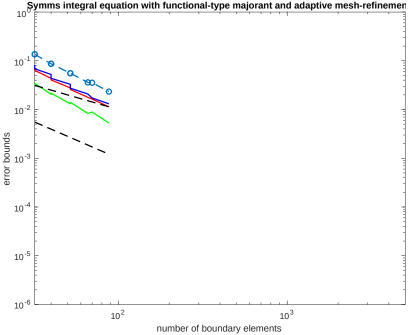
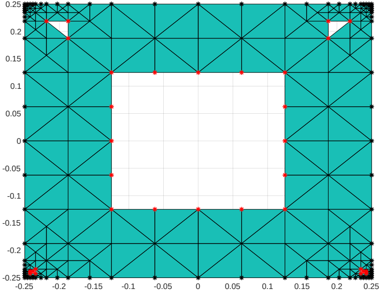


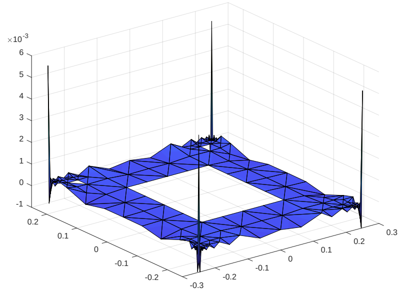







Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^2
10^3

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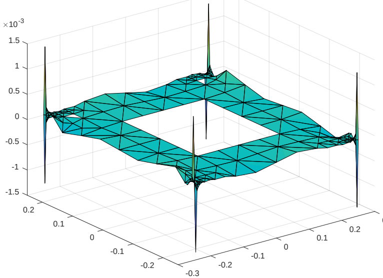
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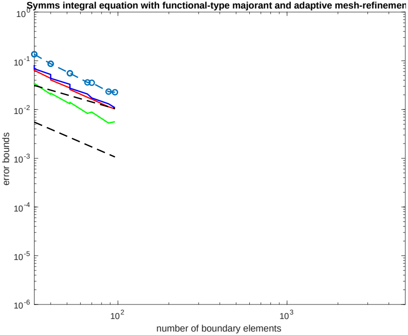
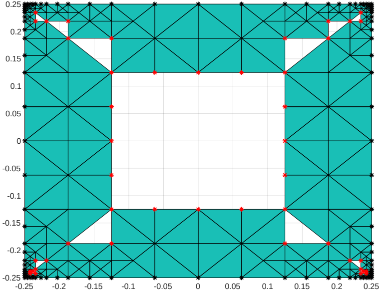
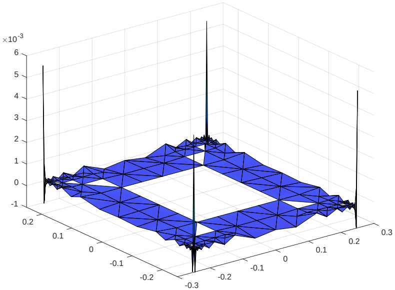
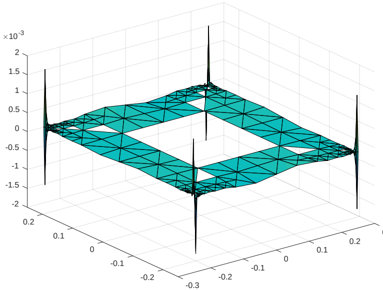
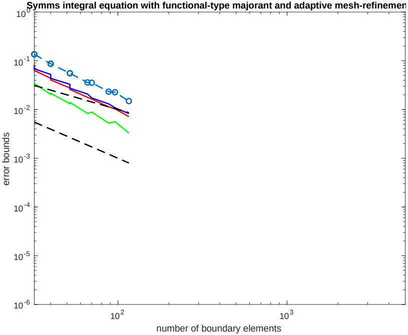
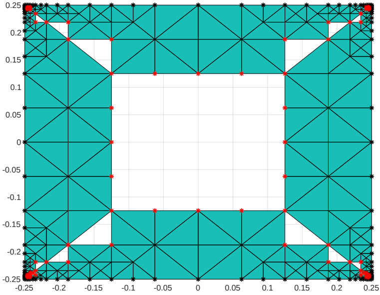

Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements

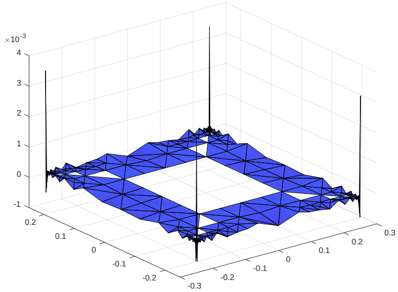







Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements

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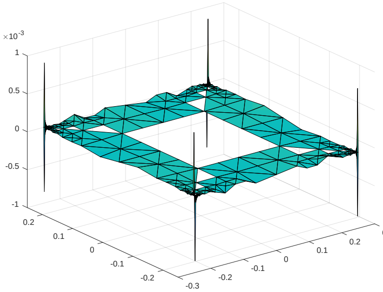
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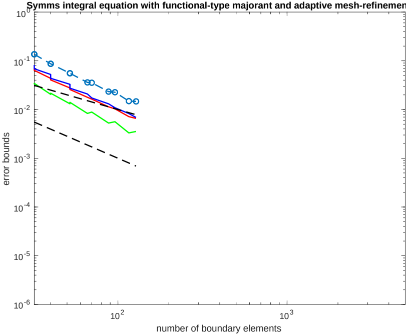
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
10^{0}
10^{-1}
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10^{-5}
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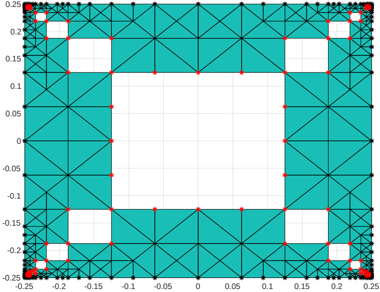
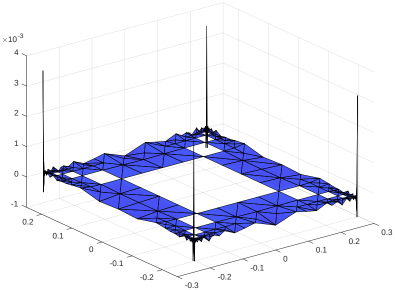
10^{2}
10^{3}

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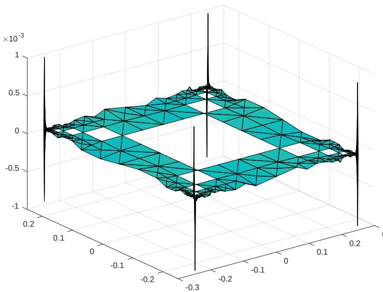
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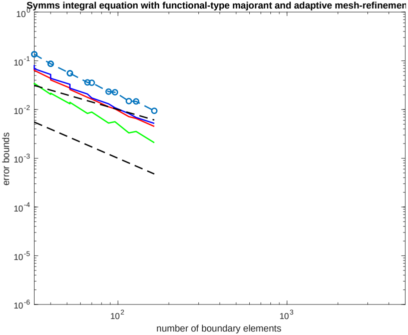
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
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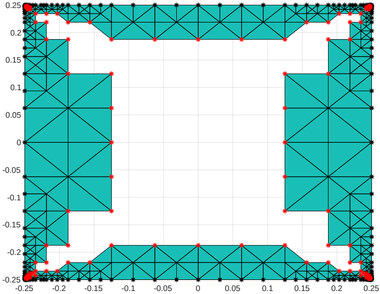
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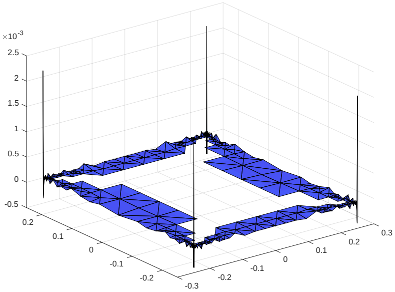
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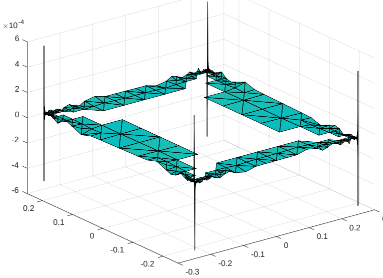
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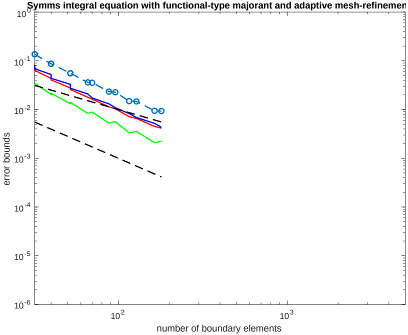
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
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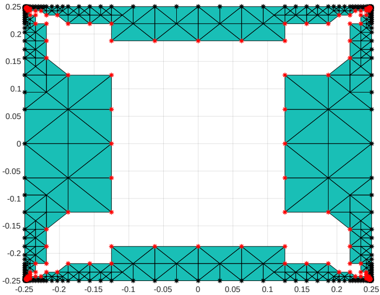
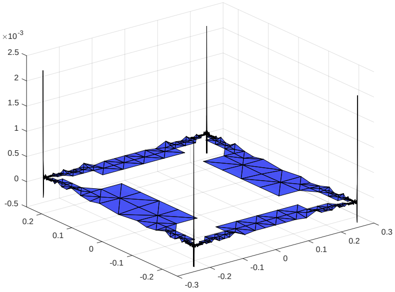
10^3

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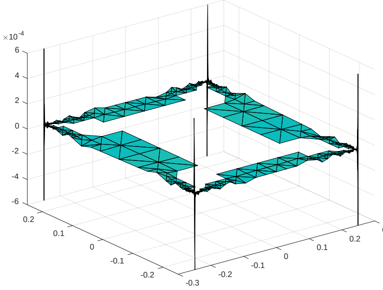
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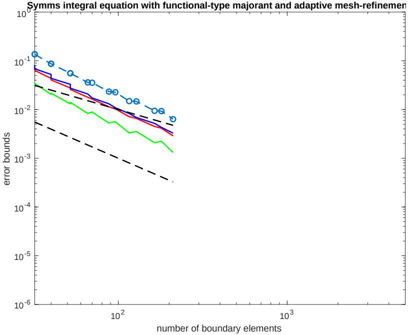
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
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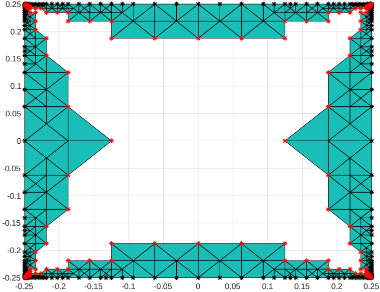
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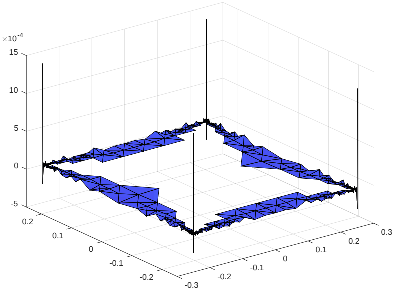
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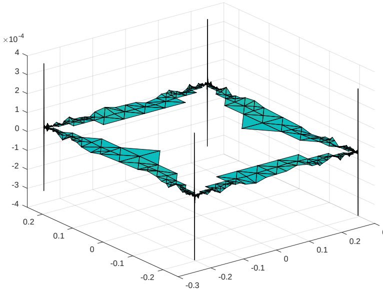
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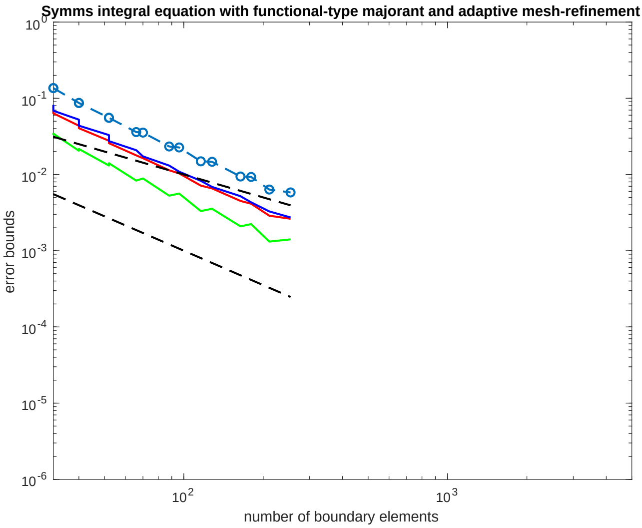
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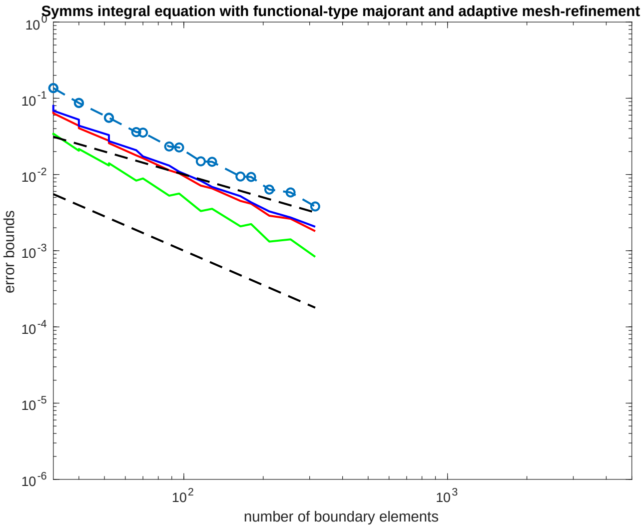
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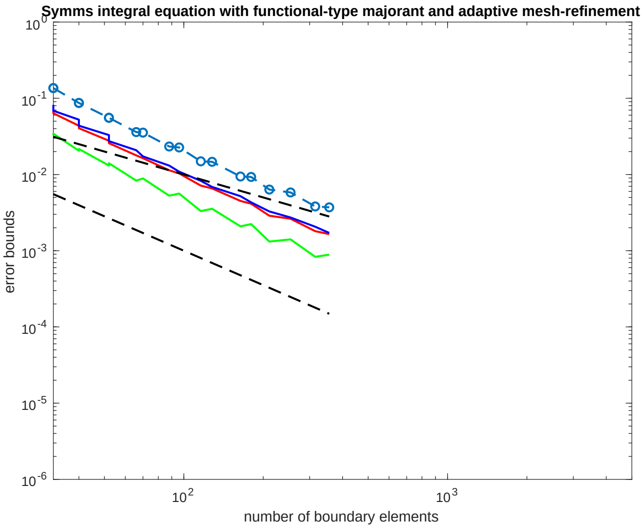
10^{3}
number of boundary elements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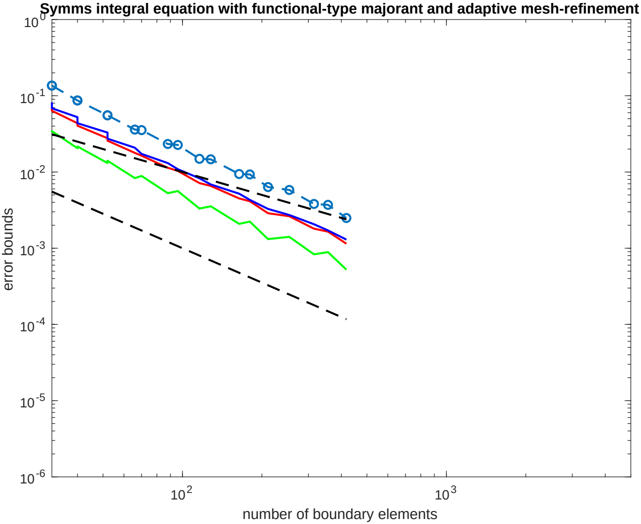
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error bounds
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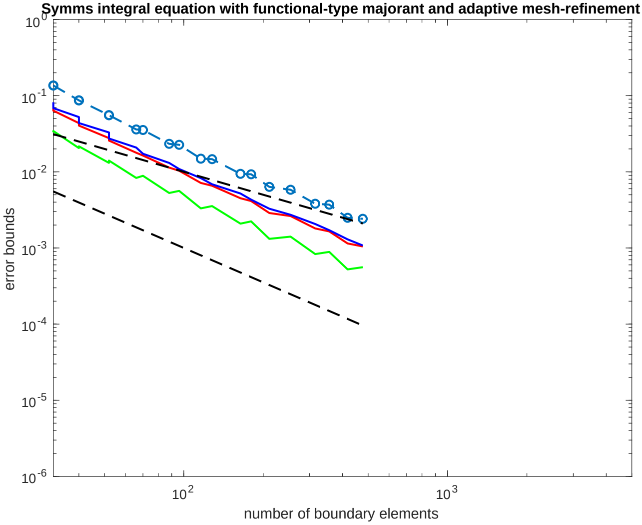
number of boundary elements


Symms integral equation with functional-type majorant and adaptive mesh-refinement
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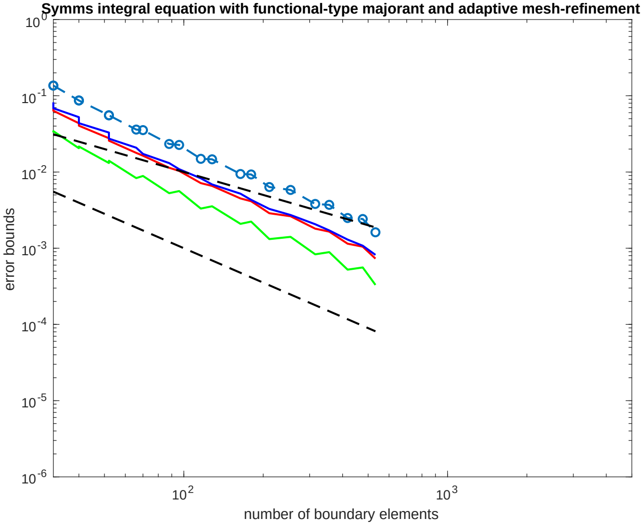
error bounds
10^2
10^3
number of boundary elements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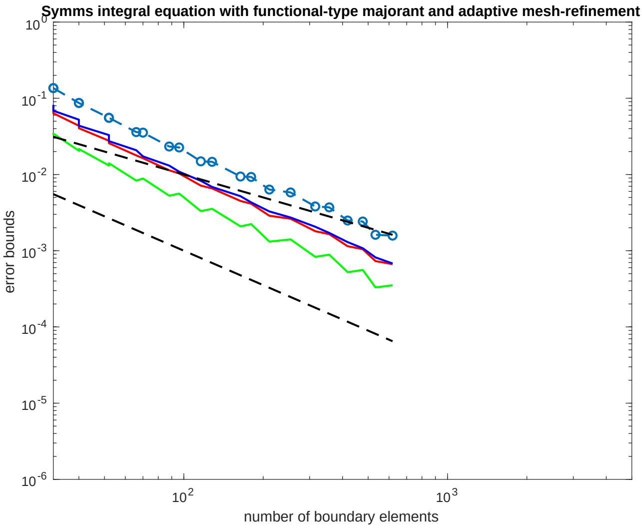
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error bounds
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10^3
number of boundary elements

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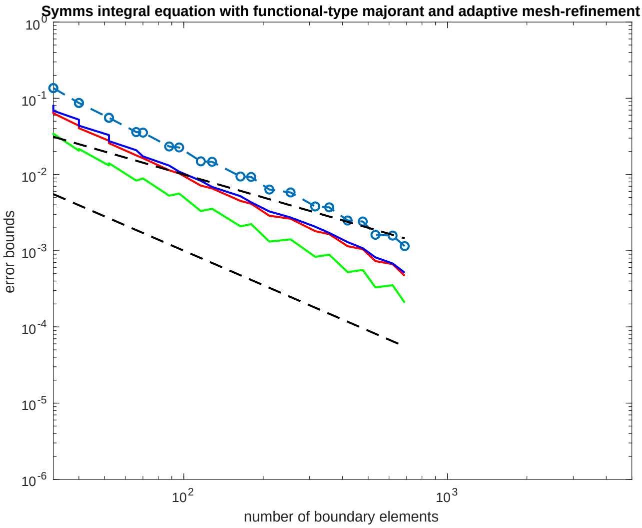

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BEM example

Motivation

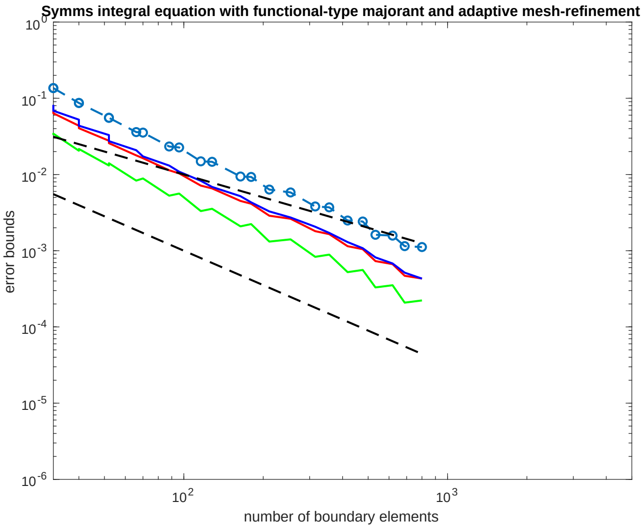
Error identity

Majorant

Minorant

Numerical experiments

Outlook



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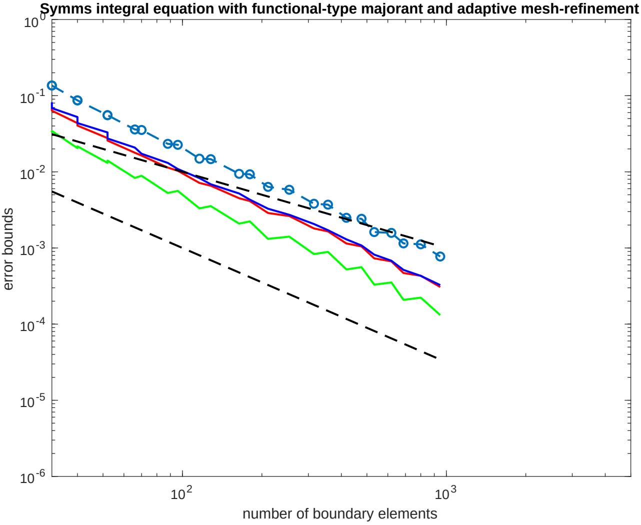

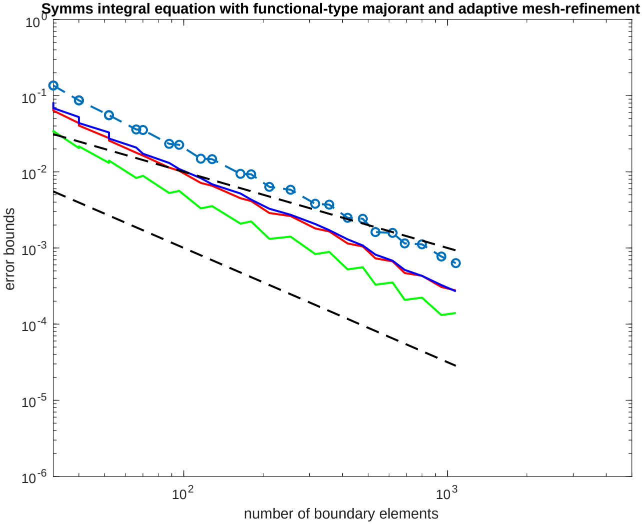
Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
number of boundary elements
10^{0}
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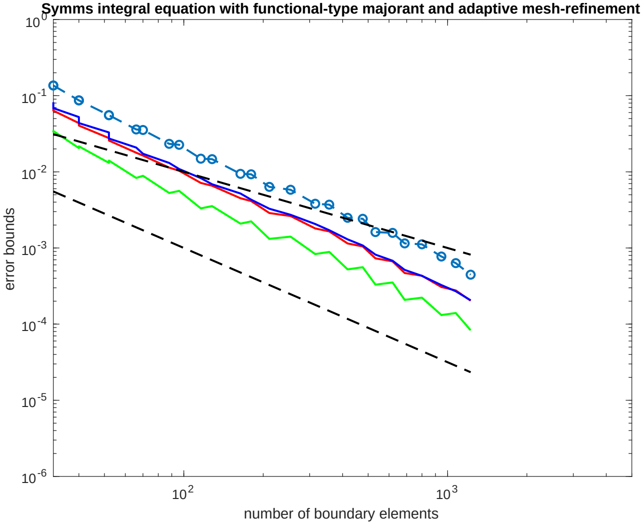

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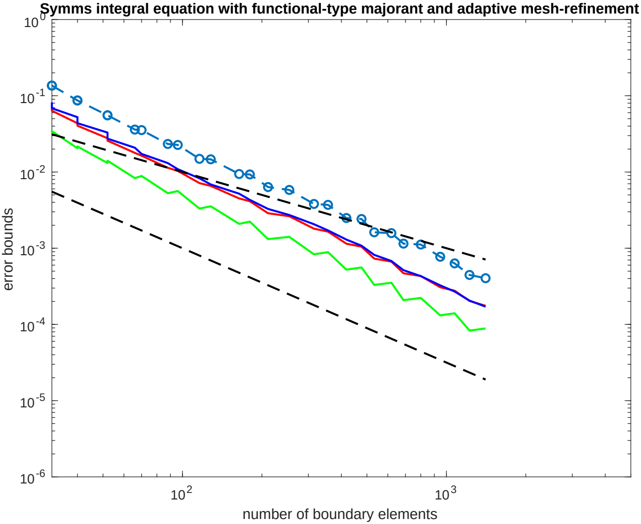

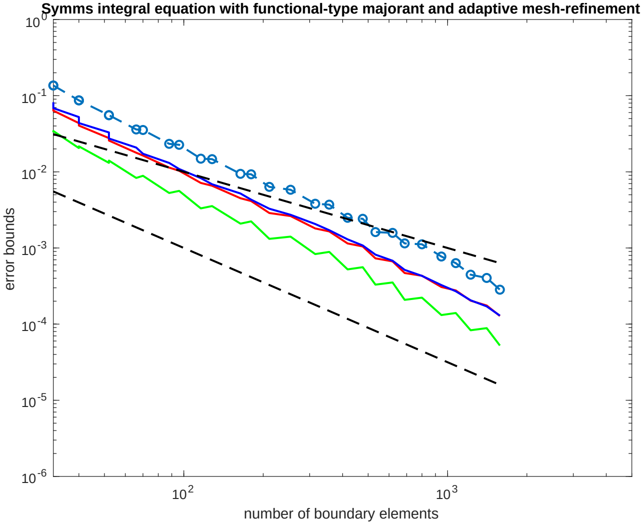
Symms integral equation with functional-type majorant and adaptive mesh-refinement
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
error bounds
10^2
10^3
number of boundary elements

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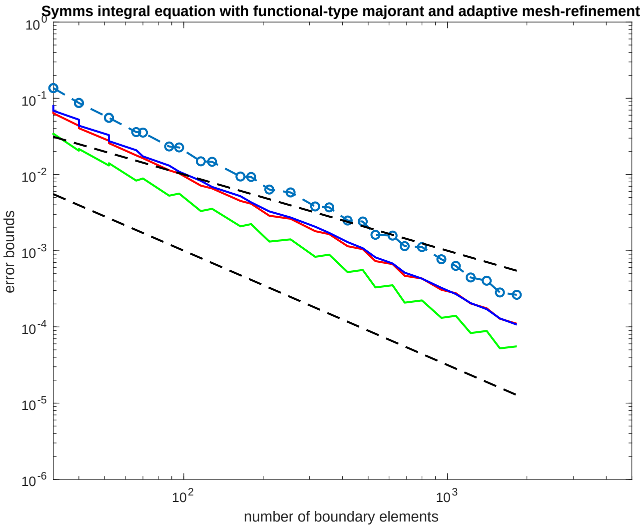

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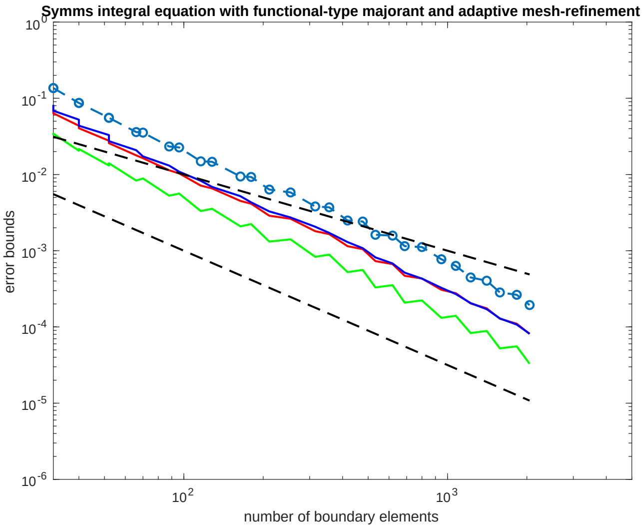

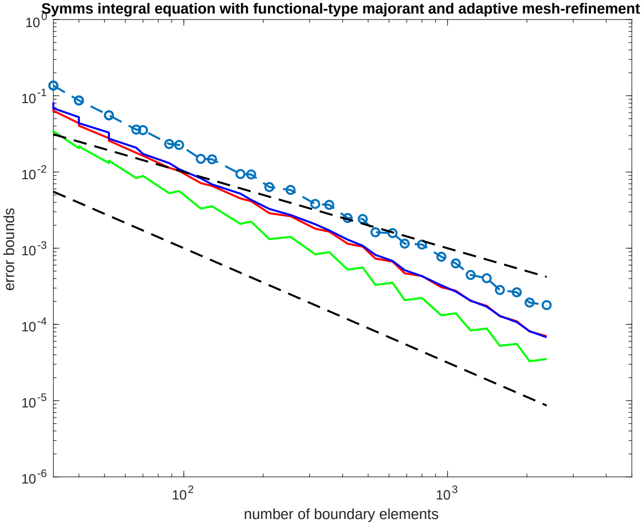
Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
number of boundary elements
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^2
10^3

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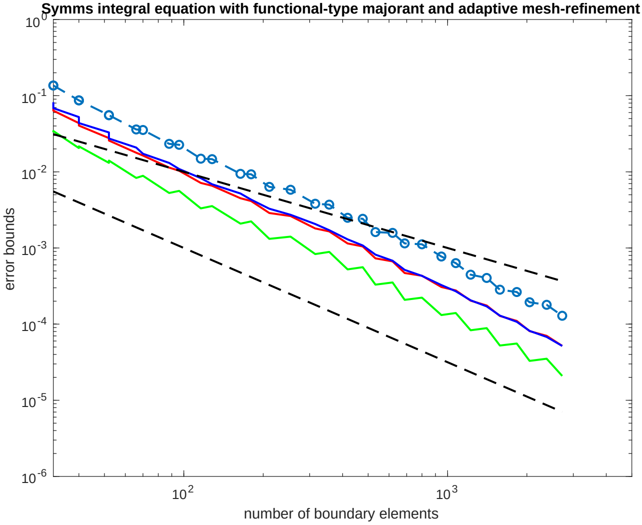

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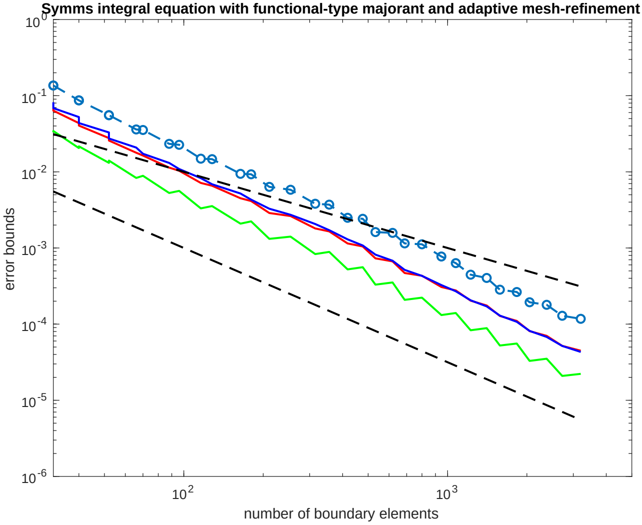

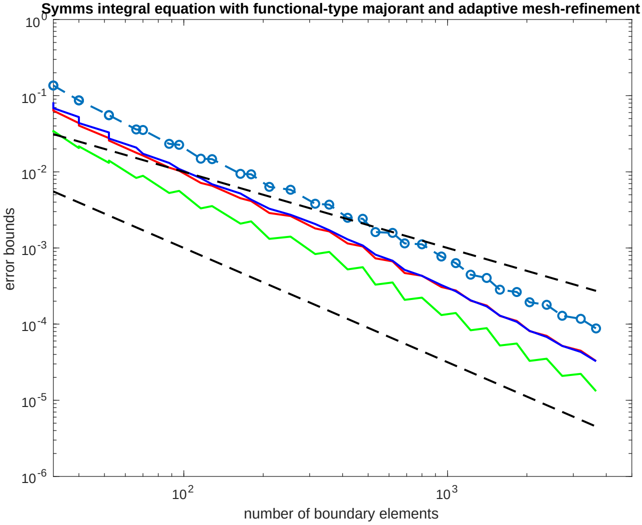
Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
number of boundary elements
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10^{-1}
10^{-2}
10^{-3}
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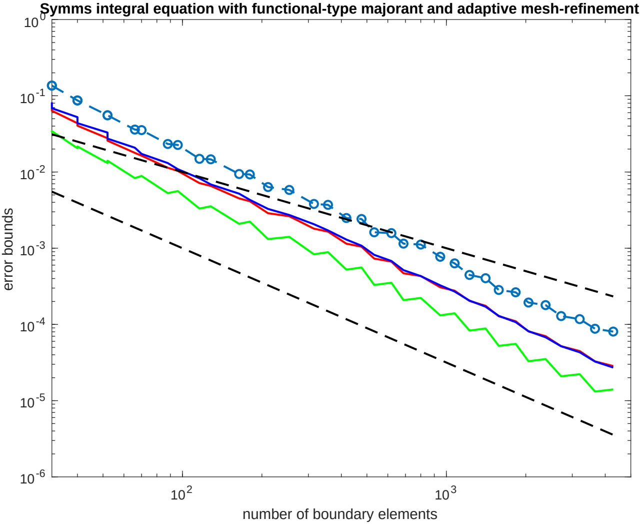
10^2
10^3


Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
number of boundary elements
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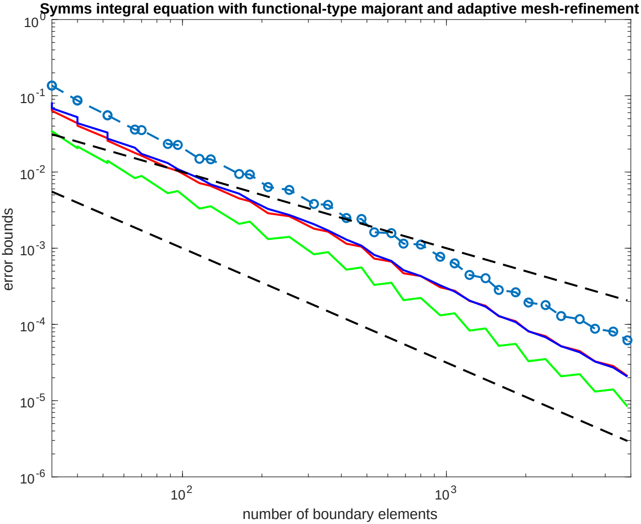
10^2
10^3


Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
number of boundary elements
10^{0}
10^{-1}
10^{-2}
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10^{2}
10^{3}

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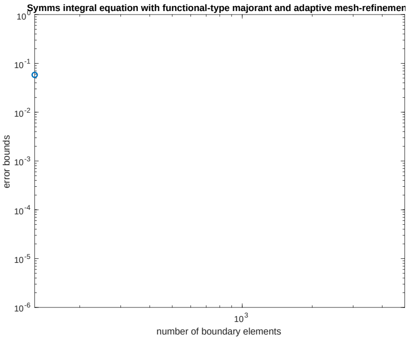
L-Shaped


Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
10^{0}
10^{-1}
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10^{-5}
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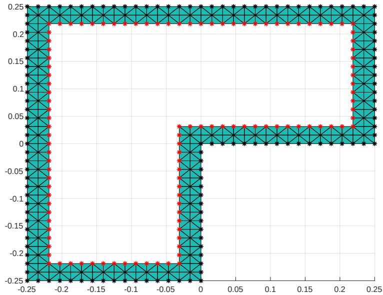
10^{3}
number of boundary elements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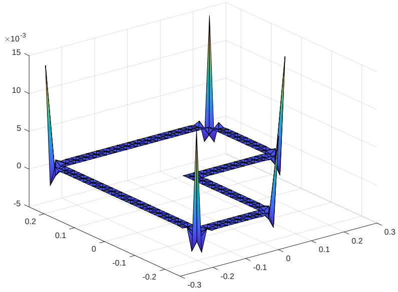
0.2
0.25


$\times 10^{-3}$
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10
5
0
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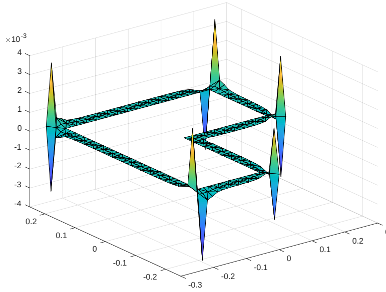
$\times 10^{-3}$
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1
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-1
-2
-3
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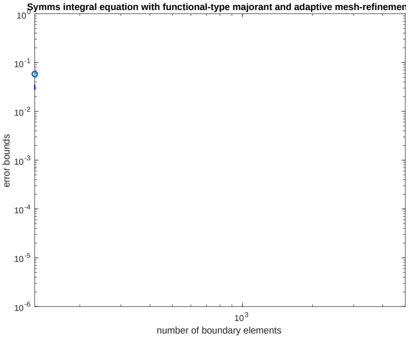
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
10^0
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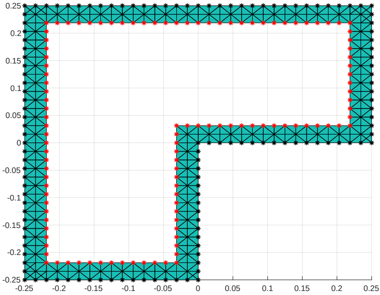
10^{-5}
10^{-6}
10^3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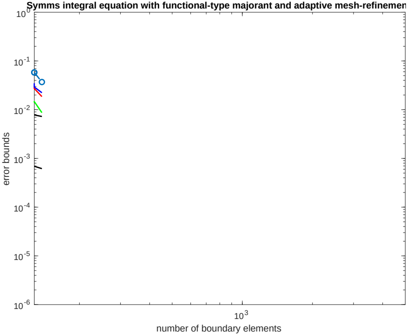

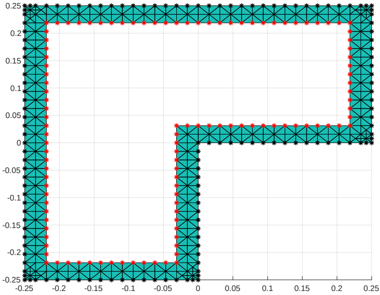

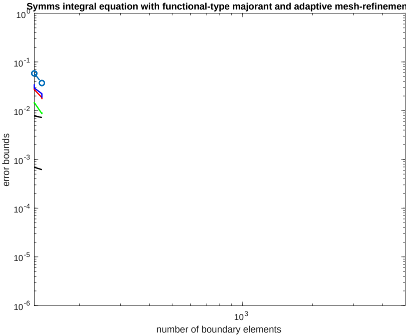
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
10^0
10^{-1}
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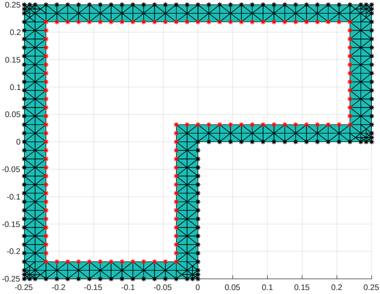
10^{-4}
10^{-5}
10^{-6}
10^3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-0.2
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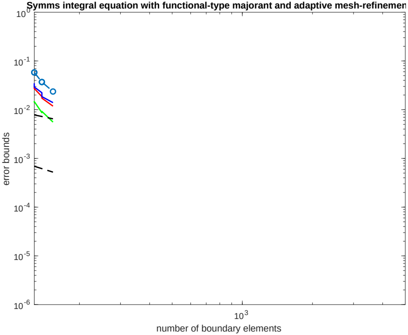
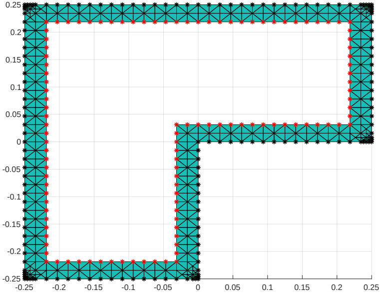

Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements



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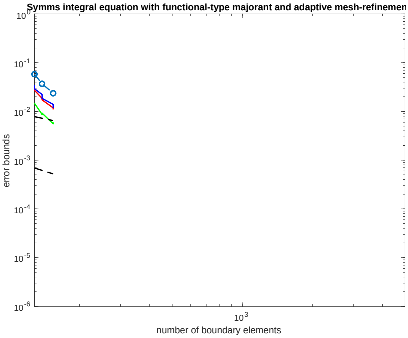

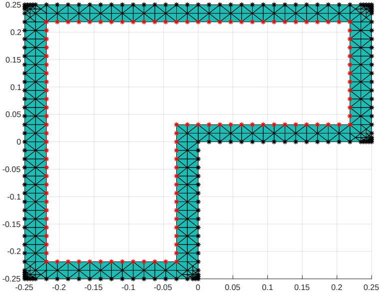

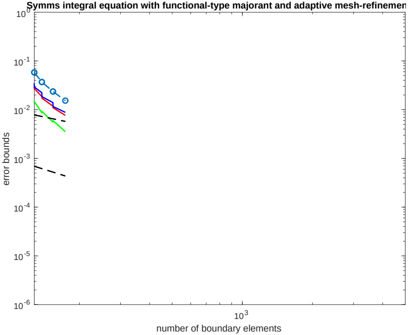
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
10^0
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10^-3
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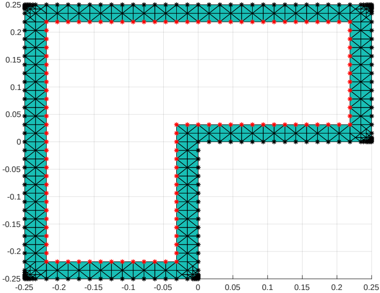
10^-5
10^-6
10^3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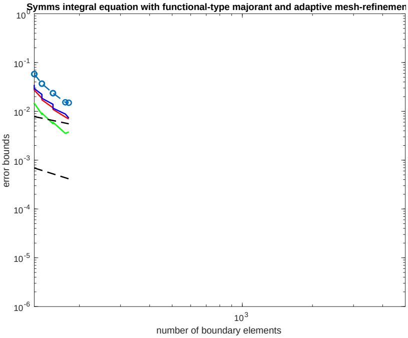
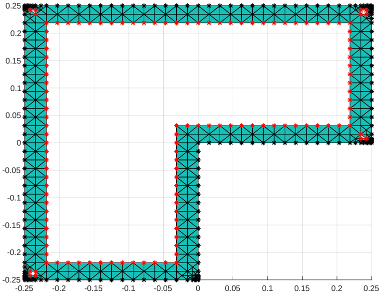

Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements



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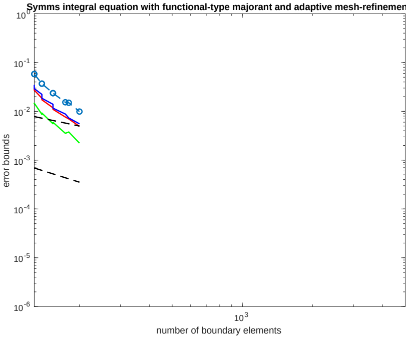

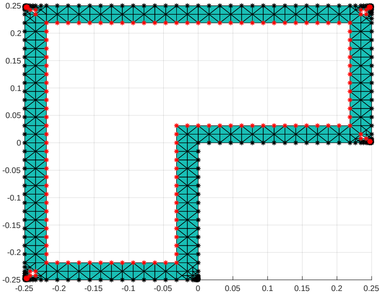

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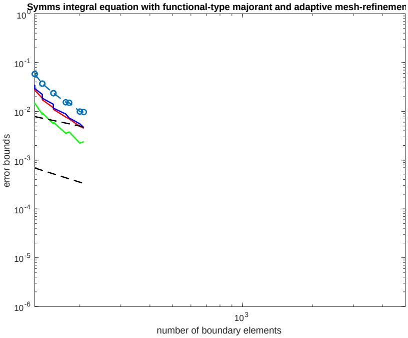

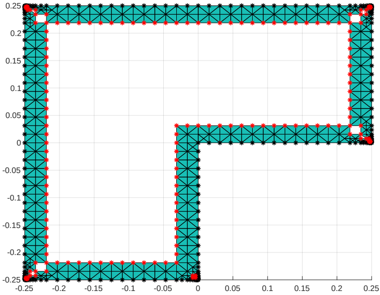

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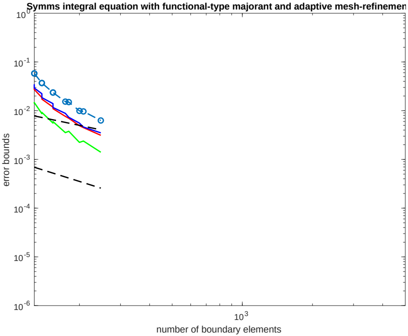

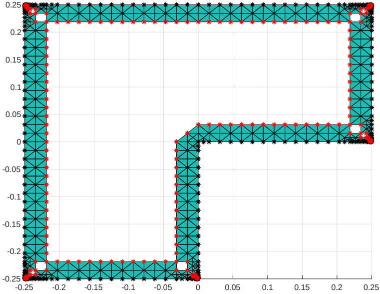

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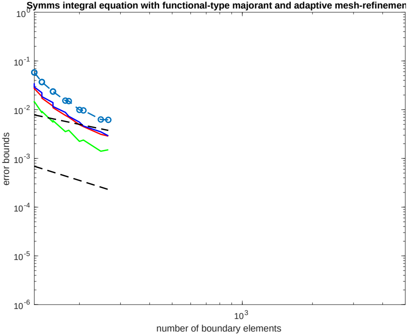

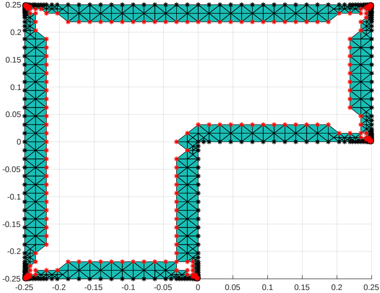

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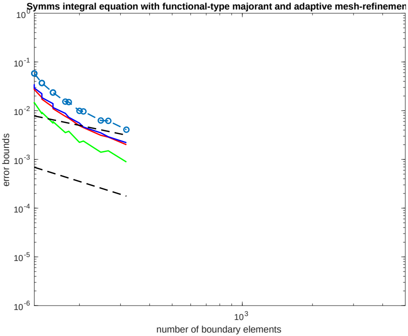

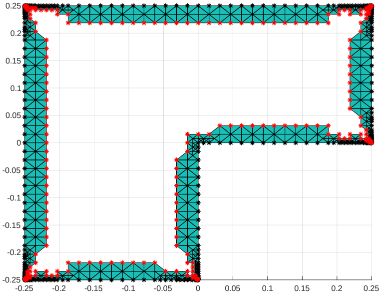

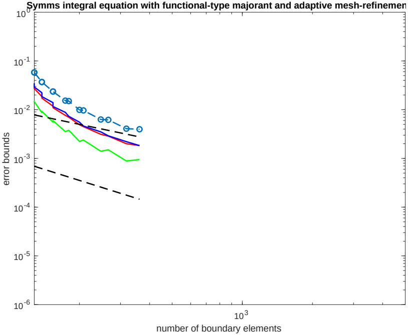
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
10^{0}
10^{-1}
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10^{-3}
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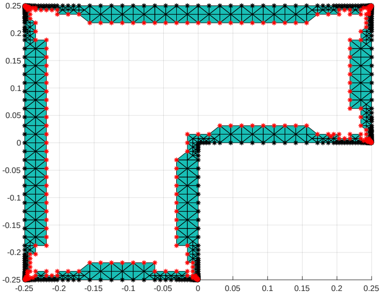
10^{-6}
10^{3}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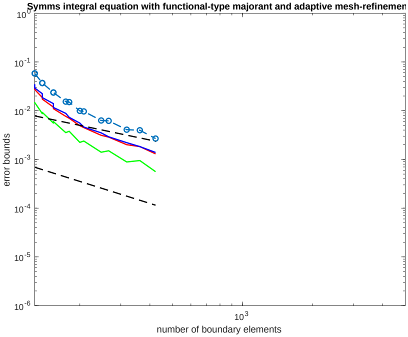

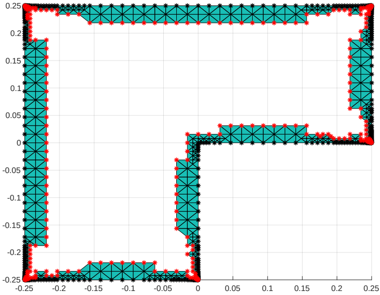

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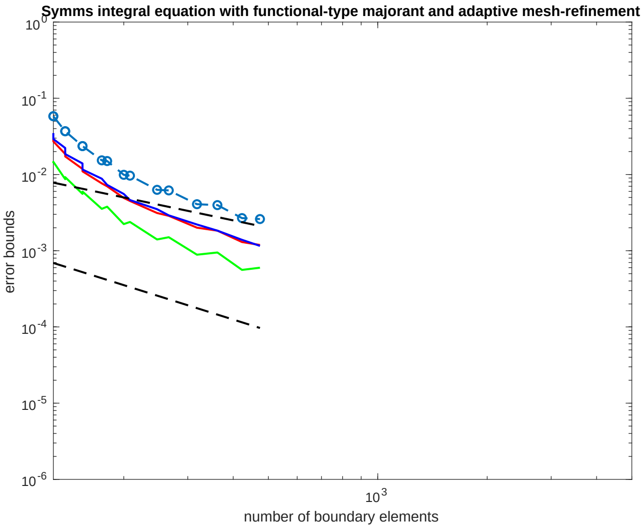

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BEM example

Motivation

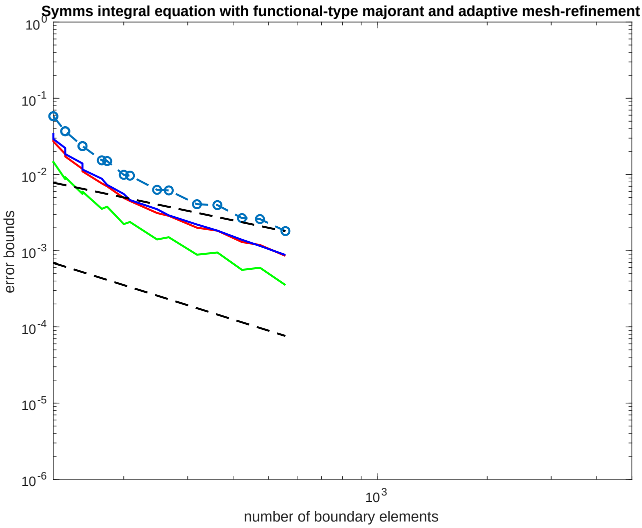
Error identity

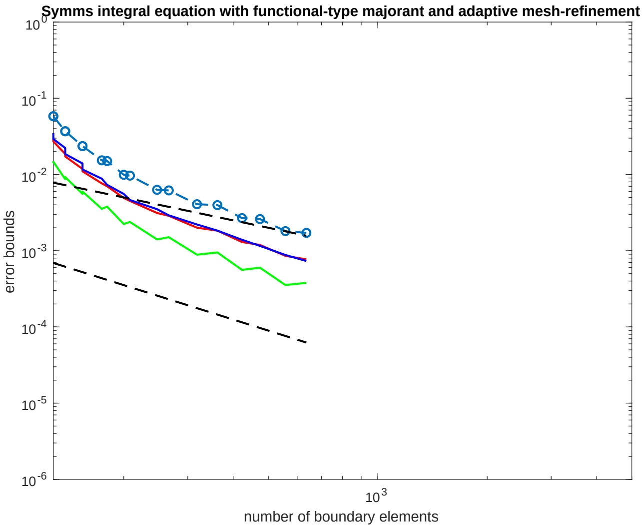
Majorant

Minorant

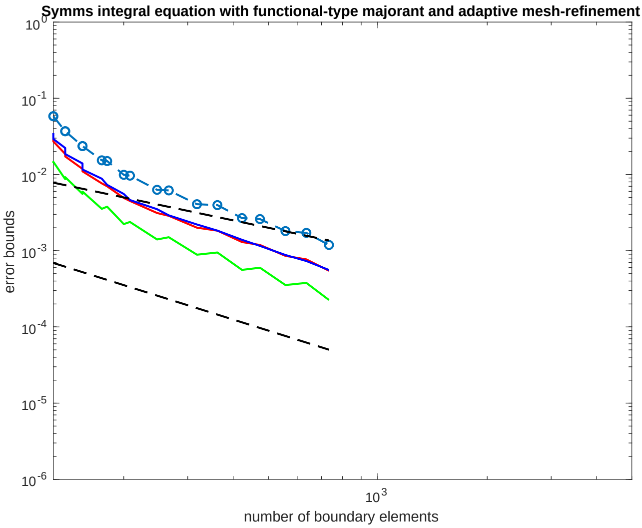
Numerical experiments

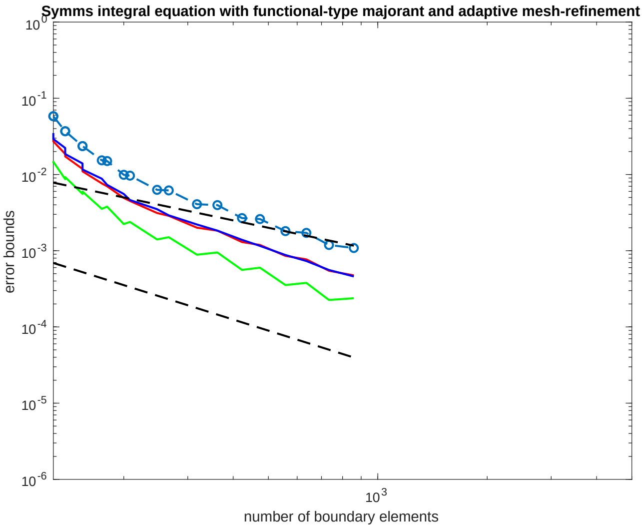
Outlook




Symms integral equation with functional-type majorant and adaptive mesh-refinement
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
error bounds
10^3
number of boundary elements

TECHNISCHE UNIVERSITÄT WIEN




Symms integral equation with functional-type majorant and adaptive mesh-refinement
10^0
10^-1
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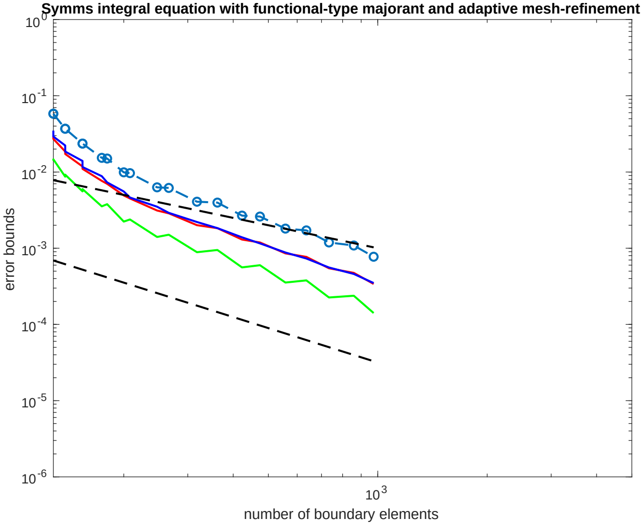
10^-3
10^-4
10^-5
10^-6
error bounds
10^3
number of boundary elements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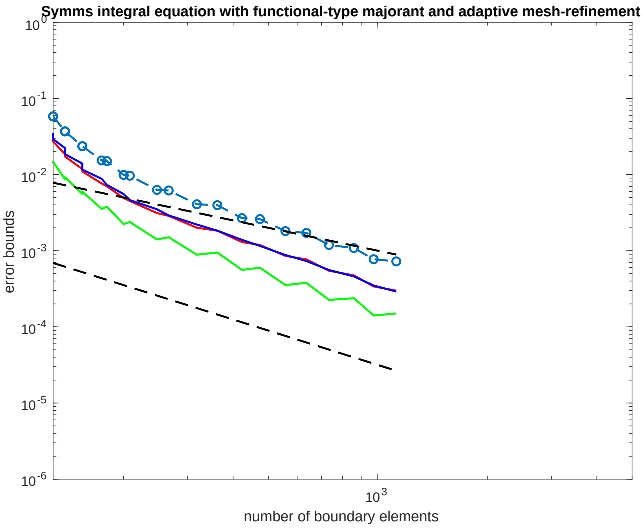
10^3


Symms integral equation with functional-type majorant and adaptive mesh-refinement
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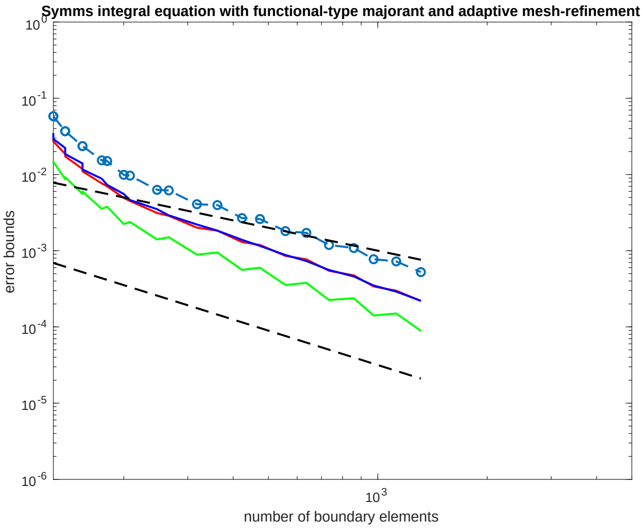
10^{-6}
10^3


Symms integral equation with functional-type majorant and adaptive mesh-refinement
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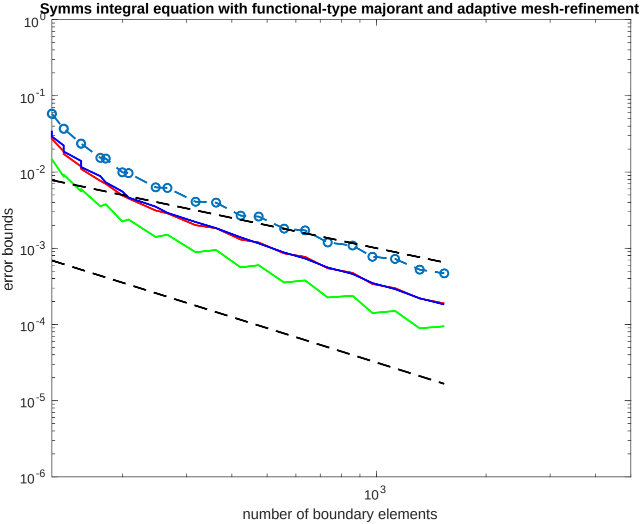
10^{-6}
10^3


Symms integral equation with functional-type majorant and adaptive mesh-refinement
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10^{-6}
10^{3}
number of boundary elements

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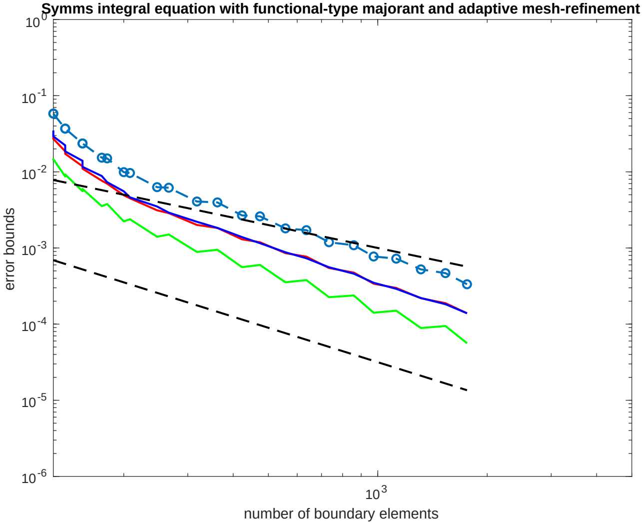

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BEM example

Motivation

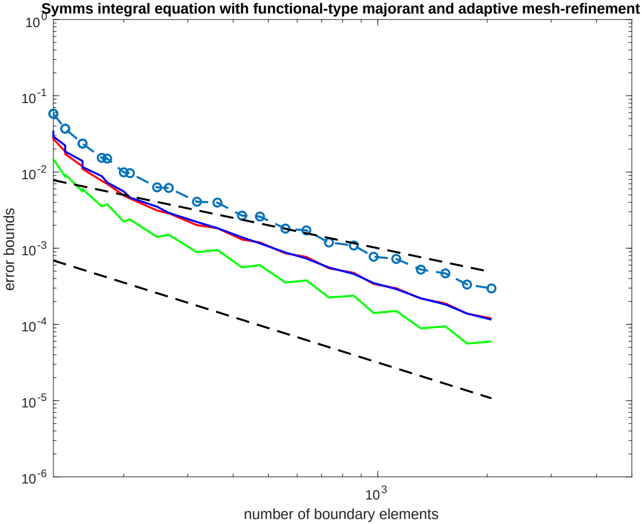
Error identity

Majorant

Minorant

Numerical experiments

Outlook



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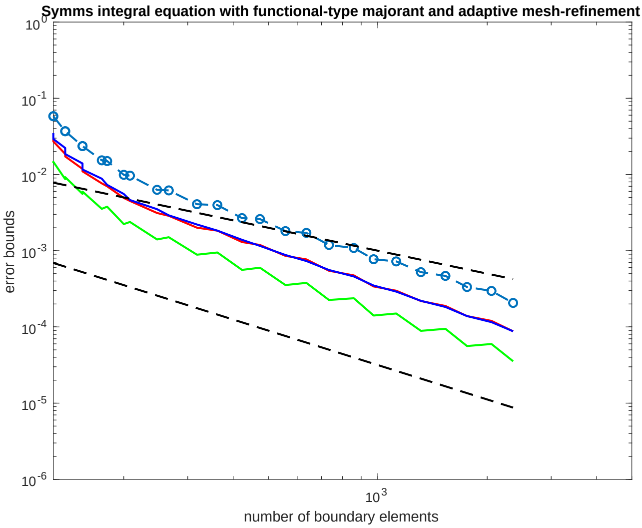
- BEM example
- Motivation
- Error identity
  - Majorant
  - Minorant
- **Numerical experiments**
- Outlook



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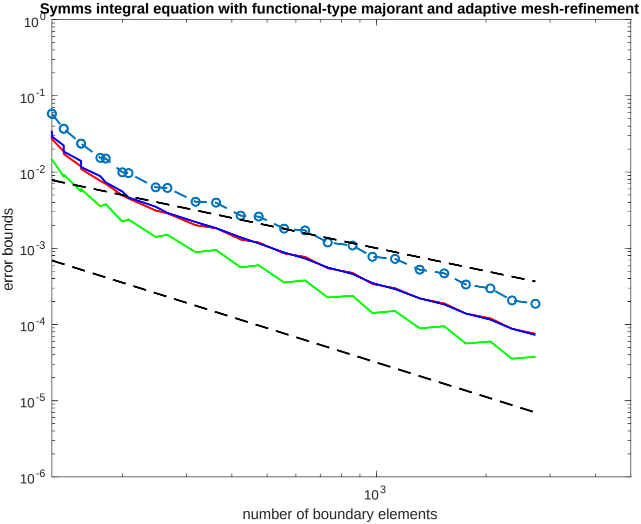
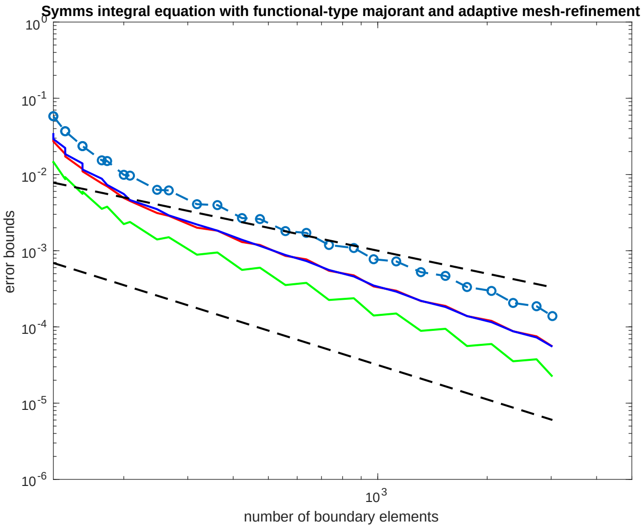
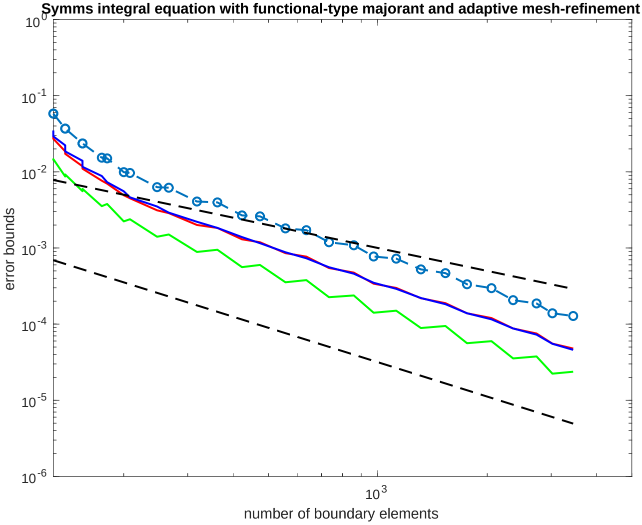



Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^3
number of boundary elements


Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
number of boundary elements
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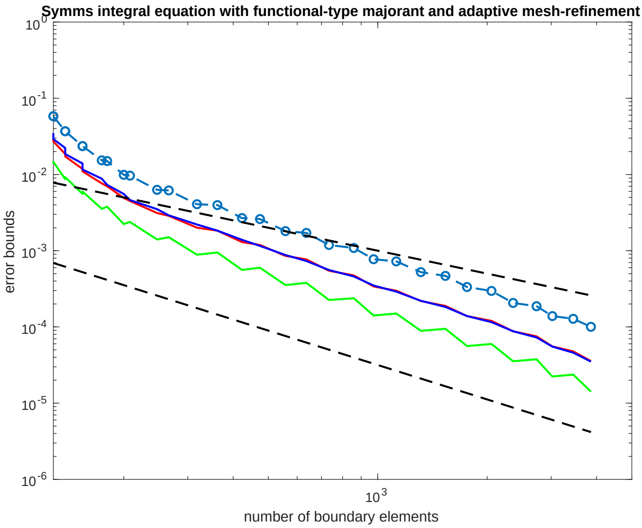
10^{-6}
10^3


Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
number of boundary elements
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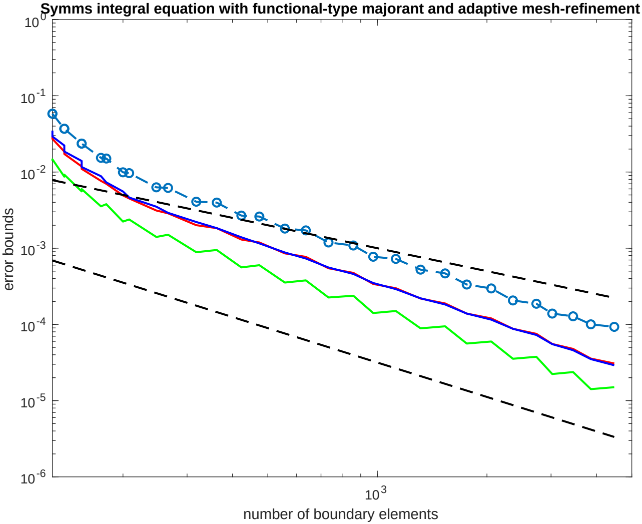
10^3


Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
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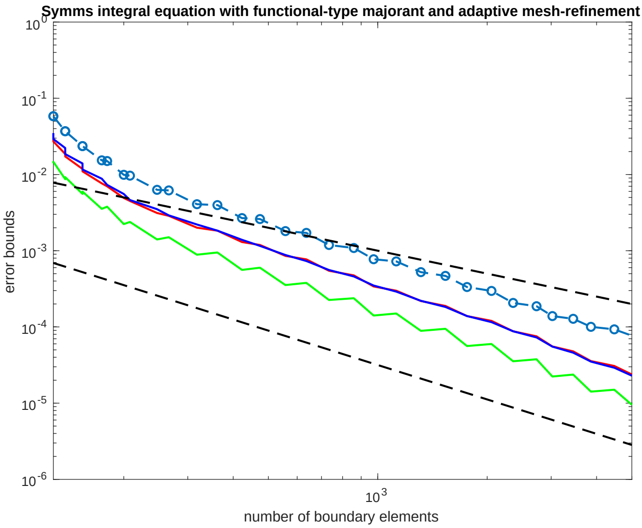
10^{3}
number of boundary elements


Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
number of boundary elements
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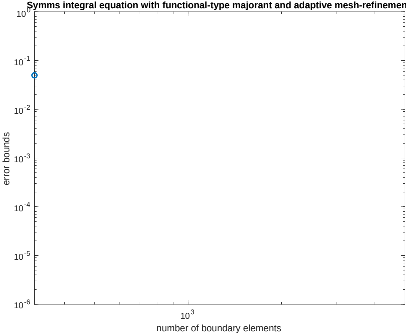
Pacman


Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
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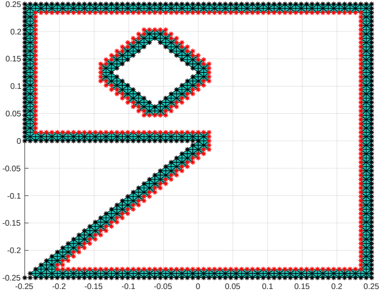
10^3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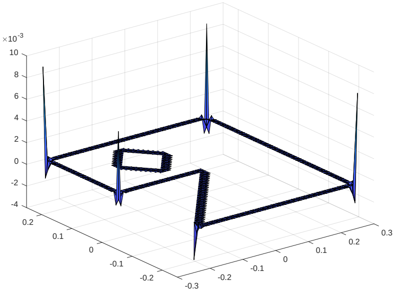
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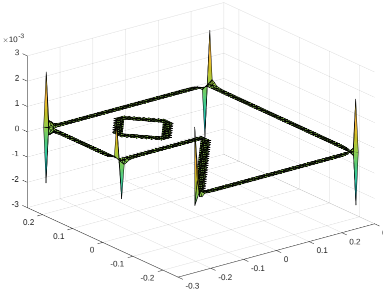
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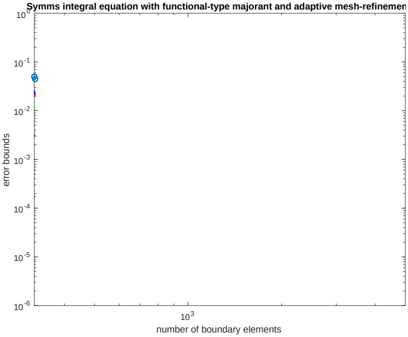
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
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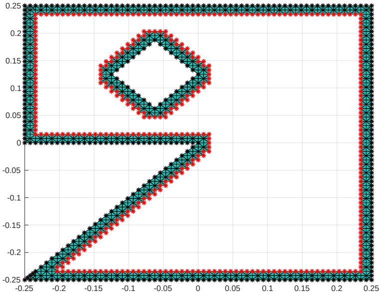

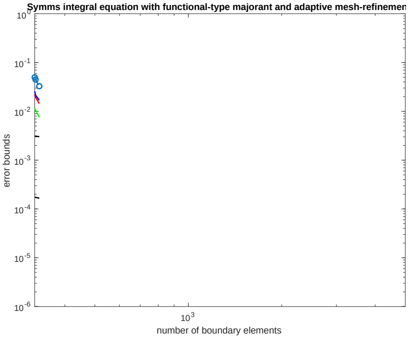
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
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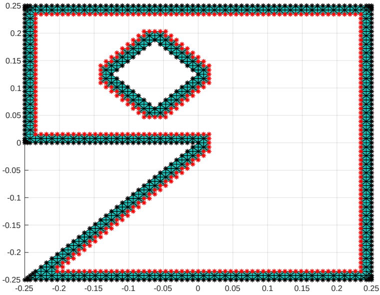
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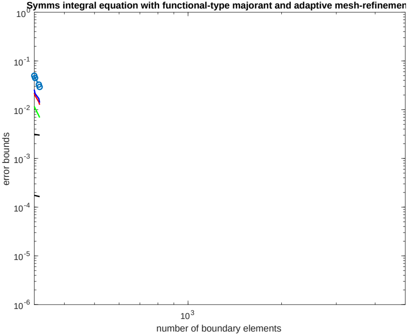
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
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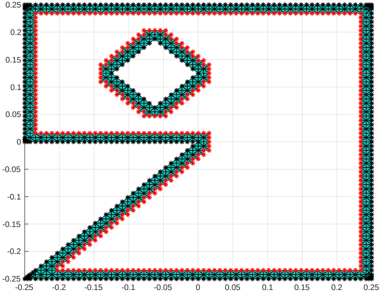
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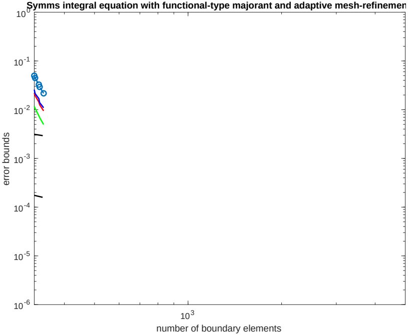

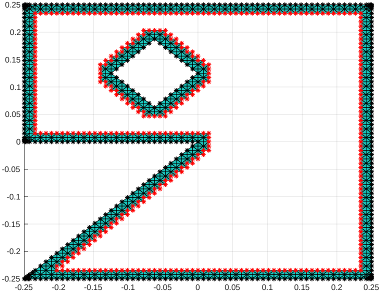

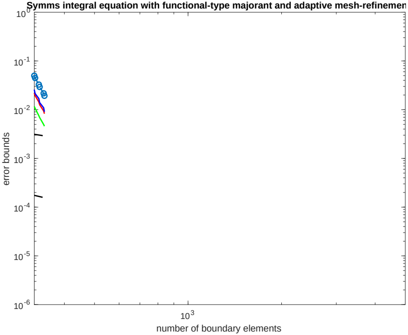
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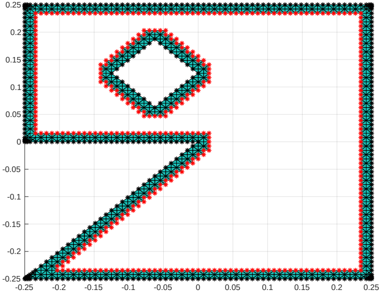
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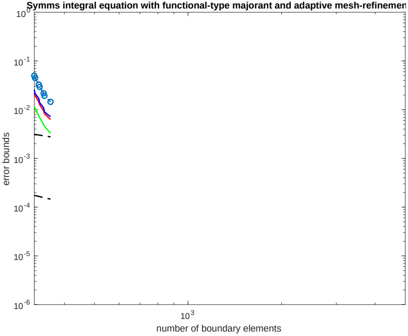
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
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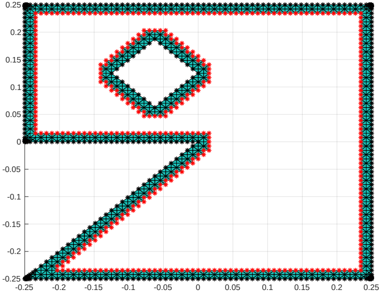
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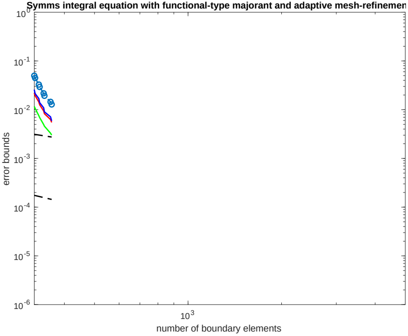
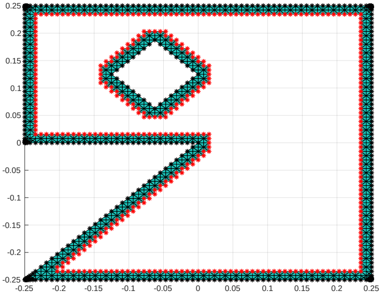
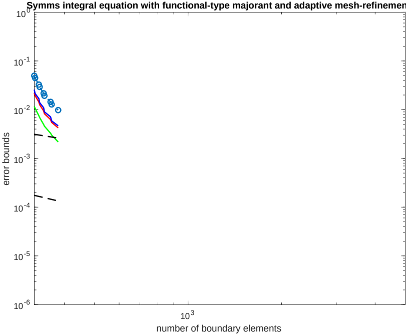
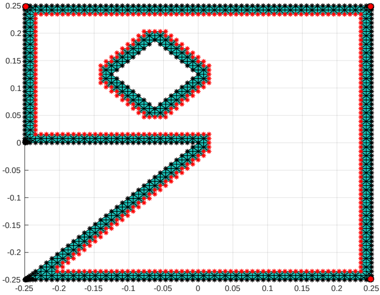
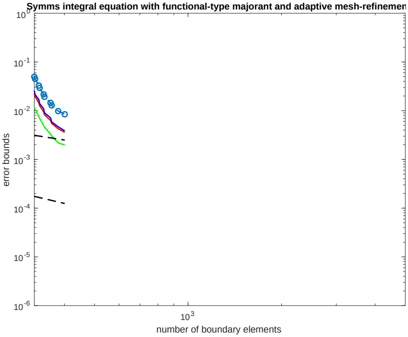





Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements




Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
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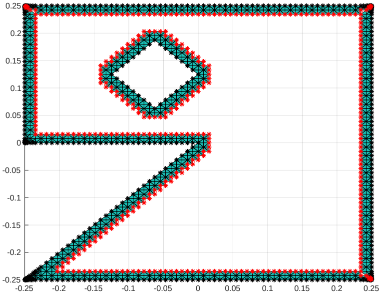
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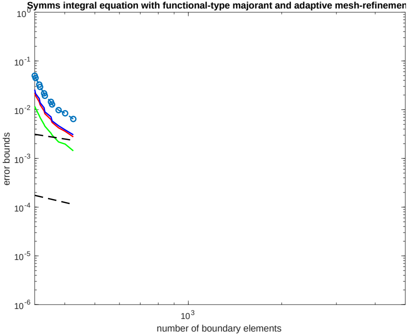
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
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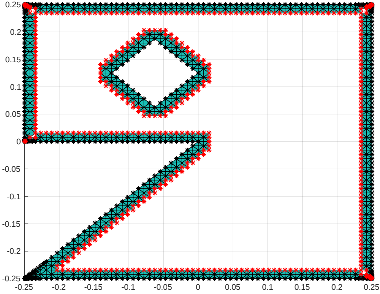
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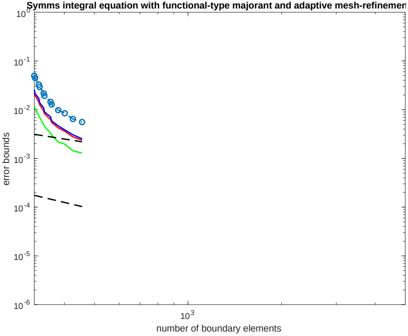

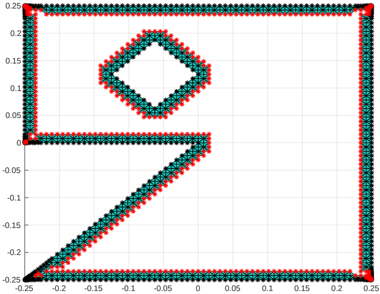

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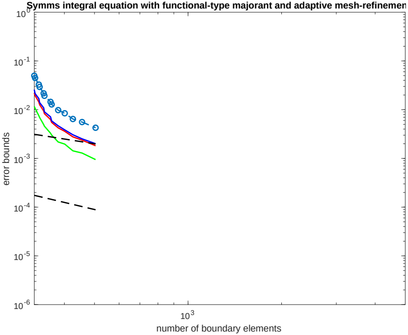

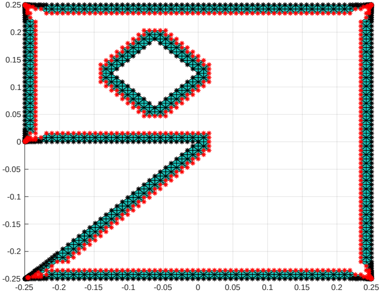

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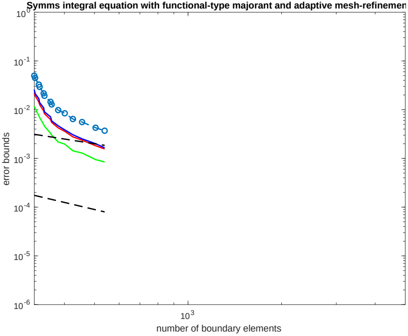

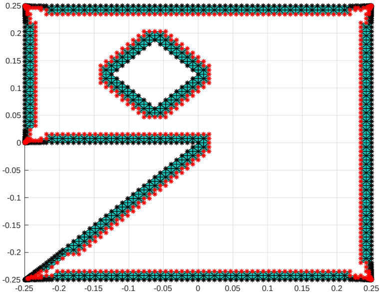

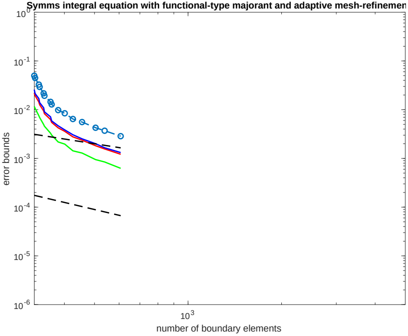
Symms integral equation with functional-type majorant and adaptive mesh-refinemen
error bounds
number of boundary elements
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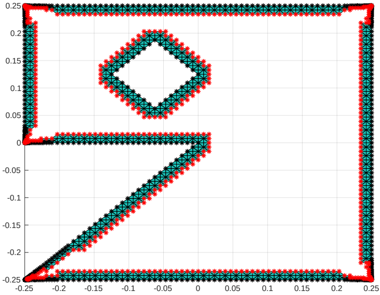
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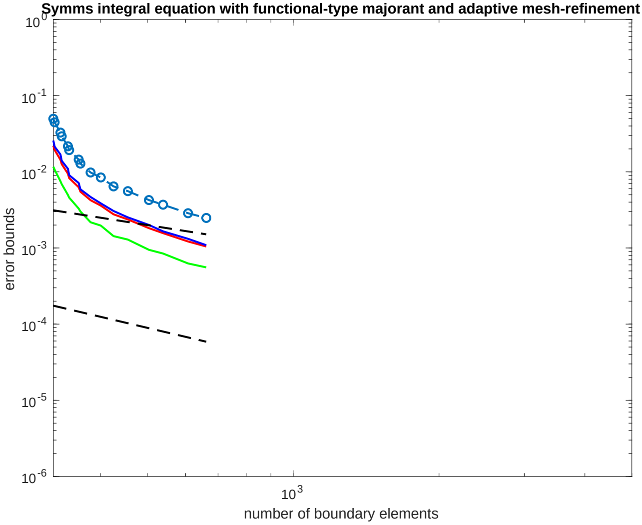
Symms integral equation with functional-type majorant and adaptive mesh-refinement
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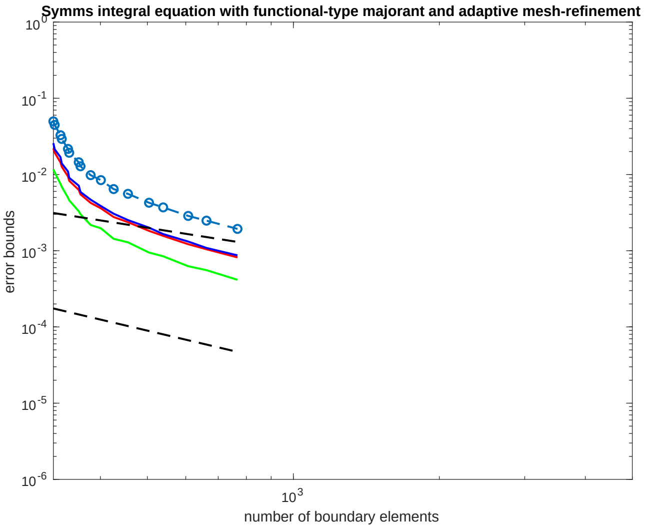
10^3
number of boundary elements


Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
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10^3
number of boundary elements

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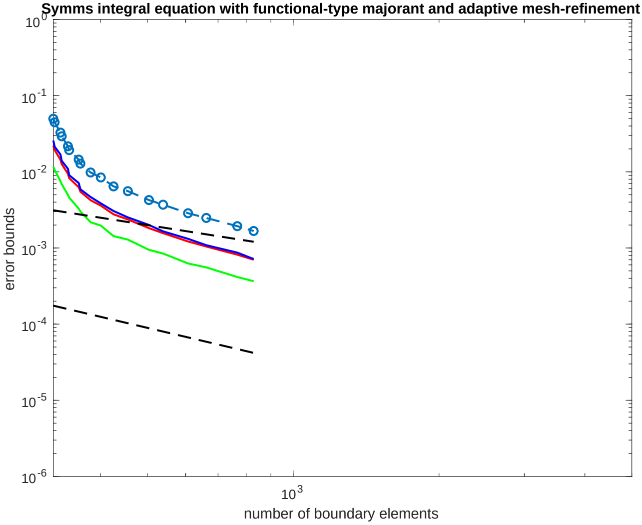

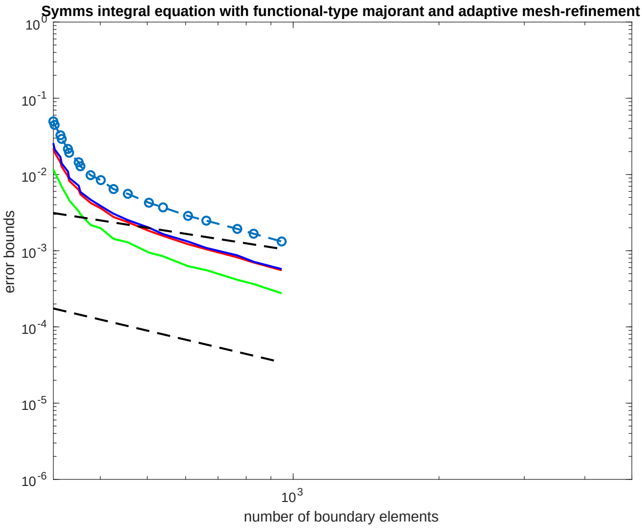
Symms integral equation with functional-type majorant and adaptive mesh-refinement
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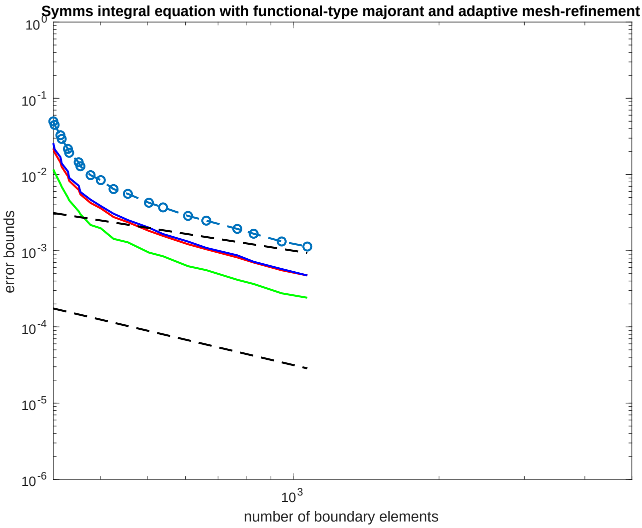
error bounds
10^3
number of boundary elements


Symms integral equation with functional-type majorant and adaptive mesh-refinement
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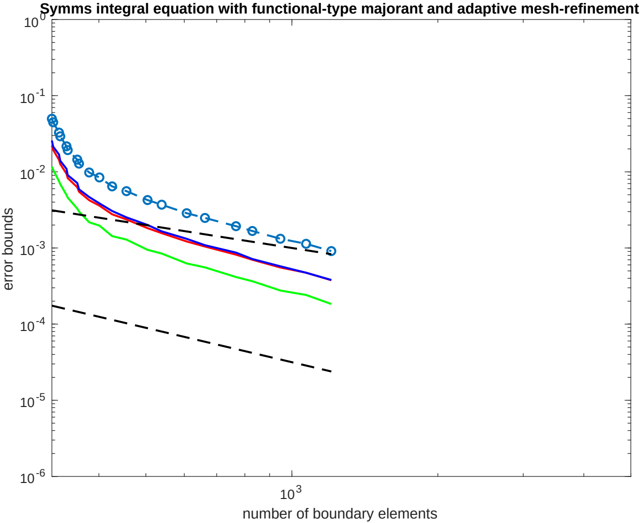
error bounds
10^{3}
number of boundary elements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BEM example

Motivation

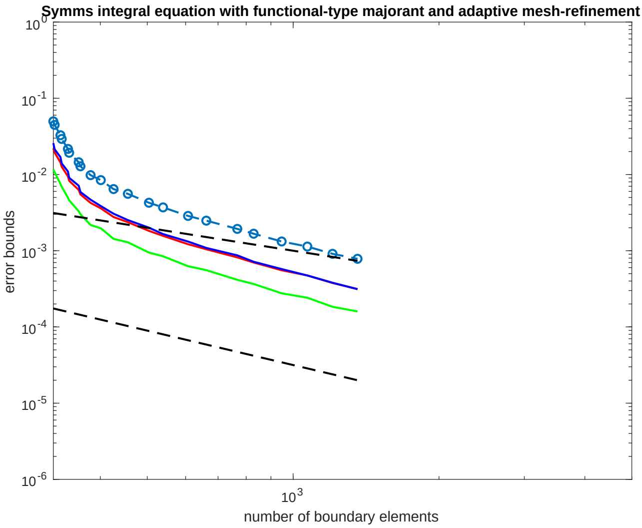
Error identity

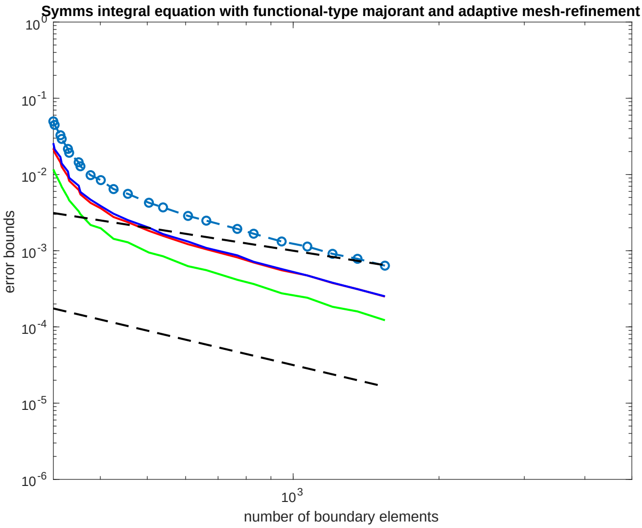
Majorant

Minorant

Numerical experiments

Outlook

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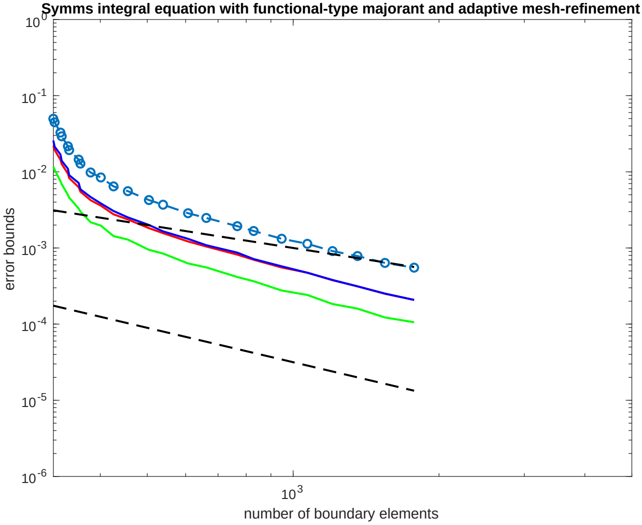
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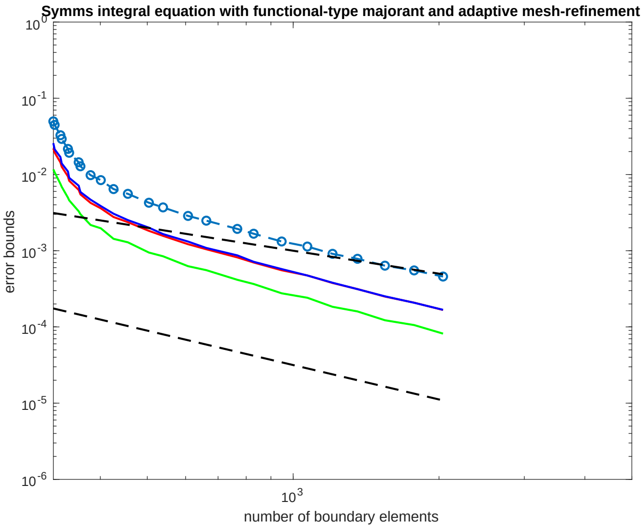
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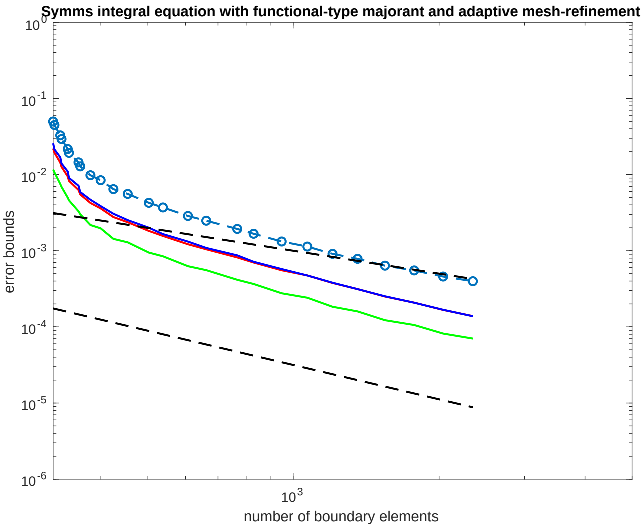

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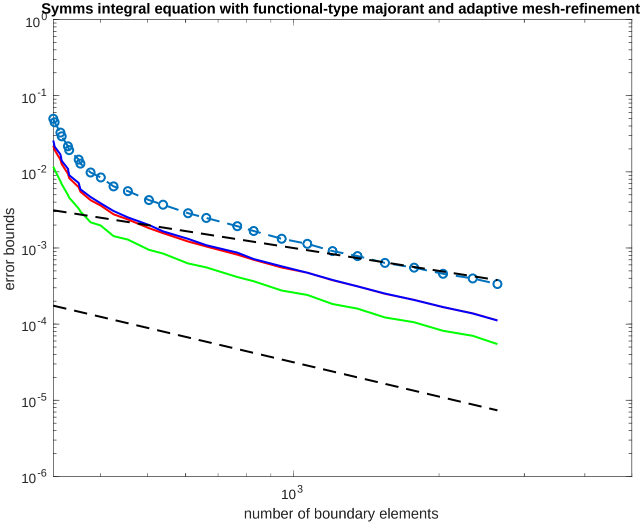

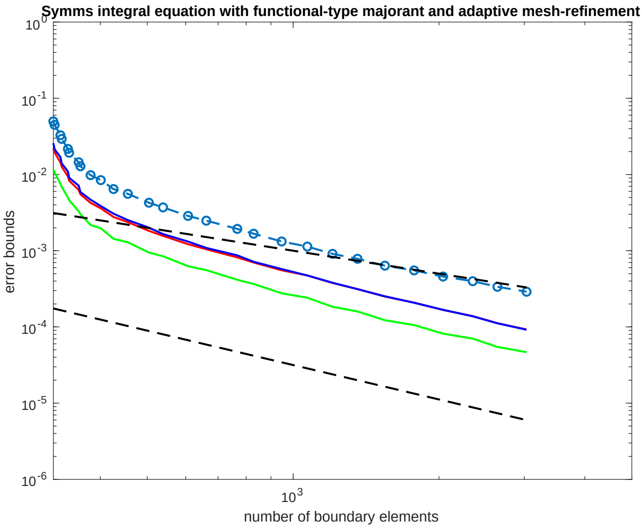
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error bounds
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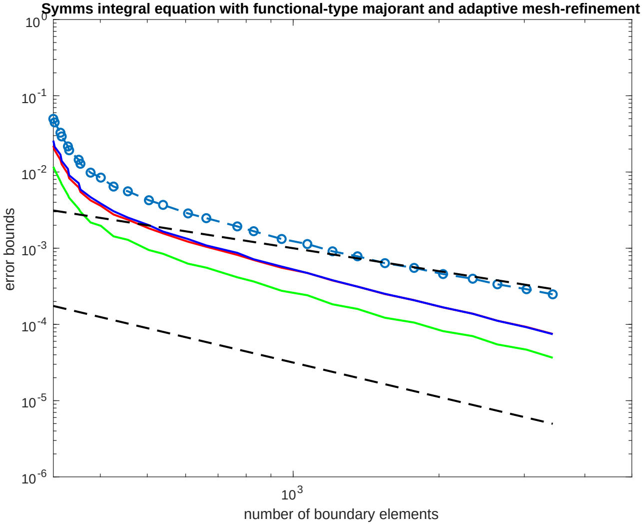

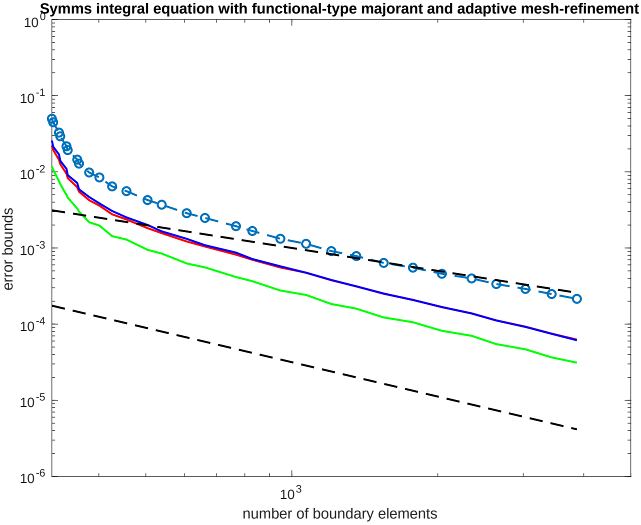
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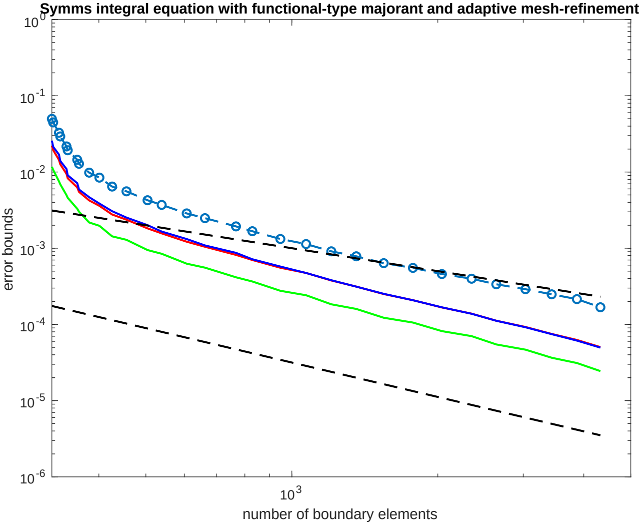
Numerical experiments

Outlook



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BEM example

Motivation

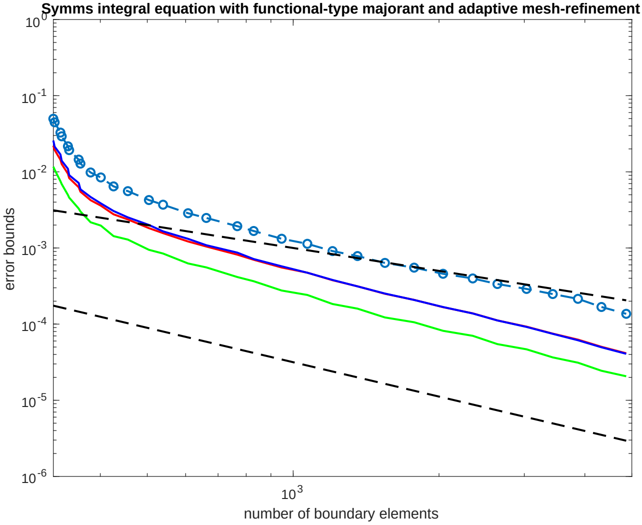
Error identity

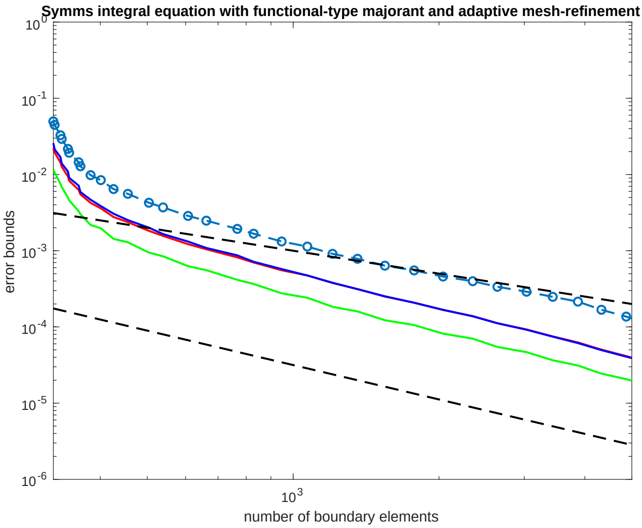
Majorant

Minorant

Numerical experiments

Outlook




Symms integral equation with functional-type majorant and adaptive mesh-refinement
error bounds
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^3
number of boundary elements

Outlook

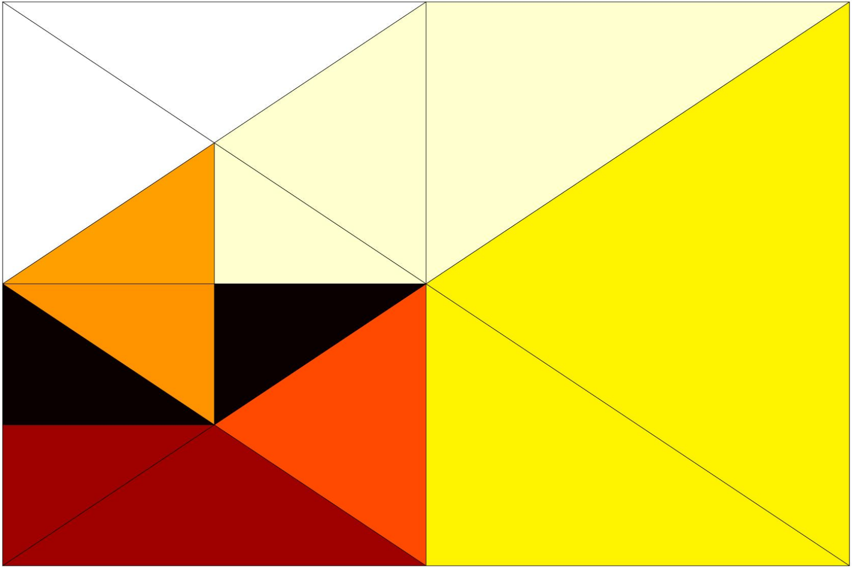
Future work

- Implementation of $P2$ AFEM (boundary residual approximation, oscillation errors)
- Solving the same (maj/min) problems with BEM?
- Implementation of direct method.
- 3D implementation

References

- S. Repin. A posteriori estimates for partial differential equations. Walter de Gruyter, Berlin, 2008.
- S. Repin and S. Kurz. Basic Introduction into the Boundary Element Method and Related Functional Error Estimates. 27.10.2017.
- D. Pauly, S. Repin, D. Praetorius and S. Kurz. Private communication.
- M. Aurada, M. Ebner, M. Feischl, S. Ferraz-Leite, T. Führer, P. Goldenits, M. Karkulik, M. Mayr, D. Praetorius. HILBERT - a Matlab implementation of adaptive BEM (Release 3). May 2013.
- S. Funken, D. Praetorius, P. Wissgott. Efficient implementation of adaptive P1-FEM in Matlab.
- C. Bahriawati, C. Carstensen Three MATLAB implementations of the lowest-order Raviart-Thomas MFEM with a posteriori error control.

Questions?



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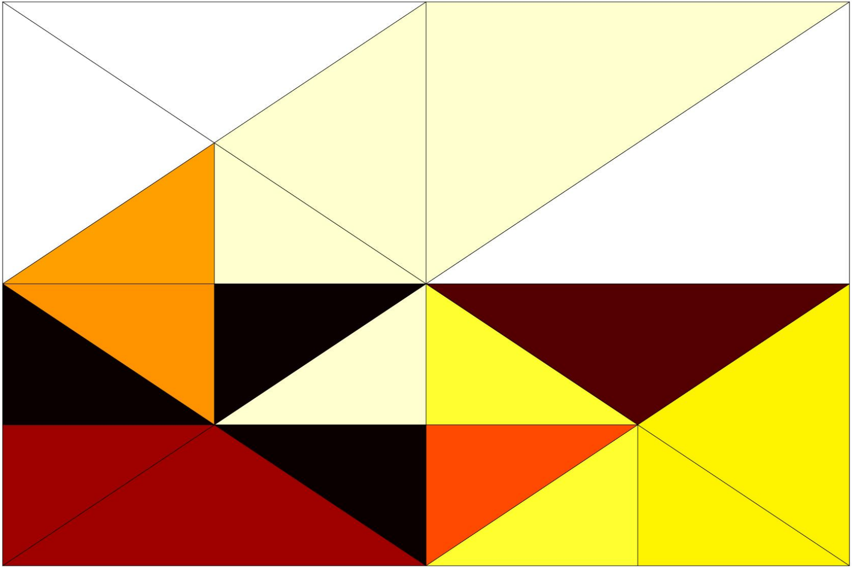

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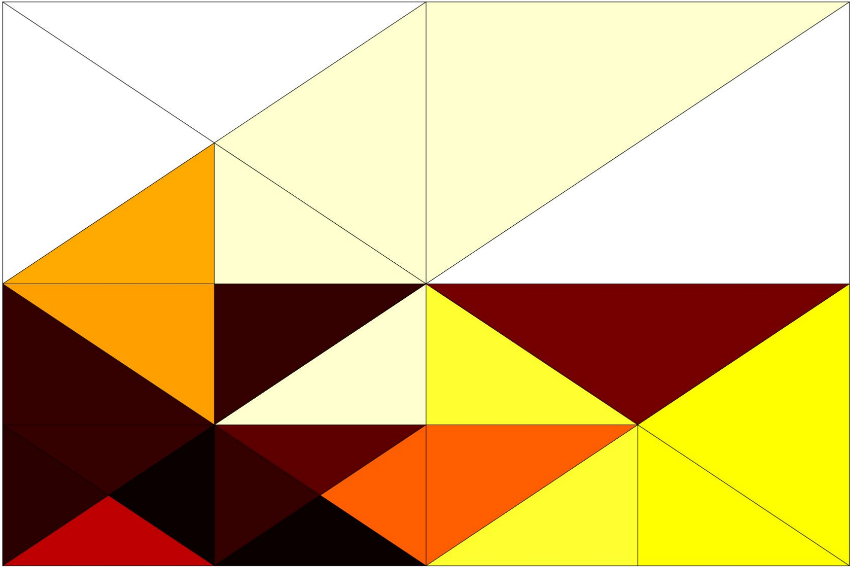

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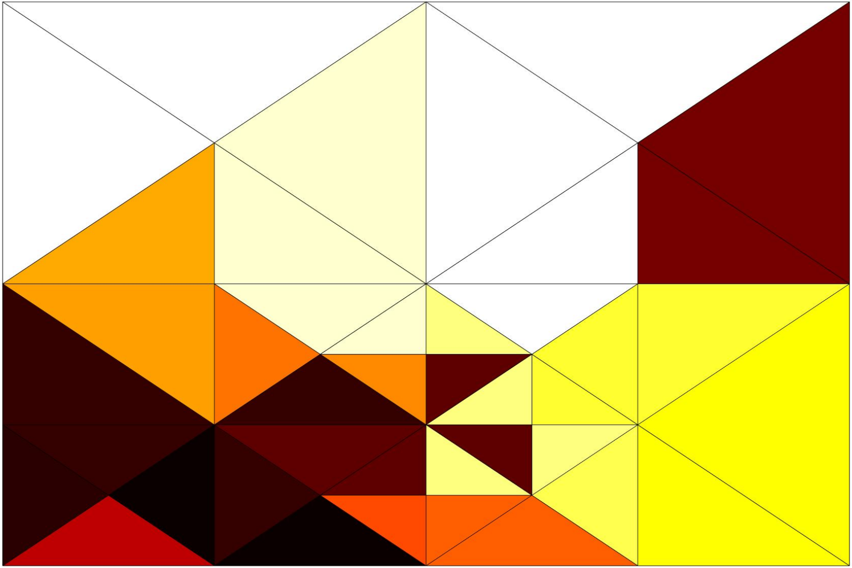

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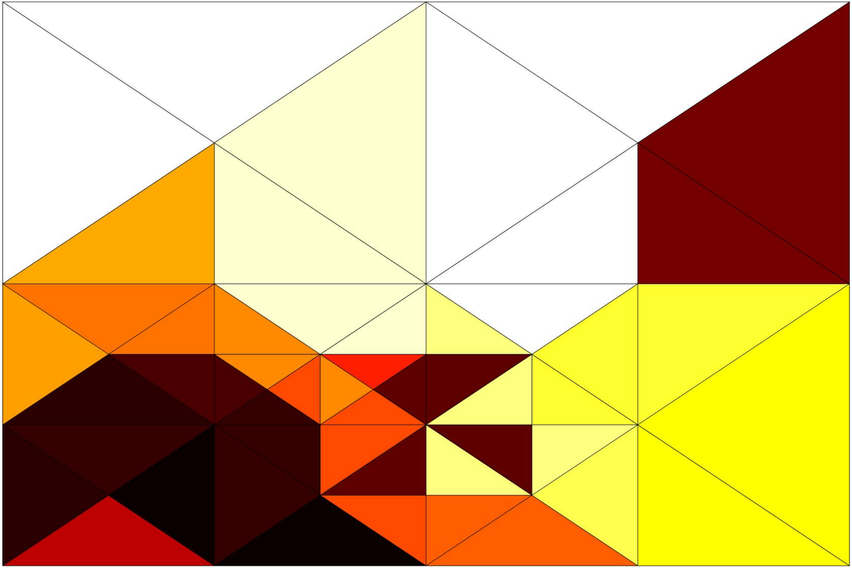

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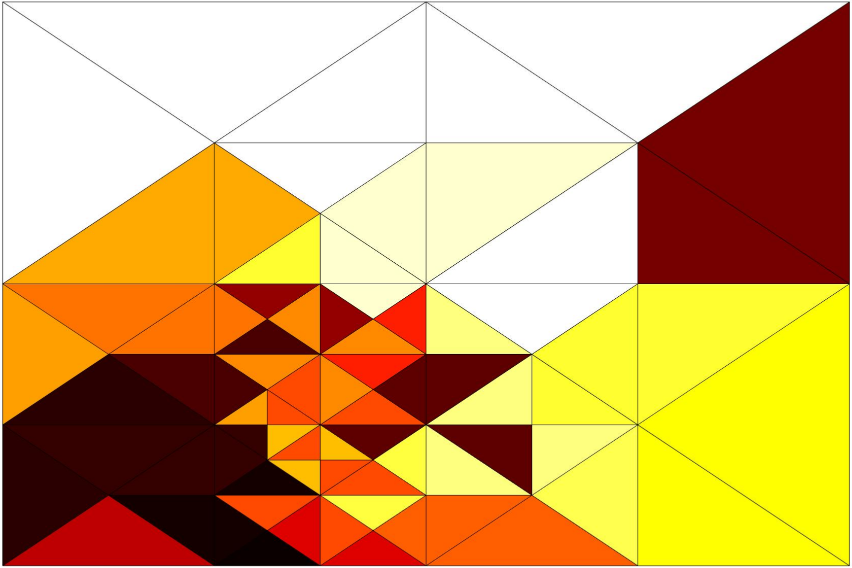

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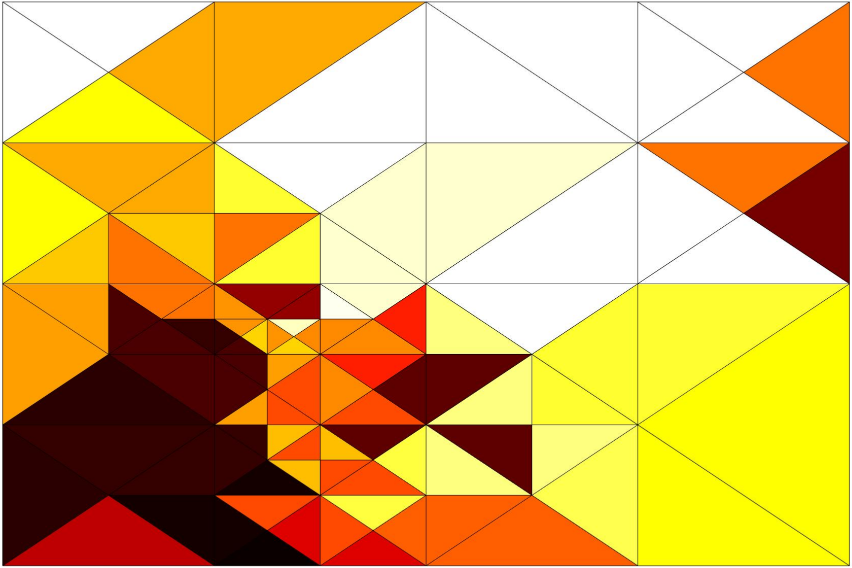

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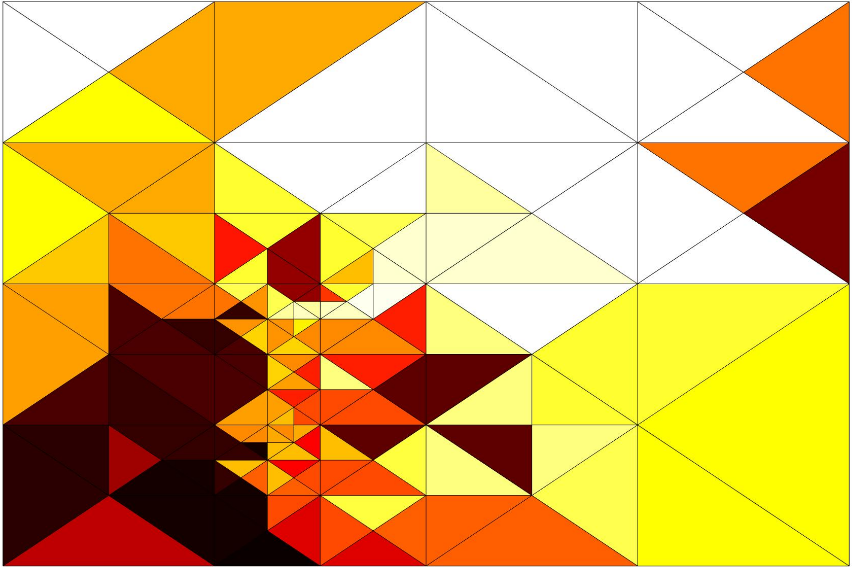

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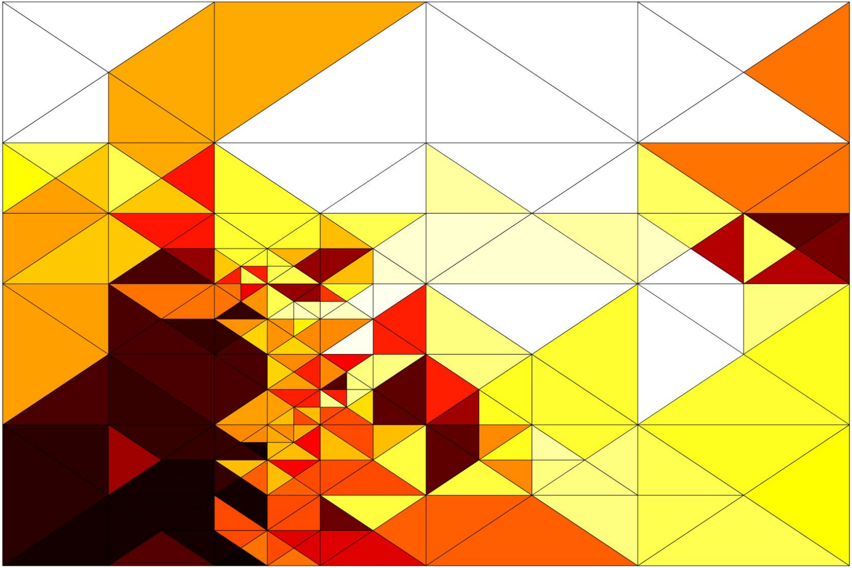

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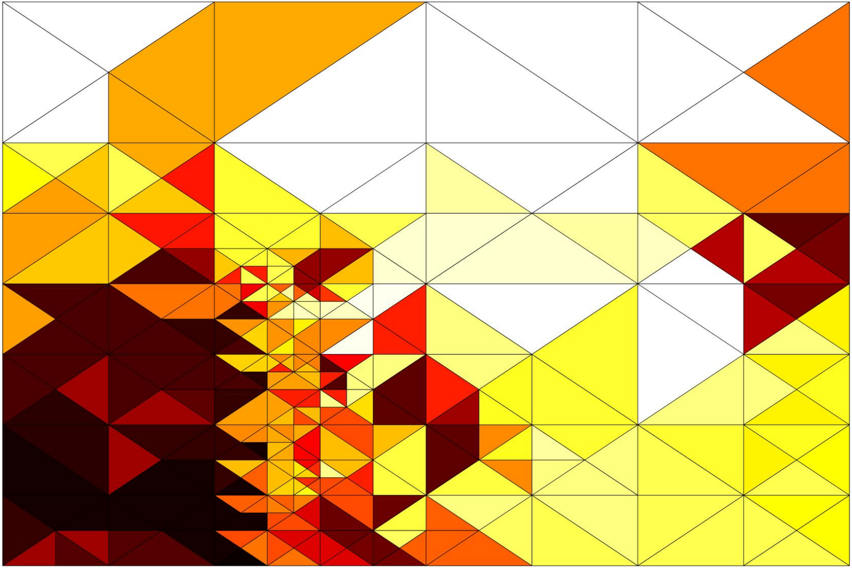

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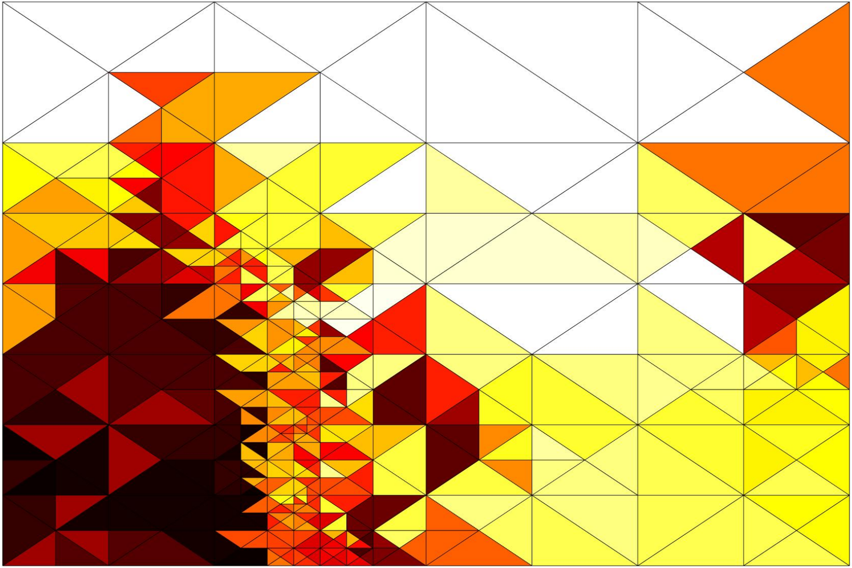

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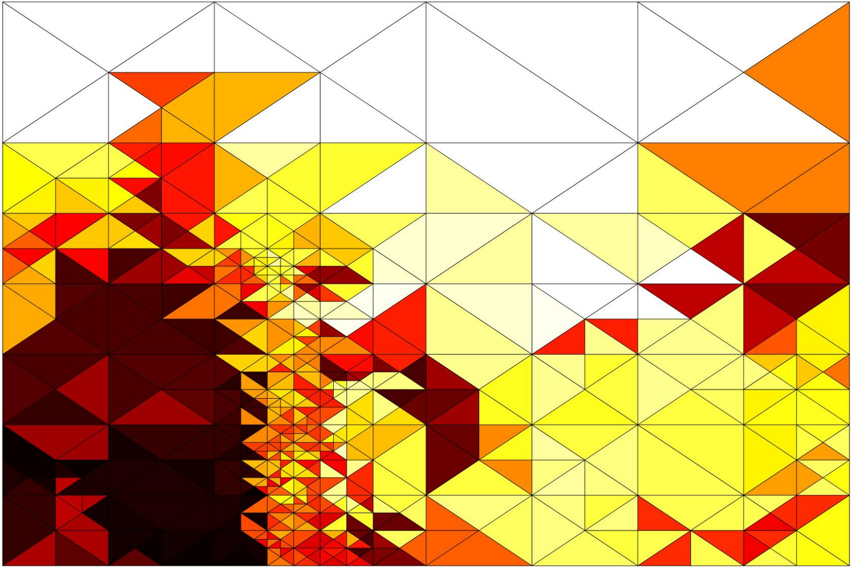

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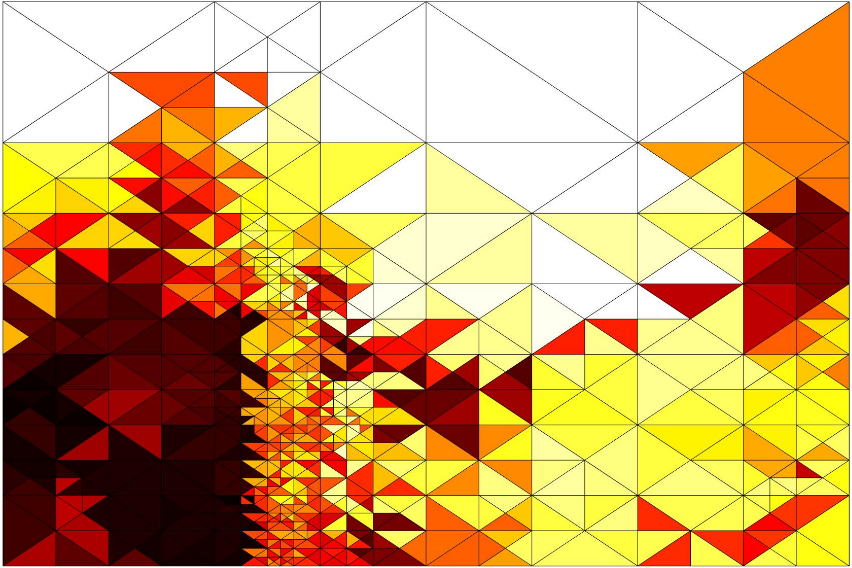

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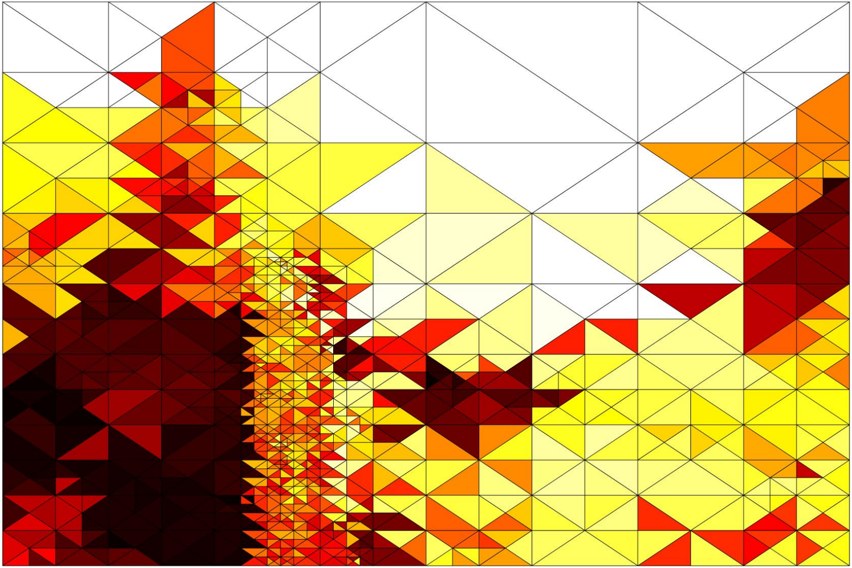

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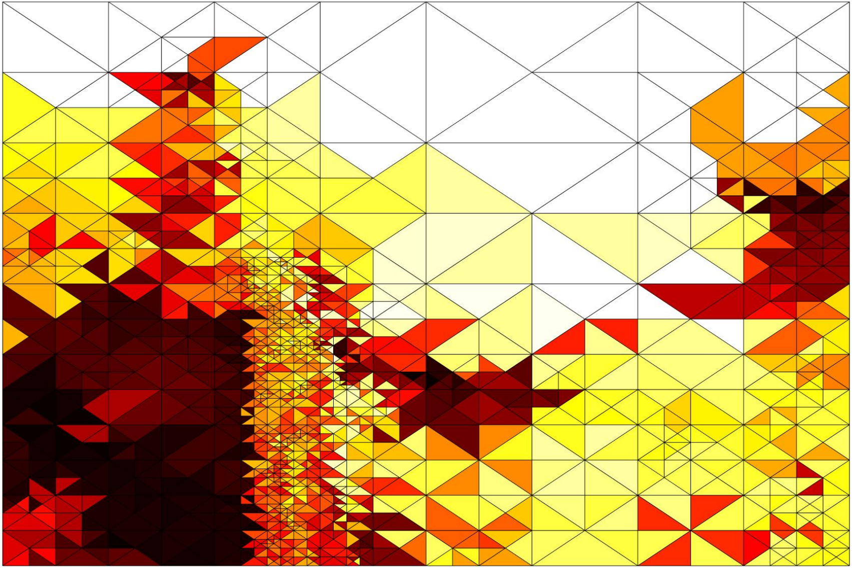

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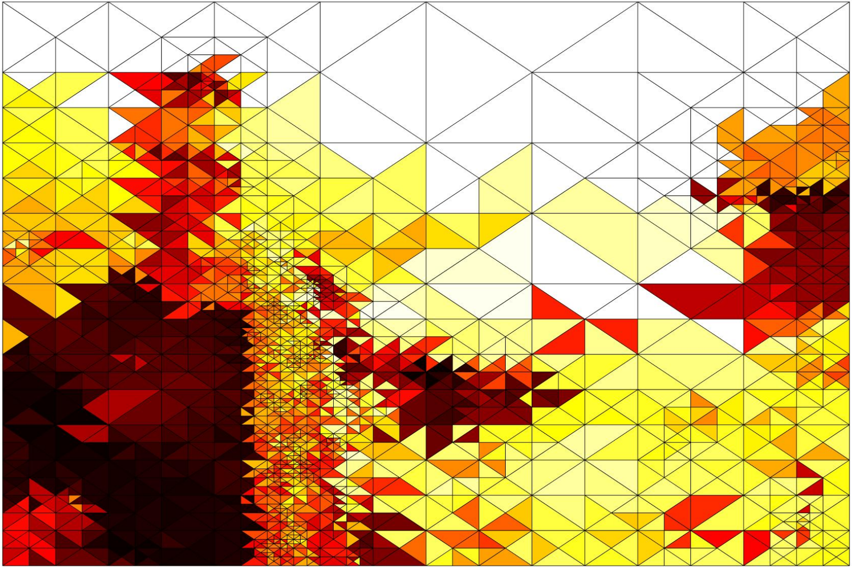

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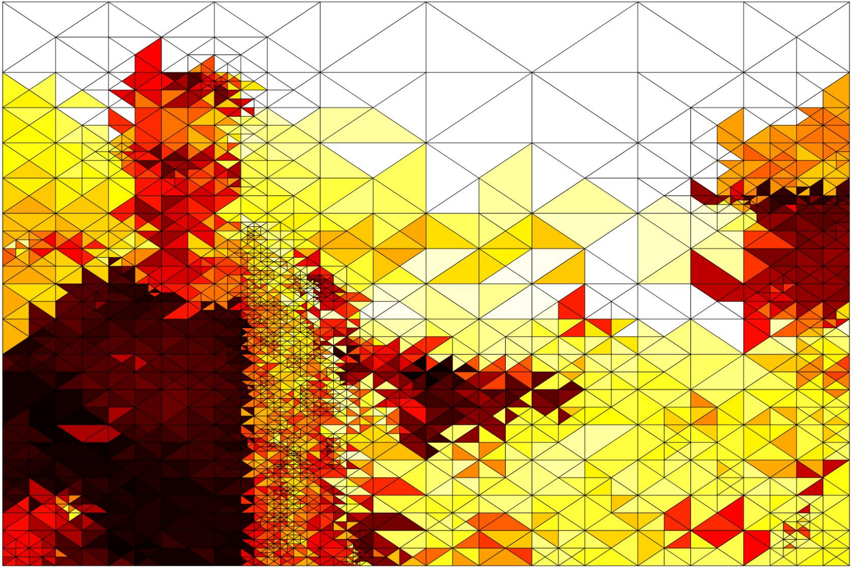

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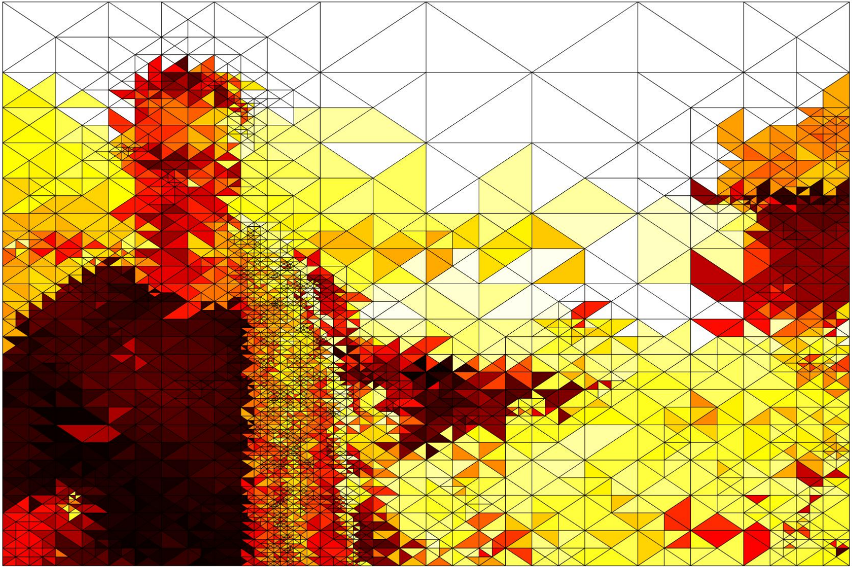

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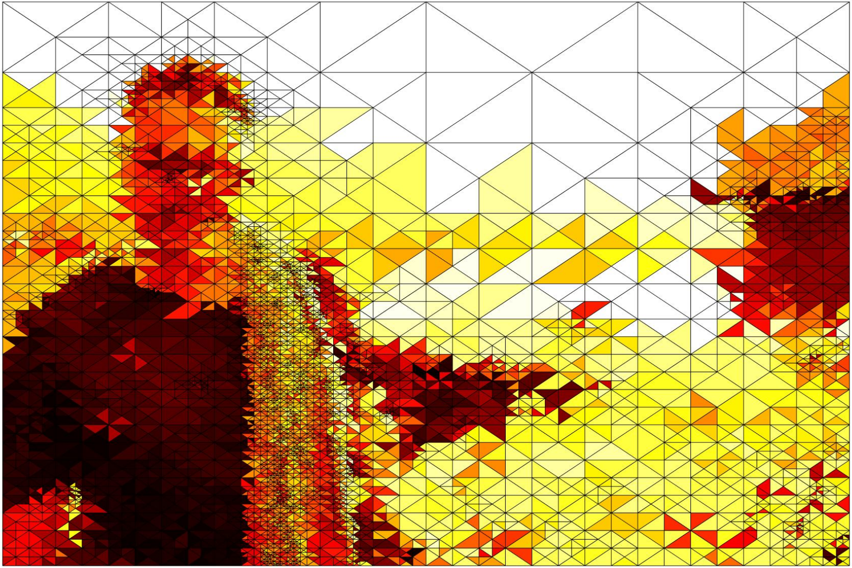

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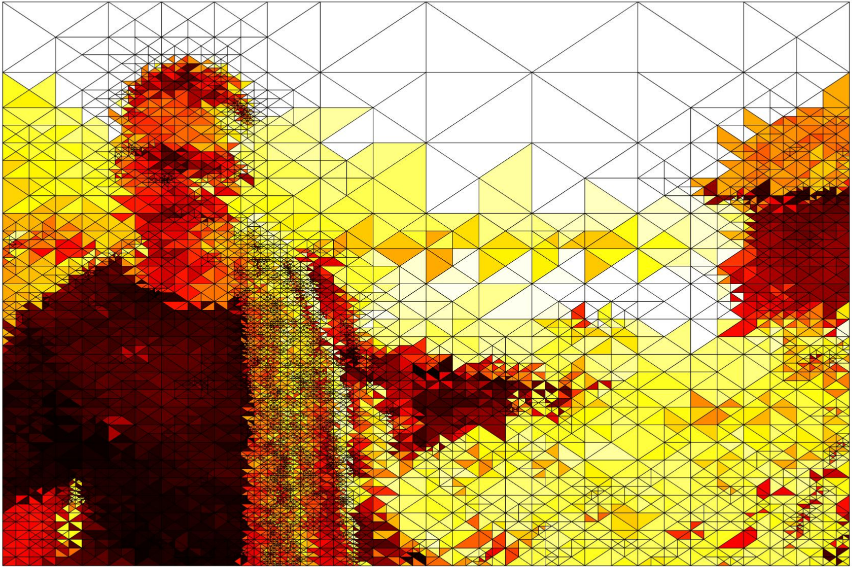

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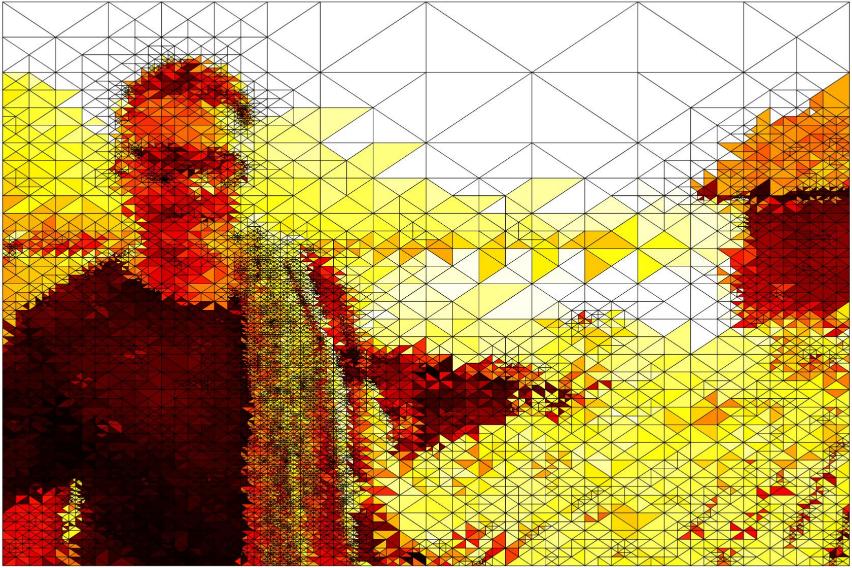

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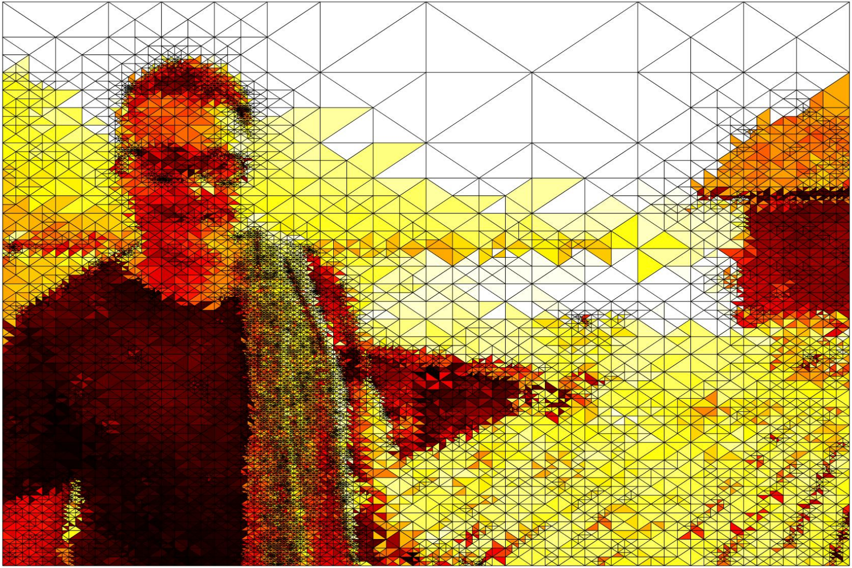

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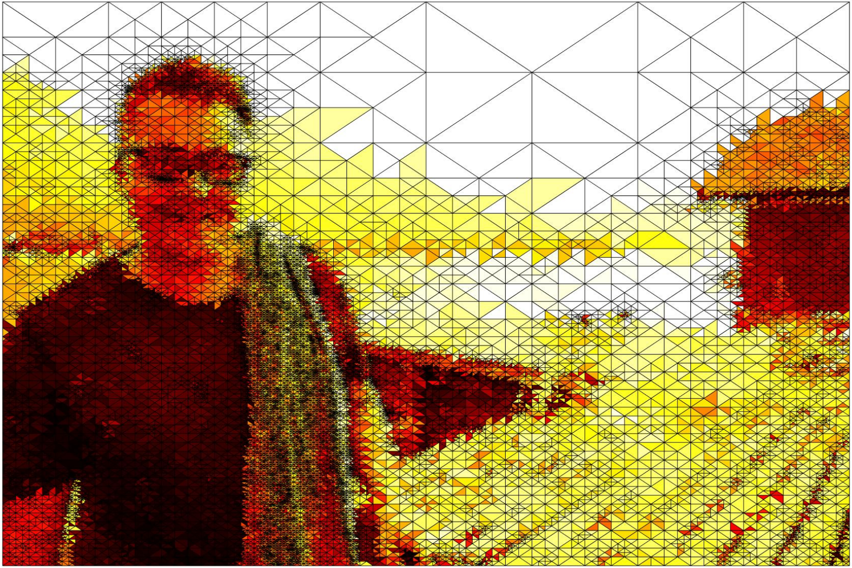

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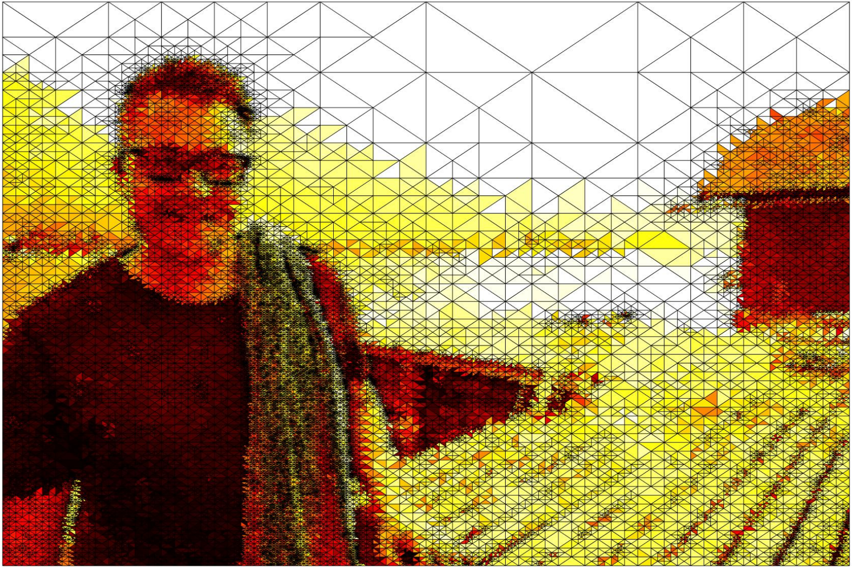

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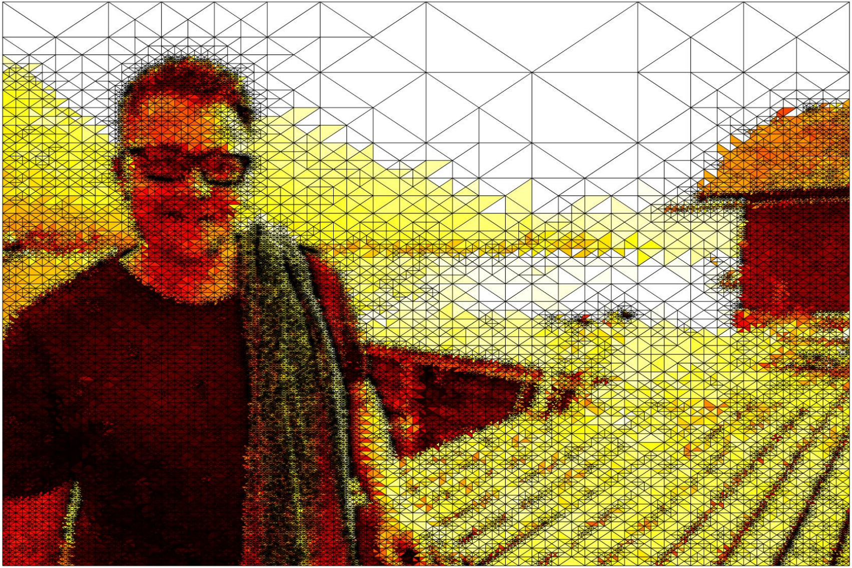

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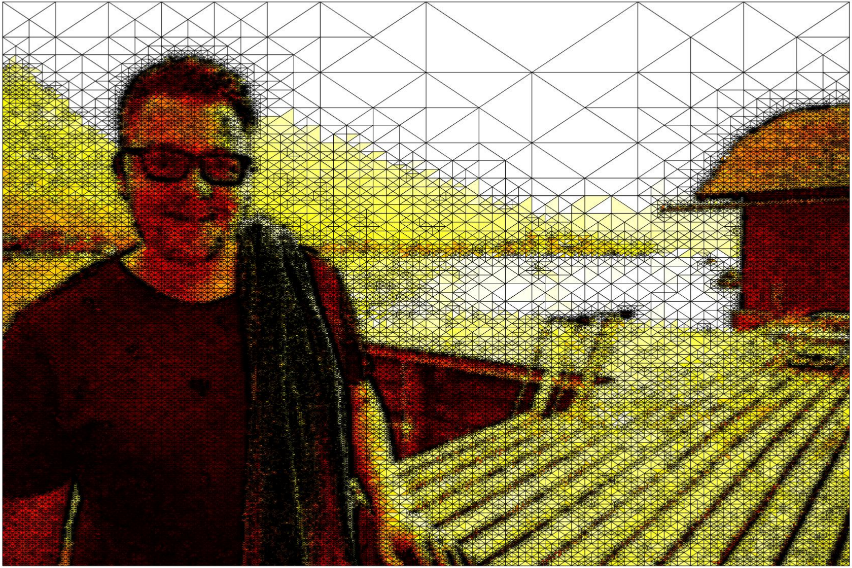

Questions? Meet me at the lake! ;-)