# CAN FUNCTIONAL-TYPE A POSTERIORI ERROR ESTIMATES UNITE ADAPTIVITY AND ERROR CONTROL IN BEM?

DANIEL SEBASTIAN

Institute for Analysis and Scientific Computing
TU Wien
Wiedner Hauptstraße 8-10, 1040 Wien
e-mail: daniel.sebastian@asc.tuwien.ac.at

**ABSTRACT**

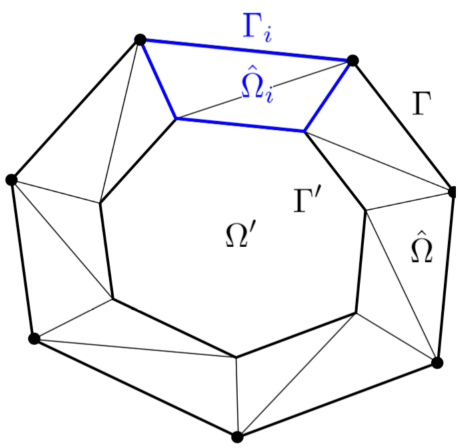
This work was intended as an attempt to motivate a new functional-type perspective on a posteriori error estimates for boundary element methods. In contrast to the state of the art, we are interested in finding *computable* bounds of the energy error $\varepsilon = ||\,\nabla(u - u_h)||_{L^2(\Omega)}$ for an approximate solution $u_h$ stemming from an either lowest order indirect or direct approach to solve the Dirichlet problem

$$\begin{aligned} \Delta u &= 0 \text{ in } \Omega \\ u_{|\Gamma} &= g \text{ on } \Gamma = \partial\Omega \end{aligned} \tag{1}$$

via Symm's Integral Equation of the first kind. One major advantage of functional-type a posteriori error estimates is, that they do not depend on crucial assumptions on the approximation method but only exploit the mathematical structure of the PDE itself. This approach turns out to be very convenient for the analysis of our energy error because the global reconstruction $u_h$ obtained by BEM is not based on a discretized domain $\Omega$ but rather on a discrete representation by means of boundary potentials. Hence, commonly demanded properties like Galerkin-orthogonality are just not available for $u_h$ on $\Omega$. BEM's distinguishing feature that approximations solve the equation exactly inside $\Omega$ is the only ingredient to conclude the sharp error bounds

$$\max_{\tau \in D_0(\Omega)} \underbrace{2\int_\Gamma (g - u_{h|\Gamma})\,\tau \cdot n - ||\tau||^2_{L^2(\Omega)}}_{\underline{\mathfrak{M}}(\tau)} = ||\,\nabla(u - u_h)||^2_{L^2(\Omega)} = \min_{\substack{\omega \in H^1(\Omega) \\ \omega_{|\Gamma} = g - u_{h|\Gamma}}} \underbrace{||\,\nabla\,\omega||^2_{L^2(\Omega)}}_{\overline{\mathfrak{M}}(\omega)},$$

which lead to minimization/maximization procedures. Instead of solving the related problems on $\Omega$, we suggest a construction via FEM on a tiny boundary layer $\hat{\Omega}$. (see figure below) We obtain full error control and as a last point reveal how an $\varepsilon$-error distribution on $\hat{\Omega}$ might indicate an adaptive mesh-refinement on $\Gamma$ to steer the BEM-algorithm.

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