IETI-DP Solvers in Simulation and Optimization of Electrical Machines

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FWF Der Wissenschaftsfonds.
GEOMETRY NFN S117 SIMULATION

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Outline

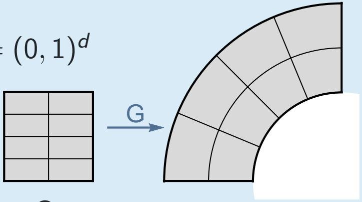
What is IgA?

- New paradigm for solving PDEs
- **Idea:** One and the same method that can be used for Computer Aided Design (CAD) and Numerical Simulation
- Exact representation of geometric objects from CAD systems without the need of meshing like in FEM

What is IgA?

- Based on splines
- Univariate splines $S_{p,k,h}(0,1)$
  - degree p
  - smoothness k
    $(S_{p,k,h}(0,1) = \{u \in C^k(0,1) : u|_{[ih,(i+1)h)} \in \mathbb{P}_p\})$
  - grid size h
- Tensor-product splines on $\hat{\Omega} := (0,1)^d$
- Global geometry function G:

  $\hat{\Omega} \to \Omega = G(\hat{\Omega})$



- IgA space in the physical domain Ω:
  $S_{p,p-1,h}(\Omega) = S_{p,p-1,h}(\hat{\Omega}) \circ G^{-1} = \{u : u \circ G \in S_{p,p-1,h}(\hat{\Omega})\}$

What is IgA?

- More complex geometries cannot be represented by one geometry map
- Use of multipatch domains:
  Per patch geometry functions G_k:
  $\overline{\Omega} = \bigcup_{k=1}^{K} \overline{G_k(\hat{\Omega})}$
- IgA space in the physical domain Ω:
  $S_{p,p-1,h}(\Omega) = \{u \in C^0(\Omega) : u \circ G_k \in S_{p,p-1,h}(\hat{\Omega}) \forall_{k=1,\dots,K}\}$

Why IgA for shape optimization?

- IgA has approximation power of a high-order method:

$$\inf_{u_h \in S_{p,p-1,h}} \|u - u_h\|_{L^2} \lesssim h^{p+1} |u|_{H^{p+1}}$$

- IgA has the problem size of a low-order method:

$$dim\ S_{p,p-1,h} \approx (n+p)^d$$

- Integration of geometry optimization in CAD environments
- No conversion of design-suitable and analysis-suitable models
  $\rightarrow$ Exact representation of the geometry
  $\rightarrow$ No systematic error

References to Shape Optimization with IgA

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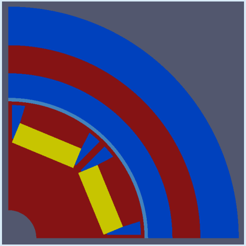
Outline

Real IPM Motor and Computational Domain

(a) Interior permanent magnet (IPM) electric motor



(b) Quarter of a cross section of an IPM motor

Figure: Real world IPM motor[1] vs. computational domain

[1] We acknowledge the permission to use this photo taken by the Linz Center of Mechatronics (LCM). The motor was produced by Hanning Elektro-Werke GmbH & Co KG.

Optimization w.r.t. Linear Magnetostatics

- Optimization of the runout performance:

$$\min_{D} J(u) := \int_{\Gamma} |B(u) \cdot n_{\Gamma} - B_d|^2 \, ds = \int_{\Gamma} |\nabla u \cdot \tau_{\Gamma} - B_d|^2 \, ds$$

subject to: find $u \in H_0^1(\Omega)$ such that

$$\langle A_D u, v \rangle = \langle F, v \rangle \qquad \forall v \in H_0^1(\Omega) \tag{1}$$

with

$$\langle A_D u, v \rangle = \int_{\Omega} \nu_D(x) \nabla u \cdot \nabla v \, dx,$$
$$\langle F, v \rangle = \int_{\Omega} J_3 v + \nu_M M^{\perp} \cdot \nabla v \, dx.$$

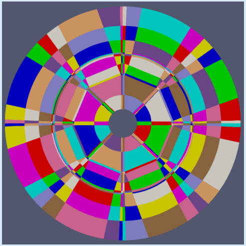
Outline

IETI-DP in a Nutshell

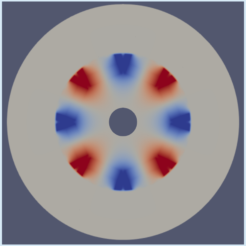
- Is a nonoverlapping domain decomposition method
- IETI-DP is an abbreviation for **D**ual-**P**rimal **I**sog**E**ometric **T**earing and **I**nterconnecting
- Similar to FETI-DP
- Basic idea:
  - Tearing of the computational domain into N subdomains
  - Posing the problem on a suitable interface space
  - DP approach requires for the solvability properly chosen primal variables, e.g., vertex evaluation, face averages, ...

Simulation with IETI-DP

- State solution in the conforming case with IETI-DP



(a) Full cross section of an IPM motor suitable for cG



(b) State on the full cross section of an IPM motor

- Multipatch domain with 372 patches and spline degree $p = 3$

Simulation with IETI-DP

- Now we can compare the solution techniques for the full cross section

# dofs	SuperLU	IETI-DP	speedup
23612	8.9 sec	5.7 sec	1.56
72 572	36.0 sec	17.0 sec	2.12
250 844	193.0 sec	69.8 sec	2.77
928 796	1943.0 sec	463.0 sec	4.20
3 570 332	–	1179.0 sec	–

Table: SuperLU vs. IETI-DP.

Simulation with IETI-DP

- Use of a parallel IETI-DP algorithm

# cores	1	2	4	8	16	32	64	128
time	1179	577	325	164	89	43	22	14
scaling	–	2.04	1.78	1.98	1.84	2.07	1.95	1.57

Table: Strong scaling with IETI-DP and 3 570 332 dofs, time in sec.

- We have an efficient PDE solver ✓

Optimization with Ipopt and IETI-DP

- Ipopt (**I**nterior **P**oint **Opt**imizer)
- Sign of the Jacobian determinant parametrization,

$$det(\mathrm{J}_G) = \sum_{k,\ell=1}^{M,N} c_{k,\ell} M_k^{2p-1}(u) N_\ell^{2q-1}(v)$$

  is used as constraint to prevent for self intersections
- Move the design variables
- Computation of the inner control coefficients from the boundary by a spring model of the mesh
  - An inner control coefficient $d_{i,j}$ satisfies

$$d_{i,j} = \frac{d_{i,j-1} + d_{i+1,j} + d_{i,j+1} + d_{i-1,j}}{4}$$

Optimization with Ipopt and IETI-DP

- Ipopt options:
  - Limited memory BFGS-method
  - NLP error tolerance: 10^{-6}
  - Relative error in the objective change: 10^{-6}
  - 3 acceptable iterations

Optimization with Ipopt and IETI-DP

- Optimization result with Ipopt:

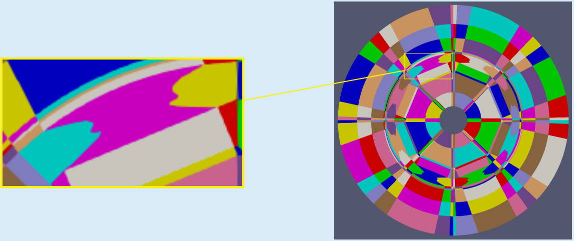


Figure: Optimized domain with Ipopt after 95 optimization iterations, the objective dropped from $4.266 \cdot 10^{-4}$ down to $2.587 \cdot 10^{-4}$

Optimization with Ipopt and IETI-DP

- Optimization result with Ipopt:

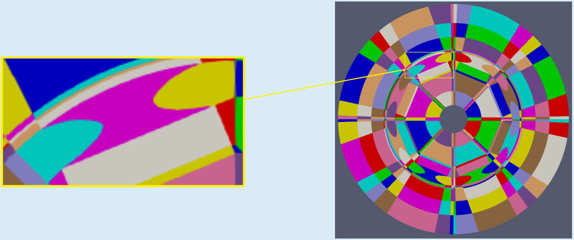


Figure: Optimized domain with Ipopt after 130 optimization iterations, the objective dropped from $4.266 \cdot 10^{-4}$ down to $2.436 \cdot 10^{-4}$

Outline

Preliminary Work to Capture the Rotation

- Hanging nodes have to be allowed
- We generalized the ideas in "Reparameterization and Adaptive Quadrature for the Isogeometric Discontinuous Galerkin method" by Seiler and Jüttler

The Reparameterization Technique

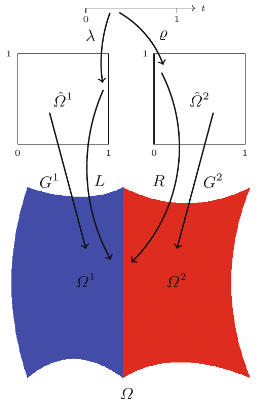


Figure: Multipatch domain Ω with geometry maps G^1, G^2

- Interface $e := G^1(1,[0,1]) = G^2(0,[0,1])$
- $L = G^1|_{(G^1)^{-1}(e)}$ and $R = G^2|_{(G^2)^{-1}(e)}$
- $\lambda : [0,1] \to \{1\} \times [0,1]$
- $\varrho : [0,1] \to \{0\} \times [0,1]$
- Relation $L \circ \lambda = R \circ \varrho$

The Reparameterization Technique

- Fix the reparameterization λ
- The reparameterization technique consists of 2 main steps:
  1. For a given number of samples, we compute

$$\varrho_i = R^{-1} \circ L \circ \lambda \left(\frac{i}{N} \right)$$

by solving

$$\varrho_i = \underset{\xi \in \{0\} \times [0,1]}{argmin} \left\| L \circ \lambda \left(\frac{i}{N} \right) - R(\xi) \right\|$$

for $i = 0, \ldots, N$

The Reparametrization Technique

- The reparametrization technique consists of 2 main steps:
  2. After choosing a suitable spline space we solve

$$\sum_{i=1}^{N}\left(\sum_{j=1}^{m} c_j N_j\left(\frac{i}{N}\right)-\varrho_i\right)^2 \rightarrow min!$$

The Reparametrization Technique

- Integration along the interface
  - Choose one master patch
  - Collect all breakpoints of the two patches
  - Compute preimages of breakpoints for the master patch
  - Put quadrature nodes between each of the breakpoints
  - Map the quadrature nodes via the reparameterization to the parameter domain of the second patch

Numerical Tests for the Reparameterization

- We consider the dG-IgA problem:

Find $u \in V_h : a(u,v) = F(v) \qquad \forall v \in V_h = \{v | v|_{\Omega^k} \in V_h^k\}$

with the bilinear form

$$a(u,v) = \sum_{k=1}^{M} a_1^k(u,v) - \frac{1}{2} \sum_{e \in \Gamma_c \cup \Gamma_D} (a_{2,1}^e(u,v) + a_{2,2}^e(u,v)) + \sum_{e \in \Gamma_c \cup \Gamma_D} a_3^e(u,v)$$

and the right-hand side

$$F(v) = \int_\Omega fv \; dx$$

Numerical Tests for the Reparameterization

- The quantities in the bilinear form are given by

$$a_1^k(u,v) = \int_{\Omega^k} \nabla u \cdot \nabla v \; dx$$

$$a_{2,1}^e(u,v) = \int_e \{\nabla u \cdot n\}^e [v]^e ds$$

$$a_{2,2}^e(u,v) = \int_e \{\nabla v \cdot n\}^e [u]^e ds$$

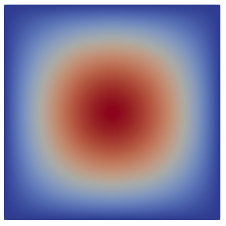
$$a_3^e(u,v) = \int_e \alpha [u]^e [v]^e ds$$

- $\{\cdot\}^e, [\cdot]^e$ denote the average and the jump across e, respectively and $\alpha \sim \frac{p^2}{h}$ is some suitably chosen parameter

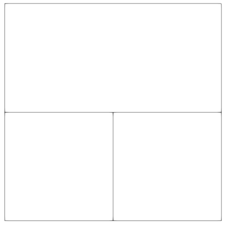
Problem Setting

- $f = 2\pi^2 sin(\pi x) sin(\pi y)$
- The exact solution of the problem is $sin(\pi x) sin(\pi y)$
- Homogeneous boundary conditions

Numerical Example 1



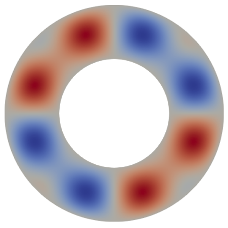
(a) Solution of the problem



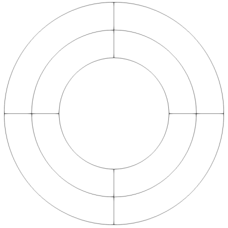
(b) Splitting of the domain

Figure: Solution with B-splines of degree 3

Numerical Example 2



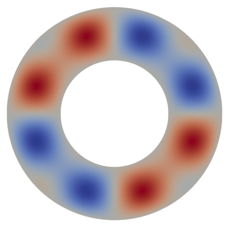
(a) Solution of the problem



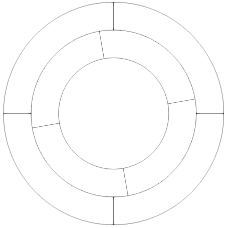
(b) Splitting of the domain

Figure: Solution with NURBS of degree 3

Numerical Example 2



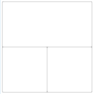
(a) Solution of the problem



(b) Splitting of the domain

Figure: Solution with NURBS of degree 3

Convergence Test of Example 1



# dofs	L^2 error	conv. rate	H^1 error	conv. rate
75	0.00108993	0	0.0544043	0
147	0.000160162	2.767	0.0113958	2.25522
363	8.21729e-06	4.285	0.000902003	3.65922
1083	4.82416e-07	4.090	8.37942e-05	3.42821
3675	2.9596e-08	4.029	8.53316e-06	3.2957
13467	1.83882e-09	4.009	9.30043e-07	3.19771
51483	1.14684e-10	4.003	1.06861e-07	3.12156
201243	7.17393e-12	3.999	1.27334e-08	3.06904

Table: L^2 and H^1 convergence rates for example 1 with B-splines of degree **3**

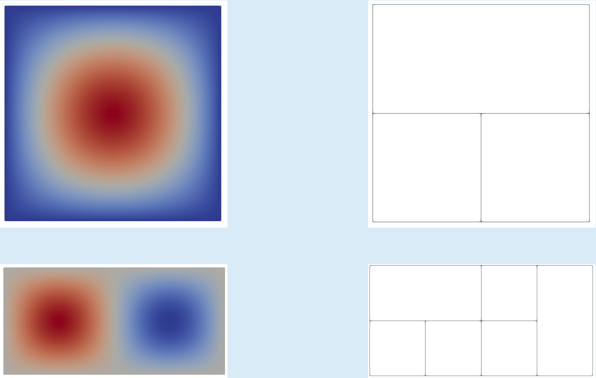
How to Handle Rotations of Elec. Machines?

- dG-IETI method
  - Would be completely new
  - No cG conforming meshes for the subdomains necessary
  - No theory so far (theory relies on matching vertices)
- Quasi p-robust multipatch multigrid
  - Developed mainly by Stefan Takacs at RICAM
  - Theory not complete
  - First experiments are promising
- ...
- ...

dG-IETI-DP

- Theory so far needs vertex values as primal variables
- Hanging nodes occur in the consideration of rotating machines
  $\rightarrow$ Vertex values are not feasible any more
  $\rightarrow$ Only edge averages are allowed

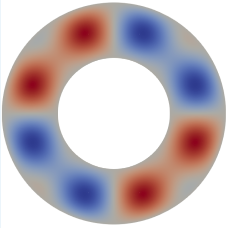
dG-IETI-DP



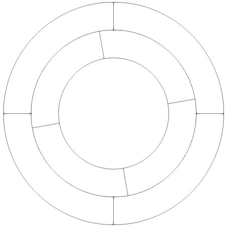
(a) Solution of the problem (b) Splitting of the domain

Figure: Solution with B-splines of degree 3

dG-IETI-DP



(a) Solution of the problem

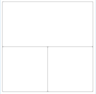


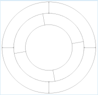
(b) Splitting of the domain

Figure: Solution with NURBS of degree 3

Tests with dG-IETI-DP

- Unit square with degree 3, NURBS circular rings with degree 3





Unit square			NURBS circular rings		
# dofs	Solving time	CG-Iterations	# dofs	Solving time	CG-Iterations
363	0.21	4	392	0.97	23
1 083	0.37	5	968	1.43	32
3 675	0.89	6	2 888	2.50	33
13 467	2.95	7	9 800	5.53	31
51 483	13.84	8	35 912	16.93	34
201 243	82.19	9	137 288	71.48	33

Table: Solving time in sec and iterations for dG-IETI-DP, Error tolerance 1e-5

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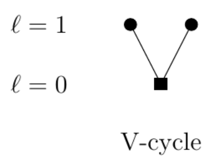
Multigrid in a Nutshell

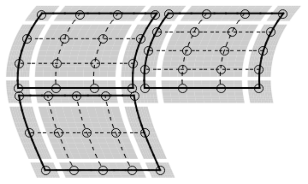
- Iterative method for solving large linear systems $Ax = f$
- Use a hierarchy of discretizations
- Basic algorithm consists of 3 steps:
  - Smoothing: $x^k \leftarrow x^k + P^{-1}(f - Ax^k)$
  - Restriction of the residual r
  - Prolongation of the correction $w = A^{-1}r$ and setting $x^{k+1} = x^k + w$



- Can be used as solver as well as preconditioner for e.g. CG

Quasi p-robust Multigrid

- Key idea is a stable splitting of the spline spaces



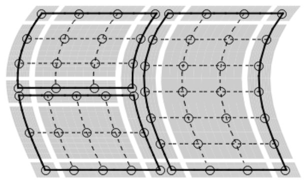


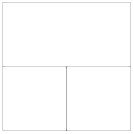
Figure: Decomposition of dofs

Quasi p-robust Multigrid

(a) Solution of the problem (b) Splitting of the domain

Tests with Multigrid

- Test on the unit square



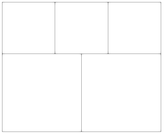
Unit square

r\p	1	2	3	4	5
2	7/44	6/70	7/102	8/140	10/184
3	8/184	7/234	8/290	9/352	12/420
4	8/752	8/850	9/954	11/1064	15/1180
5	8/3040	9/3234	10/3434	11/3640	18/3852
6	8/12224	9/12610	10/13002	12/13400	20/13804
7	9/49024	9/49794	11/50570	12/51352	20/52140
8	9/196352	10/137890	11/199434	12/200984	20/202540

Table: Iterations/#dofs, Error tolerance 1e-8

Tests with Multigrid

- Test on the rectangular domain



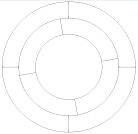
Rectangular domain

r\p	1	2	3	4	5
2	8/84	6/130	5/186	6/252	6/328
3	8/328	6/414	7/510	7/616	7/732
4	8/1296	6/1462	7/1638	8/1824	9/2020
5	8/5152	7/5478	8/5814	9/6160	10/6516
6	9/20544	7/21190	8/21846	9/22512	11/23188
7	9/82048	7/83334	8/84630	9/85936	11/87252
8	9/327936	7/330502	8/333078	10/335664	11/338260

Table: Iterations/#dofs, Error tolerance 1e-8

Tests with Multigrid

- Test on the circular rings



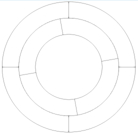
Circular rings

r\p	1	2	3	4	5
2	13/160	14/240	15/336	16/448	17/576
3	19/576	19/720	22/880	22/1056	30/1248
4	19/2176	21/2448	23/2736	26/3040	31/3360
5	20/8448	21/8976	26/9520	29/10080	36/10656
6	23/33280	23/34320	26/35376	30/36448	37/37536
7	23/132096	23/134160	27/136240	30/138336	37/140448
8	23/526336	23/530448	27/534576	30/538720	38/542880

Table: Iterations/#dofs, Error tolerance 1e-8

Tests with Multigrid

- Test on the circular rings with improved scaling



Circular rings

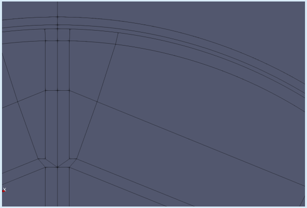
r\p	1	2	3	4	5
2	10/160	11/240	14/336	14/448	12/576
3	15/576	15/720	17/880	18/1056	27/1248
4	17/2176	16/2448	19/2736	20/3040	27/3360
5	20/8448	17/8976	20/9520	22/10080	29/10656
6	21/33280	18/34320	21/35376	22/36448	32/37536
7	22/132096	19/134160	22/136240	23/138336	32/140448
8	23/526336	19/530448	22/534576	23/538720	36/542880

Table: Iterations/#dofs, Error tolerance 1e-8

Outline

dG-IETI on a NURBS based geometry

- Solution on a NURBS based model of the IPM
- T-junctions but no rotation
- Same solution as in the conforming case



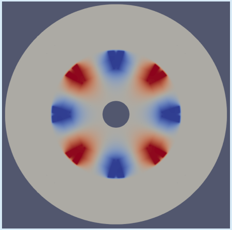


Figure: Discretization and solution of linear magnetostatics

dG-IETI on a NURBS based geometry

- We are now able to consider an arbitrary position of the motor:

Decompositon Solution

Outline

Summary

- So far only the linear static case is considered
- Need additional techniques to capture rotation
- Introduction of dG-IETI-DP and multipatch multigrid
- First results are promising
- dG-IETI-DP and multigrid still require some investigation
- Also reparametrization seems to influence the results

Outlook

- Provide a theory for dG-IETI-DP
- Provide a theory for multipatch multigrid
- Introduction of the nonlinearity in ν, i.e., $\nu = \nu(x, |\nabla u|)$
- Apply multipatch multigrid to the motorsimultation
- Proper treatment of highly distorted geometries