Space-time finite element methods for parabolic evolution problems with distributional source

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ABSTRACT

Following our previous work [1, 2], we propose an extension of our locally stabilized, consistent, conforming finite element schemes on completely unstructured simplicial space-time meshes to the numerical solution of non-autonomous parabolic evolution problems with distributional right-hand sides as, e.g., generated by permanent magnets in electromagnetics. Distributional source terms, discontinuous coefficients, non-smooth boundaries, and changing boundary conditions can lead to non-smooth solutions. We present an a priori discretization error analysis under the assumption of (local) maximal parabolic regularity, which includes low-regularity solutions arising from non-smooth data such as mentioned above. In order to avoid reduced convergence rates appearing in the case of uniform mesh refinement, we also consider adaptive refinement procedures based on residual- and functional-type a posteriori error indicators and estimators. The huge system of space-time finite element equations is then solved by means of the Generalized Minimal Residual Method, preconditioned by algebraic multigrid. Especially in the 4d space-time case that is 3d in space, simultaneous space-time adaptivity and parallelization can considerably reduce the computational time. The space-time finite element solver was implemented by means of the library MFEM. We present numerical examples with different features. The numerical results nicely confirm our theoretical findings.

References

[1] U. Langer, M. Neumüller, and A. Schafelner. Space-time Finite Element Methods for Parabolic Evolution Problems with Variable Coefficients. Chapter 13 in *Advanced Finite Element Methods with Applications - Proceedings of the 30th Chemnitz FEM Symposium 2017*, LNCSE, Springer, Berlin, Heidelberg, New York, 2019.

[2] U. Langer and A. Schafelner. Space-time finite element methods for parabolic evolution problems with non-smooth solutions. *arXiv:1903.02350* (2019), accepted for publication in the SCEE 2019 proceedings published by Springer.