On Abstract Friedrichs Systems and Some of their Applications.

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Many models of mathematical physics share a common form

$$\partial_t V + AU = F,$$

where ∂_t denotes time-differentiation and A is a maximal accretive linear operator, F a given source term. Indeed, in standard cases A is simply skew-selfadjoint in an underlying real Hilbert space H. The unknowns U, V are here linked by a so-called material law

$$V = \mathscr{M} U.$$

In a number of studies it has been illustrated, that this simple framework is indeed suitable for a large number of complex applications including even time-delay problems and fractional time derivatives. A typical and simple case, which we shall focus on here is

$$\mathscr{M} = M_0 + \partial_t^{-1} M_1, \tag{1}$$

where M_0, M_1 are continuous linear operators in H, in particular, M_0 is selfadjoint. The operator ∂_t^{-1} appearing here is forward causal time-integration, which can be properly realized in a weighted Hilbert space setting. In this case a basic well-posedness constraint for material laws of the form (1) is that

$$\rho M_0 + M_1 \geq c_0 > 0 \tag{2}$$

holds for some real number c_0 and all sufficiently large positive $\rho \in \mathbb{R}$.

For A skew-selfadjoint these systems can be transformed into operator equations of the form

$$1 + \mathscr{A},$$

where now $\mathscr{A}$ is skew-selfadjoint. Since the classical Friedrichs systems can also be brought into this formal shape, we speak of abstract Friedrichs systems. With the skew-selfadjointness of A playing a central role, we present a number of tools, useful for modeling concrete problems, to construct such operators. These tools as well as the utility of the general setting are illustrated with various applications from mathematical physics.