Convergence analysis of the Adini element on a Shishkin mesh for a singularly perturbed fourth-order problem in two dimensions

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We consider the singularly perturbed fourth-order boundary value problem $\varepsilon^2\Delta^2 u - \Delta u = f$ on the unit square $\Omega \subset \mathbb{R}^2$, with boundary conditions $u = \partial u/\partial n = 0$ on $\partial\Omega$. Here $\varepsilon \in (0,1)$ is a small parameter. The problem is solved numerically by means of Adini finite elements—a simple nonconforming finite element method for this problem. Under reasonable assumptions on the structure of the boundary layers that appear in the solution, a family of suitable Shishkin meshes with N^2 elements is constructed and convergence of the method is proved in a "broken" version of the Sobolev norm $v \mapsto \left(\varepsilon^2|v|_2^2 + |v|_1^2\right)^{1/2}$. For a particular choice of the mesh, the error in the computed solution is at most $C\left[\varepsilon^{1/2}(N^{-1}\ln N)^2 + \min\left\{\varepsilon^{1/2}, \varepsilon^{-3/2}N^{-2}\right\} + N^{-3}\right]$, where the constant C is independent of ε and N. Numerical results support our theoretical convergence rates, even for an example where not all the hypotheses of our theory are satisfied.