# Matrix-Free Multigrid Methods for Nonlinear Variational Inequalities

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Matrix-based finite element methods tend to require lots of memory, in particular, for higher-order discretizations and 3d problems. This gets even worse when we apply algebraic (AMG) or geometric multigrid (GMG) solvers, as we need to store the coarse level matrices as well. Fortunately, geometric multigrid methods are suitable for a matrix-free implementation.

Within such a matrix-free approach, we aim to assemble the matrix-vector product on the fly instead of using the standard way of assembling, storing, and performing matrix-vector multiplication. As we do no longer have all the matrix entries at our disposal, many solvers, like direct solver or AMG methods, are no longer applicable. On the other hand, GMG only relies on matrix-vector products (given an appropriate smoother), such that a matrix-free implementation is possible.

In this talk, we apply this technique to a monolithic quasi-static phase-field fracture model. The equations of interest are nonlinear and need to satisfy a variational inequality. This imposes several challenges for the implementation, which will be discussed throughout this talk. Finally, several numerical examples are presented to show the applicability and parallel scalability of the matrix-free GMG solver.