# Sharp spatial $H^1$-norm analysis of a finite element method for a time-fractional diffusion equation

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A time-fractional initial-boundary value problem $D_t^\alpha u - \Delta u = f$, where $D_t^\alpha$ is a Caputo fractional derivative of order $\alpha \in (0,1)$, is considered on the space-time domain $\Omega \times [0,T]$, where $\Omega \subset \mathbb{R}^d$ $(d \geq 1)$ is a bounded Lipschitz domain. Typical solutions $u(x,t)$ of such problems have components that behave like a multiple of $t^\alpha$ as $t \to 0^+$, so the integer-order temporal derivatives of $u$ blow up at $t = 0$. The numerical method of the paper uses a standard finite element method in space on a quasiuniform mesh and considers both the L1 discretisation and Alikhanov's L2-$1_\sigma$ discretisation of the Caputo derivative on suitably graded temporal meshes. *Optimal error bounds in* $H^1(\Omega)$ are proved; no previous analysis of a discretisation of this problem using finite elements in space has established such a bound. Furthermore, the optimal grading of the temporal mesh can be deduced from our analysis. Numerical experiments show that our error bounds are sharp.

KEY WORDS: Fractional diffusion equation, finite element method, ioptimal error in $H^1(\Omega)$, graded mesh

REFERENCES

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