

A Unified View of Some Numerical Methods for Fractional Diffusion

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In recent years, a number of numerical methods for the solution of fractional Laplace and, more generally, fractional diffusion problems have been proposed. The approaches are quite diverse and include, among others, the use of best uniform rational approximations, quadrature for Dunford-Taylor-like integrals, finite element approaches for a localized elliptic extension into a space of increased dimensions, and time stepping methods for a parabolic reformulation of the fractional differential equation. We review these methods and observe that all approaches mentioned above can, in fact, be interpreted as realizing different rational approximations of a univariate function over the spectrum of the original (non-fractional) diffusion operator.

This observation allows us to cast all described methods into a unified theoretical and computational framework, which has a number of benefits. Theoretically, it enables us to give new convergence proofs for several of the studied methods, clarifies similarities and differences between the approaches, suggests how to design new and improved methods, and allows a direct comparison of the relative performance of the various methods. Practically, it provides a single, simple to implement, efficient and fully parallel algorithm for the realization of all studied methods; for instance, this does away with the need for constructing specific multilevel methods for the efficient realization of the extension methods and lets us parallelize the otherwise inherently sequential time stepping approach.

In a detailed numerical study, we compare all investigated methods for various fractional exponents and draw conclusions from the results.