T-coercivity for practical applications

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For solving variational formulations, one classically uses the Banach-Necas-Babuska theory, that relies on the celebrated inf-sup condition. The T-coercivity approach can be seen as a reformulation of this theory, where the operator T is an explicit realization of the inf-sup condition. At the discrete level and to prove convergence of the discrete solution to the exact one, another classical tool is the Fortin lemma that allows one to prove a discrete inf-sup condition. On the other hand, in many applications one may also use the T-coercivity approach at the discrete level. The purpose of this talk is to illustrate this property and show the versatility of the T-coercivity approach.

Originally, this approach was introduced to solve wave propagation problems in electromagnetic theory around negative materials, ie. materials for which the effective response is modeled by strictly negative coefficients [4].

The outline of the talk is as follows. We start by recalling the Banach-Necas-Babuska theory and its reformulation as the T-coercivity approach. Then we provide some illustrations, starting by the diffusion equation, expressed either in primal form, or in mixed form [8]. We then consider the propagation of waves in classical media (acoustics, electromagnetics) [7]. Finally, we address the case of transmission problems with discontinuous, sign-changing coefficients that occur in the presence of negative materials surrounded by classical materials [2, 6, 3, 5, 1].

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