AN ANALYSIS OF THE GRÜNWALD–LETNIKOV SCHEME FOR INITIAL-VALUE PROBLEMS WITH WEAKLY SINGULAR SOLUTIONS

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Abstract. A convergence analysis is given for the Grünwald-Letnikov discretisation of a Riemann-Liouville fractional initial-value problem on a uniform mesh $t_m = m\tau$ with $m = 0, 1, \dots, M$. For given smooth data, the unknown solution of the problem will usually have a weak singularity at the initial time $t = 0$. Our analysis is the first to prove a convergence result for this method while assuming such non-smooth behaviour in the unknown solution. In part our study imitates previous analyses of the L1 discretisation of such problems, but the introduction of some additional ideas enables exact formulas for the stability multipliers in the Grünwald-Letnikov analysis to be obtained (the earlier L1 analyses yielded only estimates of their stability multipliers). Armed with this information, it is shown that the solution computed by the Grünwald-Letnikov scheme is $O(\tau t_m^{\alpha-1})$ at each mesh point t_m; hence the scheme is globally only $O(\tau^\alpha)$ accurate, but it is $O(\tau)$ accurate for mesh points t_m that are bounded away from $t = 0$. Numerical results for a test example show that these theoretical results are sharp.

Keywords: Riemann-Liouville derivative, Grünwald-Letnikov scheme, weak singularity, convergence analysis.

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References

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