Optimal convergence rates in L^2 for a first order system least squares finite element method

Maximilian Bernkopf[1,a], Jens Markus Melenk[1,b]

[a]maximilian.bernkopf@asc.tuwien.ac.at, [b]melenk@asc.tuwien.ac.at

[1] TU Wien

We consider a Poisson-like second order model problem written as a system of first order equations. For the discretization an $\boldsymbol{H}(\Omega, \mathrm{div}) \times H^1(\Omega)$-conforming least squares formulation is employed. A least squares formulation has the major advantage that regardless of the original formulation the linear system resulting from a least squares type discretization is always positive semi-definite, making it easier to solve. Even though our model problem in its standard $H^1(\Omega)$ formulation is coercive our methods and lines of proof can most certainly be applied to other problems as well, see [2, 3] for an application to the Helmholtz equation. A major drawback of a least squares formulation is that the energy norm is somewhat intractable. Deducing error estimates in other norms, e.g., the $L^2(\Omega)$ norm of the scalar variable, is more difficult. Numerical examples in our previous work [2] suggested convergence rates previous results did not cover. Closing this gap in the literature will be the main focus of the talk. To that end we showcase a duality argument in order to derive L^2 error estimates of the scalar variable, which was the best available estimate in the literature. We then perform a more detailed analysis of the corresponding error terms. This analysis then leads to optimal convergence rates of the method. The above procedure can then be applied to more complicated boundary conditions, for which an analogous result is a nontrivial task. As a tool, which is of independent interest, we develop $\boldsymbol{H}(\Omega, \mathrm{div})$-conforming approximation operators satisfying certain orthogonality relations. For the analysis, a crucial tool are recently developed projection based commuting diagram operators, see [4].

References

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