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Higher Regularity of the p-Poisson Equation in the Plane

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SFB 1283

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Regularity of Poisson equation

Poisson equation

$$-\mathrm{div}(\nabla u) = -\mathrm{div} F$$

Regularity of F transfers to ∇u

$$\|\nabla u\|_X \leq C\|F\|_X$$

The mapping $F \mapsto \nabla u$ is a Calderón-Zygmund operator, i.e.

$X = L^p \qquad p > 1$

$X = BMO$

$X = \mathbf{B}^s_{p,q}$ or $X = \mathbf{F}^s_{p,q}$

Formally, we can cancel div.

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p-Poisson equation

$$-\mathrm{div}(A(\nabla u)) = -\mathrm{div}(|\nabla u|^{p-2}\nabla u) = -\mathrm{div} F, \qquad 1<p<\infty$$

Similarities to non-Newtonian fluids.

Regularity of F transfers to $A(\nabla u)$

$$\|A(\nabla u)\|_X \leq C\|F\|_X$$

or $\|A(\nabla u)\|_{X(B)} \leq C\|F\|_{X(2B)}$ + lower order term of $A(\nabla u)$.

$X = L^\rho$	$\rho > p'$	Iwaniec 1986
$X = \mathrm{BMO}$	$p \geq 2$	DiBenedetto, Manfredi 1993
	$p > 1$	Diening, Kaplicky, Schwarzacher 2012
$X = C^\alpha$	$\alpha > 0$ small	Diening, Kaplicky, Schwarzacher 2012

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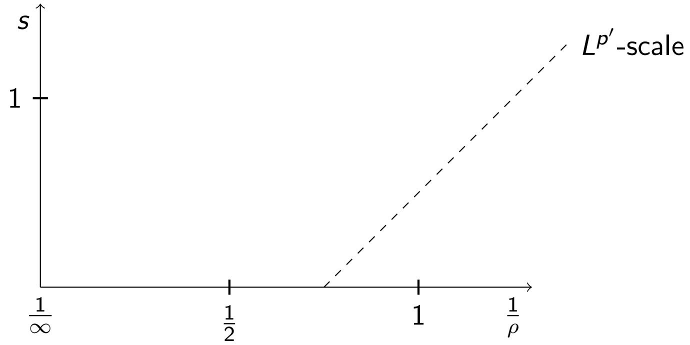
DeVore-Triebel Diagramm ($p = 4$)



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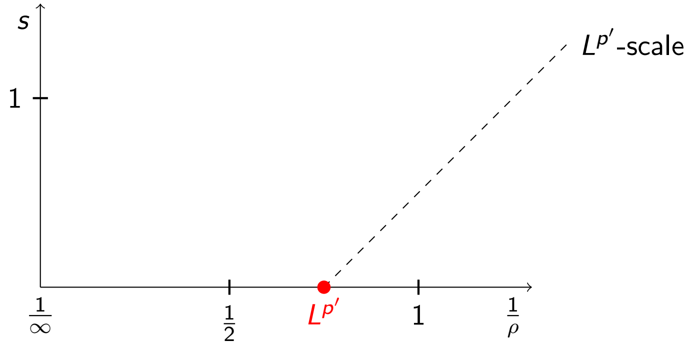

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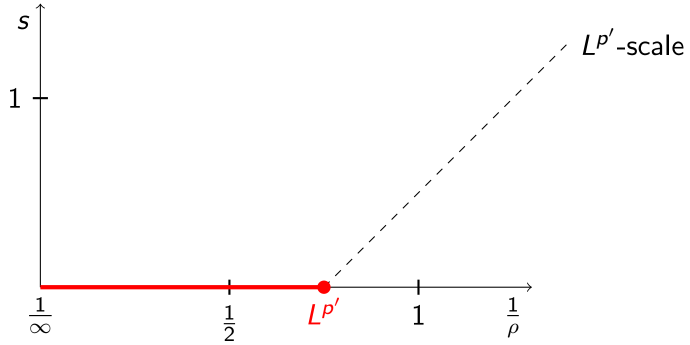

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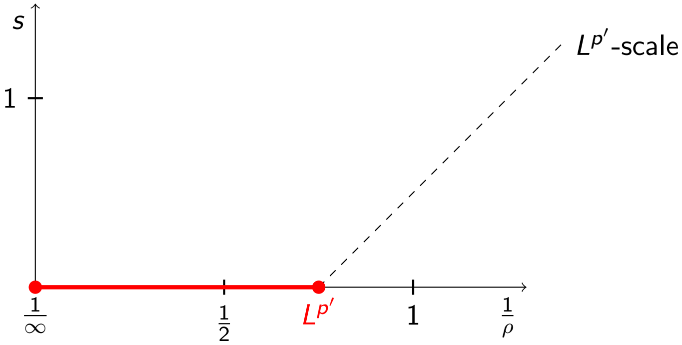

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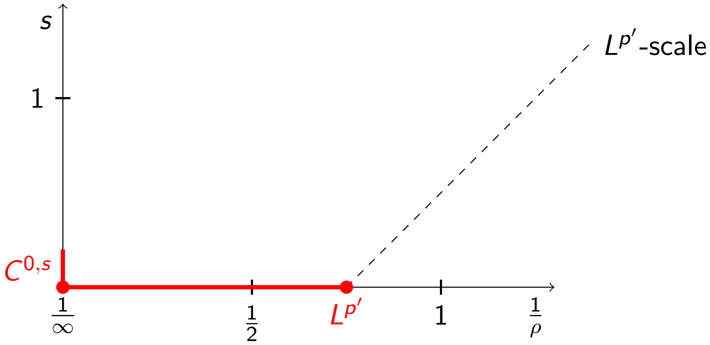

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Transfer of Higher Regularity

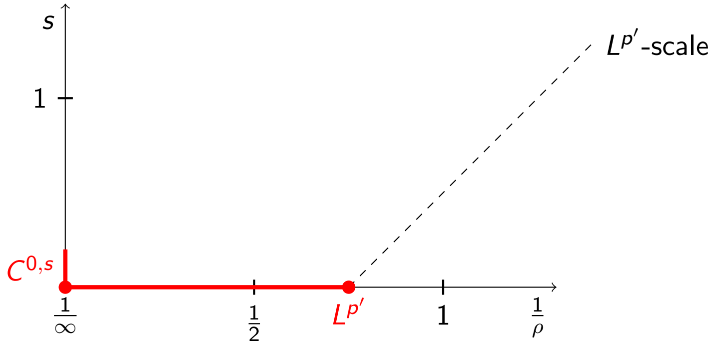
p-Poisson equation

$$-\mathrm{div}(A(\nabla u)) = -\mathrm{div}(|\nabla u|^{p-2}\nabla u) = -\mathrm{div} F = f, \qquad 1 < p < \infty$$

$\|\nabla^s A(\nabla u)\|_1 \lesssim \|f\|_1 \lesssim \|\nabla F\|_1$ — Avelin, Kuusi, Mingione 2018
local result, any $s \in (0,1)$

$\|\nabla A(\nabla u)\|_2 \lesssim \|f\|_2 \lesssim \|\nabla F\|_2$ — Cianchi, Mazya 2018
local + global result

DeVore-Triebel Diagramm ($p = 4$)



almost $X = W^{1,1}$ Avelin, Kuusi, Mingione 2018

$X = W^{1,2}$ Cianchi, Mazya 2018

Our main result:

$X = \mathbf{B}^s_{\rho,q}$ Diening, Kh. Balci, Weimar, 2019, ArXiv

$p \geq 2$, $\Omega \subset \mathbb{R}^2$, $\rho, q > 0$, $s \in (0,1)$, $\mathbf{B}^s_{\rho,q} \hookrightarrow L^{p'}$

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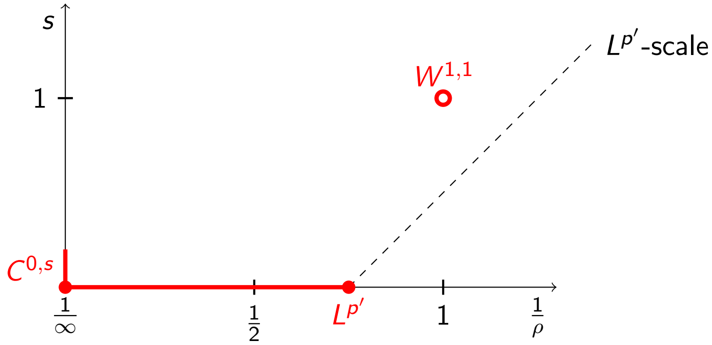

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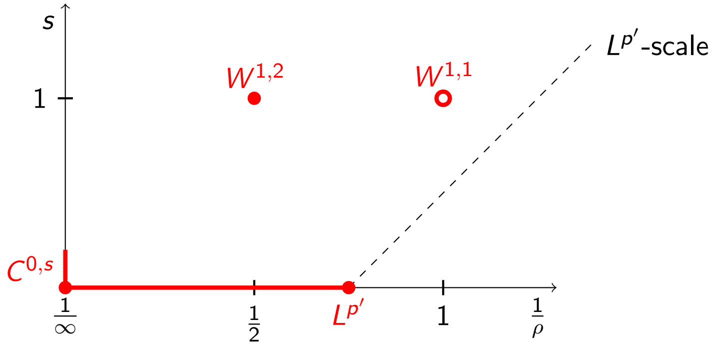

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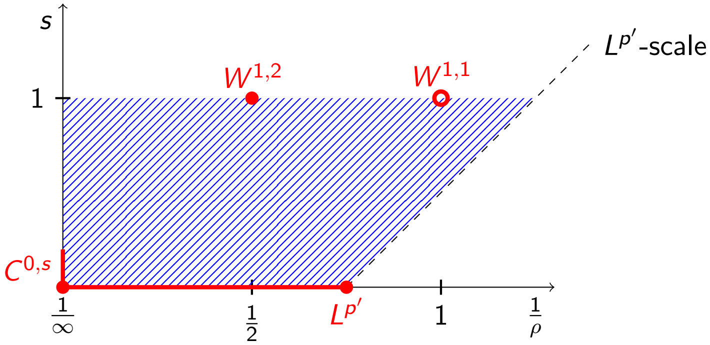

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Trade Integrability for Higher Differentiability

Dirichlet problem for Poisson equation in L-shaped domain:

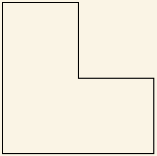

Figure: L-shaped

$$-\Delta u = 1 \quad \text{in } \Omega,$$
$$u = 0 \quad \text{on } \partial\Omega.$$

$u \in W^{1+\frac{2}{3},2} \cap W^{2,1}$ but $u \notin W^{2,2}$

Uniform mesh: $\|\nabla u - \nabla u_h\|_2 \leq c\, N^{-\frac{1}{3}} \|u\|_{W^{1+\frac{2}{3},2}}$

Adaptive methods: $\|\nabla u - \nabla u_h\|_2 \leq c\, N^{-\frac{1}{2}} \|u\|_{W^{2,1}}$

N is number of degrees of freedom.

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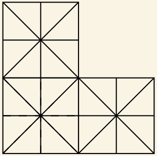

$$
\begin{aligned}
-\Delta u &= 1 \quad \text{in } \Omega, \\
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Figure: Uniform Mesh

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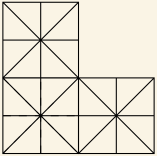

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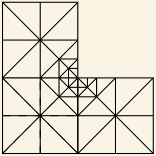

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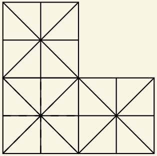

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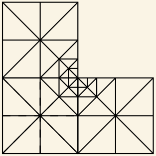

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Convergence Rate of Adaptive Methods

Adaptivity will equidistribute the error!

$$\|\nabla u - \nabla u_h\|_2^2 \approx \sum_T \int_T |\nabla u - \langle \nabla u \rangle_T|^2$$

$$\lesssim \sum_T \Big(\underbrace{\int_T |\nabla^2 u|}_{=:\delta} \Big)^2 = N\delta^2,$$

$$N\,\delta = \sum_T \int_T |\nabla^2 u| = \|\nabla^2 u\|_1,$$

$$\|\nabla u - \nabla u_h\|_2^2 \lesssim N \left(\frac{\|\nabla^2 u\|_1}{N} \right)^2 = N^{-1} \|\nabla^2 u\|_1^2.$$

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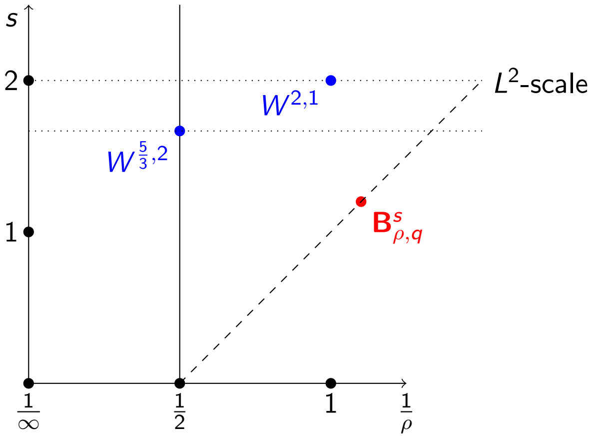
Adaptivity Scale $d = 2$



Approximability of $u \in \mathbf{B}^s_{\rho,q}$ in L^2 is $N^{-\frac{s}{d}}$. $\rho, q \in (0,1)$ possible!

Besov Regularity by Oscillations

Want to show:

$$|A(\nabla u)|_{\mathbf{B}^s_{\rho,q}} \lesssim |F|_{\mathbf{B}^s_{\rho,q}} + \text{lower order term of } A(\nabla u)$$

Besov Space

$\mathbf{B}^s_{\rho,q}$ with differentiability $s \in (0,1)$, integrability ρ and fine index q

Characterization by oscillations:

$$|g|_{\mathbf{B}^s_{\rho,q}} \eqsim \int_0^1 \left(\frac{\|\operatorname{osc}_{p'} g(\cdot,t)\|_{L_\rho}}{t^s} \right)^q \frac{dt}{t} \Big)^{\frac{1}{q}}.$$

for $0 < s < 1$ with $\mathbf{B}^s_{\rho,q} \hookrightarrow L^{p'}$.

with $$\operatorname{osc}_{p'} g(x,t) := \left(\fint_{B(x,t)} |g(y) - \langle g \rangle_{B(x,t)}|^{p'} \, dy \right)^{1/p'}$$

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From Oscillations decay to Besov Regularity

Oscillation decay:

$\operatorname{osc}_{p'} A(\nabla u)(x, \theta t) \lesssim \theta^s \operatorname{osc}_{p'} A(\nabla u)(x,t) + c_\theta \operatorname{osc}_{p'} F(x,t).$

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Regularity estimate (Main result; plane; $p \geq 2$)

$$|A(\nabla u)|_{\mathbf{B}^s_{\rho,q}(B)} \lesssim |F|_{\mathbf{B}^s_{\rho,q}(2B)} + \text{lower order term of } A(\nabla u).$$

if $0 < s < 1$, $\rho, q > 0$ and $\mathbf{B}^s_{\rho,q} \hookrightarrow L^{p'}$.

We use linear and non-linear comparison problem.

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Regularity for p-Harmonic

p-Laplace

$$-\mathrm{div}(A(\nabla h)) = 0$$

Only $h \in C^{1,\alpha}_{\mathrm{loc}}$ for some $\alpha > 0 \quad \Rightarrow \quad \nabla h \in C^{0,\alpha}_{\mathrm{loc}}, \; A(\nabla h) \in C^{0,\alpha_2}_{\mathrm{loc}}$

Ural'seva 1968, Uhlenbeck 1977, Tolksdorf 1984 ...

Best α only known for 2D ($\rightarrow$ next slide).

In 2D use complex analysis

Complex gradient $f = h_z = \frac{1}{2}(h_x - ih_y)$ is a quasiregular mapping:

$$\frac{\partial f}{\partial \bar{z}} = \left(\frac{1}{p} - \frac{1}{2}\right)\left(\frac{\bar{f}}{f}\frac{\partial f}{\partial z} + \frac{f}{\bar{f}}\frac{\overline{\partial f}}{\partial z}\right)$$

Bojarski, Iwaniec, Manfredi, Aronsson, Dobrowolski, Lindqvist

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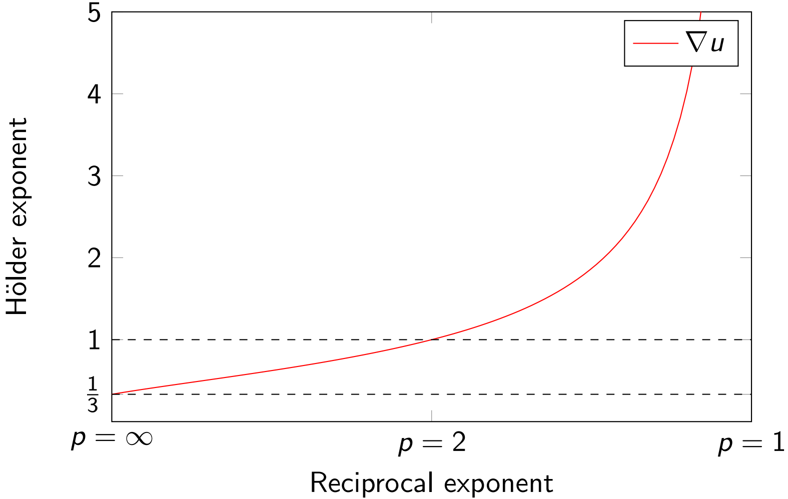
Bojarski, Iwaniec, Manfredi, Aronsson, Dobrowolski, Lindqvist

Regularity of p-harmonic Functions in 2D

$$A(\nabla u) = |\nabla u|^{p-2}\nabla u, \quad V(\nabla u) = |\nabla u|^{\frac{p-2}{2}}\nabla u$$



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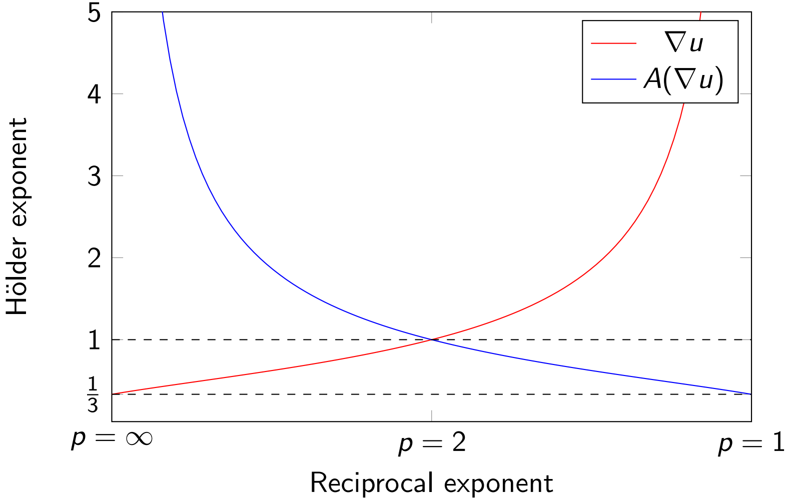
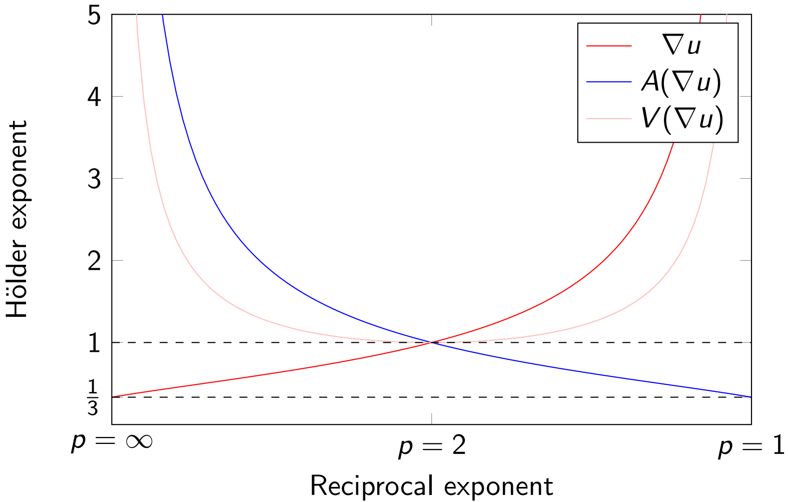


Regularity of p-harmonic Functions in 2D

$$A(\nabla u) = |\nabla u|^{p-2}\nabla u, \quad V(\nabla u) = |\nabla u|^{\frac{p-2}{2}}\nabla u$$



Improved A-Decay for p-harmonic Functions

$$-\operatorname{div}(A(\nabla h)) = 0$$

Almost linear A-decay for p-harmonic, i.e. for $\beta \in (0,1)$

$$\fint_{\theta B} |A(\nabla h) - \langle A(\nabla h)\rangle_{\theta B}|\,dx \le c_\beta \theta^\beta \fint_B |A(\nabla h) - \langle A(\nabla h)\rangle_B|\,dx$$

Non-degenerate: Like linear equation with constant coefficients:
Use linear V-decay of
Diening, Lengeler, Stroffolini, Verde '12

Degenerate: Use qualitative regularity result from
Araujo, Teixeira, Miguel Urbano 2017
based on quasi-conformal gradient estimates of
Baernstein II, Kovalev 2005

Remarks

Method inspired by pointwise estimates of
Breit, Cianchi, Diening, Kuusi, Schwarzacher 2017.

Case $1 < p < 2$

$F \in \mathbf{B}^s_{\rho,q} \;\Rightarrow\; A(\nabla u) \in \mathbf{B}^s_{\rho,q}$ fails for $p < 2$ even for $F = 0$.
Indeed, in the plane $A(\nabla h) \in C^{0,\alpha}$ with $\alpha \to \frac{1}{3}$ for $p \to 1$.

Problem if $s - \frac{2}{\rho} > \alpha$.

Summary

p-Poisson equation in the plane with $p \geq 2$

$$-\mathrm{div}(|\nabla u|^{p-2}\nabla u) = -\mathrm{div} F$$

Regularity transfer (Main result)

$$|A(\nabla u)|_{\mathbf{B}^s_{\rho,q}(B)} \lesssim |F|_{\mathbf{B}^s_{\rho,q}(2B)} + \text{lower order term of } A(\nabla u).$$

if $0 \leq s < 1$, $\rho, q > 0$ and $\mathbf{B}^s_{\rho,q} \hookrightarrow L^{p'}$.

Applications in Adaptive Finite Element Method (AFEM).

In progress: Up to the boundary.

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