On mathematical morphology, non-linear filters, and length scale control in topology optimization

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Topology optimization aims at determining the layout of material(s) within a given design domain Ω so that a given performance measure is extremized. If there are two types of materials, for convenience referred to as material and void, to fill the design domain, then the objective is to find a region $M \subset \Omega$ such that we have material in M and void in the region $V = \Omega \setminus M$. The region M occupied by material is referred to as the design. In practical applications some regularity on M (and/or V) is typically required. One such regularity constraint is that M exhibits a minimum length scale. Over the years, many different methodologies aiming at imposing a minimum length scale on the design have been introduced. However, a precise definition of the minimum length scale has rarely been used, meaning that the length scale has mostly been qualitatively assessed. We propose a definition of the minimum length scale for subsets of a bounded convex design domain, and show that the length scale so defined is tightly linked to the operators of mathematical morphology.

In practical applications, the material region is often defined by using a so-called material indicator function, or density, ρ (for now, the characteristic function of M). Material distribution topology optimization problems are generally ill-posed if no restriction or regularization method is used. A standard procedure to treat such a problem is to relax ρ to attain any value in the range $[0, 1]$ in combination with a filtering procudure and some form of penalization to promote binary designs. In a recent paper, we presented a framework of non-linear filters (fW-mean filters) that encompasses the vast majority of currently used density filters. This enables a unified analysis of a large class of material disctribution methods. We show that these filters ensure existence of solutions to a continuous version of the filtered and penalized minumum complinace topology optimization problem. Moreover, we present numerical experiments for minimum compliance problems, where we use density based topology optimization with morphology mimicking fW-mean filters based on the harmonic mean to obtain a minimum length scale on both material and void. The optimized designs are essentially binary and exhibit a minimum length scale on both material phases.