# VARIATIONAL FORMULATIONS AND FINITE ELEMENT APPROXIMATION OF THE CURL–DIV SYSTEM

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We consider the curl–div system with tangential or normal boundary condition:

$$\begin{cases} \operatorname{curl} \mathbf{u} = \mathbf{J} & \text{in } \Omega \\ \operatorname{div} \mathbf{u} = f & \text{in } \Omega \\ \mathbf{u} \times \mathbf{n} = \mathbf{a} & \text{on } \partial\Omega \end{cases} \qquad \begin{cases} \operatorname{curl} \mathbf{u} = \mathbf{J} & \text{in } \Omega \\ \operatorname{div} \mathbf{u} = f & \text{in } \Omega \\ \mathbf{u} \cdot \mathbf{n} = b & \text{on } \partial\Omega\,. \end{cases}$$

We first propose and analyze two variational formulations that rewrite the system as a saddle-point problem. Existence and uniqueness results are then an easy consequence of this approach.

Secondly, introducing suitable constrained Hilbert spaces, we devise other variational formulations that turn out to be useful for numerical approximation.

Concerning this second issue, the main novelty resides in the functional framework we adopt: we look for the solution in the space of curl-free or divergence-free vector fields. For the sake of implementation, we also describe in detail how to construct a simple finite element basis for these vector spaces; convergence of the finite element approximations is then easily shown.

Some numerical tests are also presented, illustrating the performance of the proposed approximation methods.

The results have been obtained in [1].

## REFERENCES

[1] A. Alonso Rodríguez, E. Bertolazzi, and A. Valli. The curl–div system: theory and finite element approximation. Preprint, 2016.

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