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Abstract. A reaction-diffusion initial-boundary problem with a Caputo time derivative of order $\alpha \in (0, 1)$ is considered. The solution of such a problem is shown in general to have a weak singularity near the initial time $t = 0$, and sharp pointwise bounds on certain derivatives of this solution are derived. A new analysis of a standard finite difference method for the problem is given, taking into account this initial singularity. This analysis encompasses both uniform meshes and meshes that are graded in time, and includes new stability and consistency bounds. The final convergence result shows clearly how the regularity of the solution and the grading of the mesh affect the order of convergence of the difference scheme, so one can choose an optimal mesh grading. Numerical results are presented that confirm the sharpness of the error analysis.

Reference

M.Stynes, E.O’Riordan & J.L.Gracia, *Error analysis of a finite difference method on graded meshes for a time-fractional diffusion equation*, SIAM J. Numer. Anal. 55 (2017), 1057–1079. DOI: 10.1137/16M1082329