On an Inverse Fresnel Problem.

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The main goal of GNSS-R (Global Navigation Satellite System Reflectometry), is to derive information on the composition of a portion of soil (e.g., soil moisture, snow depth, wave configurations, ...), by analyzing signals emitted by GNSS satellites, and the reflected signals captured by an antenna. To this end, one has to solve an inverse problem concerning the Fresnel polarization coefficients Γ_n and Γ_p (perpendicular and parallel). These are complex numbers depending on the incidence angle θ of the signal, and a complex parameter ε (the dielectric constant, or permittivity), which is intrinsic to the soil, and provides information on its composition and properties. In general, the antenna measures the received power of the signal, which is proportional to the modulus of Γ_n or Γ_p (or to combinations such as $\frac{1}{2}|\Gamma_n - \Gamma_p|$ or $\frac{1}{2}|\Gamma_n + \Gamma_p|$, for circular polarization). In each case, the goal is to recover the value of ε from the available measurements on Γ_n and Γ_p.

In this talk, we focus on the situation when we only have measurements on the moduli of Γ_n and Γ_p. In this case, the Fresnel formulas read

$$\left|\frac{\cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\cos\theta + \sqrt{\varepsilon - \sin^2\theta}}\right| = |\Gamma_n| \,, \tag{0.1}$$

$$\left|\frac{\varepsilon\cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}}\right| = |\Gamma_p| \,, \tag{0.2}$$

and there are two situations, according to whether the absorbing medium is dispersive (in which case $\Im(\varepsilon) \neq 0$), or not (i.e., $\Im(\varepsilon) = 0$). For dispersive media, we can regard (0.1)+(0.2) as a system in the real and imaginary parts of ε; in contrast, for non-dispersive media, system (0.1)+(0.2) is overdetermined, and is solvable only under specific compatibility conditions. We present a result on the solvability of (0.1)+(0.2) in either case.

Ideally, we would like to consider these results as the beginning of a rigorous treatment of systems of this form. The mathematics seems simple enough, but this simplicity is somewhat deceiving, in part because of a number of 'natural' conditions to be imposed on ε, and in part because of the large number of simplifying assumptions inherent to the model (0.1)+(0.2). Among these, that the underlying Maxwell equations are linear, that the electromagnetic field is time-harmonic, and that the scattering in all directions of the reflected signal is neglected, so that the fields are assumed to propagate along plane waves.