A posteriori error control for space-time in Isogeometric Analysis of parabolic problems

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We are concern with guaranteed error control of space–time Isogeometric Analysis (IgA) numerical approximations of parabolic evolution equations in fixed and moving spatial computational domains. The approach is discussed within the paradigm of classical *linear parabolic initial-boundary value problem (I-BVP)* as model problem: find $u : \overline{Q} \rightarrow \mathbb{R}^d$ such that

$$\partial_t u - \Delta_x u = f \text{ in } Q, \qquad u = 0 \text{ on } \Sigma, \qquad u = u_0 \text{ on } \overline{\Sigma}_0, \tag{1}$$

where $\overline{Q} := Q \cup \partial Q$, $Q := \Omega \times (0, T)$, denote the space-time cylinder with a bounded domain $\Omega \subset \mathbb{R}^d$, $d \in \{1, 2, 3\}$, having a Lipschitz boundary $\partial\Omega$, and $(0, T)$ is a given time interval, $0 < T < +\infty$. Here, the cylindrical surface is defined as $\partial Q := \Sigma \cup \overline{\Sigma}_0 \cup \overline{\Sigma}_T$ with $\Sigma = \partial\Omega \times (0, T)$, $\Sigma_0 = \Omega \times \{0\}$ and $\Sigma_T = \Omega \times \{T\}$.

Following the spirit of the paper from Langer, Moore, and Neumüller, 2016, we consider a *stable IgA space-time scheme* for variation formulation of (1), which is obtained by testing it with auxilary function $v_h + \delta_h \, \partial_t v_h$, $\delta_h = \theta \; h$, $v_h \in V_{0h} \subset H^1_{0,\underline{0}}(Q)$, where θ is a positive constant and h is the global mesh-size parameter (with mesh denoted by $\mathcal{K}_h$). The obtained discrete bilinear forms are V_{0h}-coercive on the IgA space with respect to corresponding discrete energy norms, which together with boundedness property, consistency and approximation results for the IgA spaces provides an a priori discretization error estimates.

Finally, we derive the functional a posteriori error estimates for the discussed schemes (see Repin, 2012), which apart from the quantitatively efficient indicators provides the reliable and sharp error estimates. This type of error estimates can exploit the higher smoothness of NURBS basis functions to its advantage. Since the obtained approximations are generally C^{p-1}-continuous (provided that the inner knots have the multiplicity 1), this automatically provides that its gradients are in $H(\Omega, \mathrm{div})$ space. Therefore, there is no need in projecting it from $\nabla u_h \in L^2(\Omega, \mathbb{R}^d)$ into $H(\Omega, \mathrm{div})$.

The efficiency of the obtained error bounds is analysed from both the error estimation (indication) and the computational expenses points of view. Several examples illustrate that functional error estimates (alternatively referred to as the *majorants* and *minorants* of deviation from an exact solution) perform a much sharper error control than, for instance, residual-based error estimates. Simultaneously, assembling and solving routines for an auxiliary variable reconstruction which generate the majorant of an error can be executed several times faster than the routines for a primal unknown.