Pathfollowing and a Posteriori Error Estimators for An Optimal Control Problem With BV-Functions

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Abstract: In this talk we consider an optimal control problem, or rather a regularization, with a state $y \in H_0^1(\Omega)$ and a control $u \in BV(\Omega)$ of bounded variation on a sufficiently smooth domain $\Omega \subset \mathbb{R}^n$, $n \in \{2,3\}$:

$$\begin{cases} \min\limits_{(y,u)\in H_0^1(\Omega)\times BV(\Omega)} \frac{1}{2}\|y-y_\Omega\|_{L^2(\Omega)}^2 + \beta|u|_{BV(\Omega)}, \\ \text{such that } -\Delta y = u \text{ and } y|_{\partial\Omega} = 0. \end{cases} \tag{P}$$

Here $y_\Omega \in L^2(\Omega)$ is some desired state and $\beta > 0$. This problem suffers from two non-smoothnesses: The term $|u|_{BV(\Omega)} = \int_\Omega d|\nabla u|$ and $u \in BV(\Omega)$. For $\gamma > 0$ and $\delta > 0$ we introduce a regularization of (P)

$$\begin{cases} \min\limits_{(y,u)\in H_0^1(\Omega)\times H^1(\Omega)} \frac{1}{2}\|y-y_\Omega\|_{L^2(\Omega)}^2 + \beta\int_\Omega \sqrt{\delta + |\nabla u|_2^2}\, dx + \frac{\gamma}{2}\|u\|_{H^1(\Omega)}^2, \\ \text{such that } -\Delta y = u \text{ and } y|_{\partial\Omega} = 0. \end{cases} \tag{$\mathrm{P}_{\gamma,\delta}$}$$

The first optimality conditions for ($\mathrm{P}_{\gamma,\delta}$) show that optimal solutions of ($\mathrm{P}_{\gamma,\delta}$) have higher regularity than just $H^1(\Omega)$. We show that this implies locally quadratic convergence of the Newton method for solving ($\mathrm{P}_{\gamma,\delta}$). The smooth formulation also allows us to deduce two a posteriori error estimators for the errors caused by γ and δ respectively. Based on these error estimators and locally quadratic convergent Newton method we present a pathfollowing strategy solving (P) up to a desired error.

We will provide numerical examples illustrating the structure of the problems,the algorithmic performance and the effectivity of the error estimators for the regularization errors.