CONVERGENCE OF A FINITE VOLUME SCHEME FOR A CROSS-DIFFUSION MODEL FOR ION TRANSPORT

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We study the numerical approximation of a cross-diffusion system describing ion transport through biological membranes. The ion concentrations solve strongly coupled, nonlinear diffusion equations with a drift term involving the electric potential, which is coupled to the concentrations through a Poisson equation. We propose an implicit Euler discretization in time and a finite volume discretization in space based on upwind fluxes. In the simplified case of equal diffusion constants, we prove the existence of solutions to the nonlinear system of equations arising from the numerical scheme. Under the additional assumption of vanishing potential, we prove the uniqueness of the approximate solution, a discrete entropy inequality and the convergence of the scheme. The uniqueness proof makes use of the E-monotonicity technique of Gajewski. The main difficulty for the convergence proof is the degeneracy of the a priori estimates for vanishing solvent concentration. For this reason, a new version of a discrete Aubin-Lions lemma is needed to establish the compactness of a sequence of approximate solutions. Finally, a numerical simulation of a calcium-selective ion channel is presented.

This is a joint work with Ansgar Jüngel (Vienna University of Technology), Claire Chainais-Hillairet and Clément Cancès (Université Lille 1).

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