Efficient approximation-schemes for Schrödinger-type equations including turning points

June 28, 2017

Kirian Döpfner Institute of Analysis and Scientific Computing
Wiedner Hauptstr. 8, 1040 Wien
kirian.doepfner@tuwien.ac.at

PhD advisor:
Anton Arnold Institute of Analysis and Scientific Computing
Wiedner Hauptstr. 8, 1040 Wien
anton.arnold@tuwien.ac.at

We are concerned with the highly oscillatory regime of a stationary (1D) Schrödinger equation including so-called turning points, i.e. zeros of the coefficient function. The numerical integration of a highly oscillating differential equation usually demands high computational cost. To avoid that, we introduce a hybrid method using a transformation to a smoother problem and then approximating the resulting oscillatory integrals yielding an efficient numerical scheme. In order to do so we make an approach which is motivated by asymptotic expansions found in [1] and a numerical scheme from [2]. A Hankel function-ansatz is used to transform the ODE hence to eliminate the dominant oscillations. This ODE can then be numerically approximated using asymptotic expansions in the rescaled Planck constant ε and the step-size of the spatial grid h. Overall this method yields an asymptotically correct scheme that is first order consistent with no necessity for a fine spatial grid.

References

[1] R. E. Langer. On the asymptotic solutions of ordinary differential equations, with an application to the bessel functions of large order. *Transactions of the American Mathematical Society*, 1931.

[2] A. Arnold, N. Ben Abdallah, and C. Negulescu. Wkb-based schemes for the oscillatory 1d schrdinger equation in the semiclassical limit. *SIAM Journal on Numerical Analysis*, 2011.