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⊛ $\dot{u}(t) + Bu(t) = F(u(t))$ $-B \sim$ semigroup.

F: continuous $X \to X$ Banach space

$x_e \in D(B)$

$F(x_e) = 0$, Frechét-derivative of F exists in x_e

Q: If $\dot{v}(t) + Bv(t) = F'[x_e]v(t)$ is asymptotically exp. stable

$\Rightarrow$ ⊛ locally exp. stable at x_e ?

Ex.: $\dot{v}(t) + Bv(t) = F(v_t)$ (DDE)

& $F: E \to X$, E = BUC (past prehistories)

Need: (Apparently) $D(F) = X$ (globally defined), globally Frechét-diff'ble, $x \mapsto F'[x]$ locally Lipschitz

Desch / Schappacher '86: $S(t): C \to C \subseteq X$ (strongly continuous non-linear operator), B-space

$S(t)x_e = x_e$, $\exists\ \tilde{S}(t)$ C_0-sem.: $\tilde{S}(t) = S(t)'[x_e]$

$\tilde{S}(t)$ exp. stable $\Rightarrow$ x_e loc. exp. stable

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As for (⊛): $B - F = A \rightsquigarrow (S(t))_{t \geq 0}$ s. cont. nl. semigr.

$B - \tilde{F} = \tilde{A} \rightsquigarrow (\tilde{S}(t))_{t \geq 0}$

Seek: Cond's on $A, \tilde{A}$ & s.t. $\tilde{S}(t) = S(t)'[x_e]$?

$A \subseteq X \times X$ ω- accretive :

$\forall (x_i, y_i) \in A: \quad (1-\lambda\omega)\|x_1 - x_2\| \leq \|x_1 - x_2 + \lambda(y_1 - y_2)\|$

$J_\lambda := (1 + \lambda A)^{-1}$ $\quad \frac{1}{1-\lambda\omega}$ - Lipschitz.

(C-L) $(R(I+\lambda A) \supset cl\, D(A)) \Rightarrow S(t)x = \lim_{n\to\infty} J_{t/n}^n x$

$0 \in \dot{u}(t) + A\,u(t)$

$A, \hat{A} \subseteq X \times X$

$B - F = A \sim J_\lambda^A \quad \tilde{J}_\lambda \sim B \tilde{-} \tilde{F}$

$J_\lambda^A(x) = J_\lambda^B(x + \lambda F(J_\lambda^A x))$

$J_\lambda^A(x) - x_e \overset{!}{=} \lambda J_\lambda^B [F(J_\lambda x) - F x_e - \hat{F}(J_\lambda x - x_e)]$

$+ \lambda J_\lambda^B \tilde{F} [J_\lambda x - x_e - \tilde{J}_\lambda(x - x_e)]$

(*) $\left[\begin{array}{l} \forall \varepsilon > 0 \ \exists \delta > 0 \quad x \in D(A), \|x - x_e\| < \delta \\ \Rightarrow \|Fx - Fx_e - \hat{F}(x - x_e)\| < \varepsilon \|x - x_e\| \end{array} \right.$

(*) $\Rightarrow \forall \varepsilon > 0 \ \exists \delta > 0, 0 < \lambda_0: \ x \in cl\, D(A), \|x - x_0\| < \delta$

$\Rightarrow \| J_\lambda x - J_\lambda x_0 - \tilde{J}_\lambda (x - x_0)\| \leq \varepsilon\lambda \|x - x_0\|$
$+ y(x,\lambda)$
$(\forall \lambda < \lambda_0)$

Def. $\tilde{A}$ r. diff'ble of A at $x_0 \in D(A)$: $\exists \lim_{(x,\lambda)\to(x_0,0)} y(x,\lambda) = 0$

$$\boxed{\tilde{A} \text{ (RD) } A \text{ at } x_e \Rightarrow \hat{S}(t) = S(t)'[x_e]}$$

If $(0,0) \in \tilde{A}$ and $\tilde{S}$ exp. stable $\Rightarrow x_e$ loc. exp. stable