Dirk

$$\begin{array}{ccccc} & A_0 & & A_1 & \\ H_0 & \longrightarrow & H_1 & \longrightarrow & H_2 \\ & \longleftarrow & & \longleftarrow & \\ & A_0^* & & A_1^* & \end{array}$$

Lemma (div-curl-lemma)

$A_0 : D(A_0) \subset H_0 \to H_1$, $A_1 : D(A_1) \subset H_1 \to H_2$, ldddc

$R(A_0) \subset N(A_1)$ cplx

Let $D(A_1) \cap D(A_0^*) \hookrightarrow\hookrightarrow H_1$

Then If $(x_n) \overset{bd}{\subset} D(A_1)$, $(y_n) \overset{bd}{\subset} D(A_0^*)$

$\Rightarrow \exists\ x \in D(A_1)$, $y \in D(A_0^*)$, s.t.

$x_{\pi n} \xrightarrow{D(A_1)} x$, $y_{\pi n} \xrightarrow{D(A_0^*)} y$ and

$$\langle x_{\pi n}, y_{\pi n} \rangle_{H_1} \longrightarrow \langle x, y \rangle_{H_1}$$

$$H_1 = R(A_0) \oplus N(A_0^*)$$

$$D(A_1) = R(A_0) \oplus \boxed{D(A_1) \cap N(A_0^*)}$$

$$x_{\pi n} = \underset{bd}{A_0 z_{\pi n}} + \underset{\text{bd in } H_1}{\tilde{x}_{\pi n}}$$

$z_{\pi n} \in D(A_0)$

$(z_{\pi n}) \overset{bd}{\subset} D(A_0) \hookrightarrow\hookrightarrow H_0$

$A_1 x_{\pi n} = A_1 \tilde{x}_{\pi n}$

| $D(A_1) \cap D(A_0^*) \hookrightarrow\hookrightarrow H_1$
| $\Downarrow$
| $D(A_0) \hookrightarrow\hookrightarrow H_0$
| $R(A_0) = R(A_0)$ ~~
| $\Rightarrow \exists c_0\ \forall x \in D(A_0)$
| $|x| \le c_0 |A_0 x|$
| $(z_{\pi n}) \overset{bd}{\subset} H_0$

$$= \langle z_{\pi n}, A_0^* y_{\pi n} \rangle$$

$$\langle x_{\pi n}, y_{\pi n} \rangle = \langle A_0 z_{\pi n}, y_{\pi n} \rangle + \langle \tilde{x}_{\pi n}, y_{\pi n} \rangle$$

$$\longrightarrow \langle z, A_0^* y \rangle + \langle \tilde{x}, y \rangle$$

$$= \langle Az, y \rangle\ \langle \tilde{x}, y \rangle$$

$$= \langle x, y \rangle$$

$z_{\pi n} \xrightarrow{H_0} z \in D(A_0)$

$\tilde{x}_{\pi n} \in D(A_1) \cap N(A_0^*)$

$\tilde{x}_{\pi n} \xrightarrow{H_1} \tilde{x}$