AANMPDE-9-16

Strobl, Austria, July 4–7, 2016

9th Workshop on Analysis and Advanced Numerical Methods for Partial Differential Equations (not only) for Junior Scientists

and

Schafberg Special Session, July 8, 2016

AANMPDE-9-16: Monday, July 4, 2016

10:00–10:20 "Registration"

10:20–10:30 Workshop Opening

Key Note Lecture. Chair: Sergey Repin

10:30–11:30	*Wolfgang Wendland*	On Potential Methods for Porous Media Flows

Session I. Chair: Sergey Repin

11:35–12:15	*Walter Zulehner*	A New Approach to Mixed Methods for Biharmonic Problems in 2D and 3D

12:15–13:00 Free Time

13:00–14:00 Lunch Break

Session II. Chair: Kees Osterlee

14:00–14:20	*Svetlana Matculevich*	A Posteriori Error Estimates For a Poroelastic Medium
14:20–14:40	*Immanuel Anjam*	Functional A Posteriori Error Control for Conforming Mixed Approximations of Parabolic Problems
14:40–15:00	*Olga Chistiakova*	Implementation of Functional Approach to A Posteriori Error Control for Curvilinear Timoshenko Beams
15:00–15:20	*Robert Schorr*	An Adaptive Non-Symmetric Finite Volume and Boundary Element Coupling Method for a Fluid Mechanics Interface Problem

15:20–15:40 Coffee Break

Session III. Chair: Martin Costabel

15:40–16:20	*Frank Osterbrink*	Time-harmonic Maxwell equations for radiation problems with mixed boundary conditions in exterior domains
16:20–16:40	*Monika Wolfmayr*	Space-Time Methods for Optimal Control Models in Pedestrian Dynamics
16:40–17:00	*Tytti Saksa*	Computational Performance of Controllability Techniques for Time-Periodic Waves Using Discrete Exterior Calculus

17:00–17:20 Coffee Break

Session IV. Chair: Wolfgang Wendland

17:20–17:40	*Svetoslav Nakov*	Functional A Posteriori Error Estimate for the Nonlinear Poisson-Boltzmann Equation
17:40–18:00	*Marjaana Nokka*	A Posteriori Error Bounds for Approximations of the Stokes Problem with Nonlinear Boundary Conditions
18:00–18:20	*Maxim Olshanskii*	Conservative Galerkin Methods for the Incompressible Navier-Stokes Equations

AANMPDE-9-16: Tuesday, July 5, 2016

KEY NOTE LECTURE. CHAIR: DIRK PAULY

10:00–11:00	*Martin Costabel*	Approximation of the LBB Constant on Corner Domains

SESSION I. CHAIR: DIRK PAULY

11:05–11:45	*Rainer Picard*	On Some Models of Thermo-Piezo-Electro-Magnetism

11:45–13:00 Free Time

13:00–14:00 Lunch Break

SESSION II. CHAIR: HERBERT EGGER

14:00–14:20	*Raffael Casagrande*	An Enriched Discontinuous Galerkin Method for Resolving Eddy Current Singularities by P-Refinement
14:20–14:40	*Anna Zubkova*	On Generalized Poisson-Nernst-Planck Equations with Inhomogeneous Boundary Conditions
14:40–15:00	*Marco Zank*	An Adaptive Space-Time Boundary Element Method for the Wave Equation

15:00–15:40 Coffee Break

SESSION III. CHAIR: MARCUS WAURICK

15:40–16:20	*Sascha Trostorff*	On Evolutionary Inclusions
16:20–16:40	*Thomas Wick*	Goal Functional Evaluations for Phase-Field Fracture Using PU-based DWR Mesh Adaptivity
16:40–17:00	*Jan Valdman*	Computations of Elasto-Plasto-Damage Model

17:00–17:20 Coffee Break

SESSION IV. CHAIR: MONIQUE DAUGE

17:20–17:40	*Sarah Eberle*	Boundary Integral Representation for the Elastic Wave Equation
17:40–18:00	*Daniel Walter*	Inverse Point Source Location for the Helmholtz Equation

AANMPDE-9-16: Wednesday, July 6, 2016

Key Note Lecture. Chair: Rainer Picard

10:00–11:00	*Monique Dauge*	Analytic Anisotropic Weighted Regularity in Polyhedra and Exponential Convergence of h-p Methods

Session I. Chair: Rainer Picard

11:05–11:45	*Marcus Waurick*	On the Eddy Current Approximation for Non-Autonomous Maxwell's Equations

11:45–13:00 Free Time

13:00–14:00 Lunch Break

14:00–19:00 Excursion

AANMPDE-9-16: Thursday, July 7, 2016

Key Note Lecture. Chair: Johannes Kraus

10:00–11:00	*Kees Oosterlee*	Fourier, Wavelet and Monte Carlo Methods in Computational Finance

Session I. Chair: Johannes Kraus

11:05–11:45	*Herbert Egger*	Damped Wave Systems on Networks: Exponential Stability and Uniform Approximations

11:45–13:00 Free Time

13:00–14:00 Lunch Break

Session II. Chair: Sascha Trostorff

14:00–14:20	*Thomas Kugler*	Structure Preserving Model Reduction for Damped Wave Propagation on Networks
14:20–14:40	*Bogdan Radu*	Superconvergence and Postprocessing for Mixed Finite Element Approximations of the Wave Equation
14:40–15:00	*Bernhard Stiftner*	An Extended Midpoint Scheme for the Landau-Lifshitz-Gilbert Equation in Computational Micromagnetics
15:00–15:20	*Ioannis Toulopoulos*	Numerical Methods for P-Power Law Diffusion Problems

15:20–15:40 Coffee Break

Session III. Chair: Maxim Olshanskii

15:40–16:00	*Stefan Takacs*	Robust multigrid methods for Isogeometric Analysis
16:00–16:20	*Christoph Hofer*	Dual-Primal Isogeometric Tearing and Interconnecting Solvers for Continuous and Discontinuous Galerkin IgA Equations
16:20–16:40	*Robert Gruhlke*	An Adaptive hp-XFEM Method for a Hardy Problem Featuring an Inverse Square Point Potential
16:40–17:00	*Joe Eyles*	A Finite Element Approach to Solving a Mathematical Model for Tumour Evolution

17:00–17:20 Coffee Break

Session IV. Chair: Walter Zulehner

17:20–17:40	*Duy Phan*	Riccati Based Feedback Stabilization to Trajectories for Parabolic Equations
17:40–18:00	*Sarah-Lena Bonkhoff*	Time Fractional Diffusion Equation
18:00–18:20	*Qingguo Hong*	A Sharp Korn's Inequality and Related Finite Elements

18:20–18:30 Workshop Ending

Schafberg Special Session: Friday, July 8, 2016

12:25–13:08 Ship

13:30–14:05 Train to Schafberg

Schafberg Afternoon Session. Chair: Dirk Pauly

15:00–15:45	*Sergey Repin*	Sharp Estimates of Constants in Functional Inequalities and A Posteriori Error Estimates for PDEs
16:00–16:30	*Christoph Koutschan*	Inverse Inequality Estimates with Symbolic Computation: Part One
16:30–17:00	*Cristian-Silviu Radu*	Inverse Inequality Estimates with Symbolic Computation: Part Two

ABSTRACTS: AANMPDE-9-16

Key Note Lecture I
Monday, July 4, 10:30–11:30

On Potential Methods for Porous Media Flows

► Wolfgang Wendland
(Universität Stuttgart, Germany)

Joint work with: Mirela Kohr (Babeş Bolyai Univ., Cluj–Napoca), Massimo Lanza de Cristoforis (Univ. Padua), and Sergey E. Mikhailov (Brunel University, Uxbridge)

We obtain existence and uniqueness results in Sobolev spaces for transmission problems for the nonlinear Darcy–Forchheimer–Brinkman system in a Lipschitz domain and the linear Stokes system in the exterior in $\mathbb{R}^3$ and also on compact Riemannian manifolds. We apply a layer potential method for the Stokes and Brinkman systems combined with a fixed point argument to obtain existence. Uniqueness is also obtained for sufficiently small given data.

Key Note Lecture II
Tuesday, July 5, 10:00–11:00

Approximation of the LBB Constant on Corner Domains

► Martin Costabel
(Université de Rennes 1, France)

In this talk we present recent results about corner singularities and approximation of the inf-sup constant of the divergence, also known as LBB constant, see the references below.

The inf-sup constant of the divergence (LBB constant) of a domain is a quantitative property that plays an important role in the analysis and numerical analysis of the Stokes and Navier-Stokes systems. It governs pressure stability estimates, convergence of iterative algorithms, and in its discrete form as Babuška-Brezzi condition, the stability and convergence of finite element methods and of the solution of associated linear systems. Because of this practical importance in theoretical and numerical fluid dynamics, an enormous body of literature exists that provides estimates for continuous and discrete LBB constants. It has also long been known that the LBB condition is closely related to other important inequalities such as Korn's inequality and to non-standard eigenvalue problems (Cosserat or Stokes). Not much is known about the numerical approximation of the value of the constant for a given domain. This is a difficult problem for several reasons that will be explained in the talk. One reason is that the eigenvalue problems associated with the constant are not of standard elliptic type, but have an essential spectrum. Another reason is the appearance of strong corner singularities in this eigenvalue problem. Non-H^1 singular functions are related to the presence of a continuous spectrum in the Cosserat eigenvalue problem. Only recently have we found sufficient conditions for the convergence of finite element approximation of the LBB constant, but many questions still remain open.

References

[1] C. Bernardi, M. Costabel, M. Dauge, and V. Girault, *Continuity properties of the inf-sup constant for the divergence*, SIAM J. Math. Anal., 48 (2016), pp. 1250–1271.

[2] M. Costabel, *Inequalities of Babuška-Aziz and Friedrichs-Velte for differential forms*. To appear in Operator Theory, Advances and Applications, arXiv: 1507.08464.

[3] M. Costabel, M. Crouzeix, M. Dauge, and Y. Lafranche, *The inf-sup constant for the divergence on corner domains*, Numer. Methods Partial Differential Equations, 31 (2015), pp. 439–458.

[4] M. Costabel and M. Dauge, *On the inequalities of Babuška-Aziz, Friedrichs and Horgan-Payne*, Archive for Rational Mechanics and Analysis, 217 (2015), pp. 873–898.

[5] M. Dauge, C. Bernardi, M. Costabel, and V. Girault, *On Friedrichs constant and Horgan-Payne angle for LBB condition*, in Twelfth International Conference Zaragoza-Pau on Mathematics, vol. 39 of Monogr. Mat. García Galdeano, Prensas Univ. Zaragoza, Zaragoza, 2014, pp. 87–100.

Analytic Anisotropic Weighted Regularity in Polyhedra and Exponential Convergence of h-p Methods

► Monique Dauge
(Université de Rennes 1, France)

Weighted spaces with analytic type control of derivatives were first introduced by Ivo Babuška and Benqi Guo for 2D polygonal domains in the late 80'ies [1, 2] with the name "countably normed spaces". They have proved that such a regularity holds for several model problems. These spaces allow exponentially fast approximation by piecewise polynomials in the framework of the *h-p* version of finite elements.

Benqi Guo has introduced the corresponding relevant spaces in 3D polyhedra in 1993 [5]. Their special feature (absent in 2D) is their anisotropy along edges. The proof that solutions of the Laplace equation with analytic right hand sides belong to such spaces, was still pending.

Then came 20 years later the result by Costabel, Dauge and Nicaise [3, 4]. A full zoology of weighted spaces is introduced in order to give the best account of the natural regularity of solutions when the right hand side is an analytic function, and the boundary conditions are either Dirichlet or Neumann. Step weighted anisotropic norms have to be used. We will motivate these definitions by the structure of singularities along edges and at corners of a polyhedron.

Finally we will present recent results by Schötzau, Schwab and Wihler that prove the exponential convergence of the *h-p* version of finite elements, first for DG (discontinuous Galerkin) methods [8, 9, 10], then for continuous (H^1 conforming) elements [6, 7].

References

[1] I. Babuška and B. Guo, *Regularity of the solution of elliptic problems with piecewise analytic data. I. Boundary value problems for linear elliptic equation of second order*, SIAM J. Math. Anal., 19 (1988), pp. 172–203.

[2] ———, *Regularity of the solution of elliptic problems with piecewise analytic data. II. The trace spaces and application to the boundary value problems with nonhomogeneous boundary conditions*, SIAM J. Math. Anal., 20 (1989), pp. 763–781.

[3] M. Costabel, M. Dauge, and S. Nicaise, *Analytic regularity for linear elliptic systems in polygons and polyhedra*, Math. Models Methods Appl. Sci., 22 (2012), pp. 1250015, 63.

[4] ———, *Weighted analytic regularity in polyhedra*, Comput. Math. Appl., 67 (2014), pp. 807–817.

[5] B. Guo, *The h-p version of the finite element method for solving boundary value problems in polyhedral domains*, in Boundary value problems and integral equations in nonsmooth domains (Luminy, 1993), M. Costabel, M. Dauge, and S. Nicaise, eds., Dekker, New York, 1995, pp. 101–120.

[6] D. Schötzau and C. Schwab, *Exponential convergence for hp-version and spectral finite element methods for elliptic problems in polyhedra*, Math. Models Methods Appl. Sci., 25 (2015), pp. 1617–1661.

[7] ———, *Exponential convergence of hp-FEM for elliptic problems in polyhedra: Mixed boundary conditions and anisotropic polynomial degrees*, SAM Report, ETH Zürich, 2016-05 (2016).

[8] D. Schötzau, C. Schwab, and T. P. Wihler, *hp-dGFEM for second-order elliptic problems in polyhedra I: Stability on geometric meshes*, SIAM J. Numer. Anal., 51 (2013), pp. 1610–1633.

[9] ———, *hp-DGFEM for second order elliptic problems in polyhedra II: Exponential convergence*, SIAM J. Numer. Anal., 51 (2013), pp. 2005–2035.

[10] ———, *hp-dGFEM for second-order mixed elliptic problems in polyhedra*, Math. Comp., 85 (2016), pp. 1051–1083.

Key Note Lecture IV
Thursday, July 5, 10:00–11:00

Fourier, Wavelet and Monte Carlo Methods in Computational Finance

► Kees Oosterlee
(CWI - Center for Mathematics and Computer Science, Amsterdam, Netherlands; Delft University of Technology, Netherlands)

In this presentation we will explain how we can solve linear, semi-linear as well as nonlinear partial differential equations by the concept of conditional expectations and (backward) stochastic differential equations, Fourier cosine expansions, wavelets and Monte Carlo simulation.

We also present typical research questions in finance from a mathematical perspective. We will discuss in some detail the highly efficient pricing of financial options in the Fourier context. Also the use of the Stochastic Grid Bundling Method (SGBM) which is a Monte Carlo technique in the financial context of high-dimension option pricing and risk management is explained and detailed.

Session I (1 talk)
Monday, July 4, 11.35–12.15

A New Approach to Mixed Methods for Biharmonic Problems in 2D and 3D

▶ Walter Zulehner
(Johannes Kepler Universität Linz, Austria)

A new variant of a mixed variational formulation for a biharmonic problem is presented, which involves a non-standard Sobolev space for the Hessian of the original unknown. Based on a Helmholtz-like decomposition of this non-standard Sobolev space, the fourth-order problem can be rewritten as a sequence of three (consecutively to solve) second-order problems.

On the discrete level this approach can be exploited in 2D to reformulate the well-known Hellan-Herrmann-Johnson method in such a way that the assembling of the discretized equations involves only standard Lagrangian elements. Similar to the continuous level a decomposition of the discretized problem into three discretized second-order problems is available, which substantially simplifies the construction of efficient solution techniques on the discrete level.

Possible extensions to more general classes of fourth-order problems will also be shortly discussed.

Session II (4 talks)
Monday, July 4, 14.00–15.20

A Posteriori Error Estimates For a Poroelastic Medium

▶ Svetlana Matculevich
(RICAM, Austria)

The work is dedicated to the topic of a posteriori error estimates for poroelastic models, in particular the so-called Biot system (see, e.g., [1, 6]), which contributes to a wide range of application areas including simulation of oil reservoirs, preventing environmental changes, predicting the liquefaction of the soil in earthquake engineering as well as various studies in biomechanics.

In particular, it deals with a construction of the upper bound of the approximation error in iterative algorithms for coupling of the flow and the geomechanics [2]. The resulting bound is based on combination of Ostrowski's estimates [3] (derived for error control in the contractive iteration algorithm) as well as the a posteriori error majorants of functional type (see [4, 5]).

References

[1] M.A. Biot, *General theory of three-dimensional consolidation*, Journal of applied physics, 12(2), pp. 155–164, 1941.

[2] A. Mikelić and M. F. Wheeler, *Convergence of iterative coupling for coupled flow and geomechanics*, Comput. Geosci., 17(3), pp. 455–461, 2013.

[3] A. Ostrowski, *Les estimations des erreurs a posteriori dans les procédés itératifs*, C.R. Acad.Sci. Paris Sér. A–B, 275, pp. A275–A278, 1972.

[4] S. Repin, *A posteriori estimates for partial differential equations*, Radon Series on Computational and Applied Mathematics, p. xii+316, 2008.

[5] , P. Neittaanmäki and S. Repin, *Reliable methods for computer simulation*, Studies in Mathematics and its Applications, Elsevier Science B.V., p. x+305, 2004.

[6] R. E. Showalter, *Diffusion in poro-elastic media*, J. Math. Anal. Appl., 251(1), pp. 310–340, 2000.

Functional A Posteriori Error Control for Conforming Mixed Approximations of Parabolic Problems

► Immanuel Anjam
(Universität Duisburg-Essen, Germany)

Joint work with: D. Pauly (Universität Duisburg-Essen, Germany)

We demonstrate a simple method to derive a posteriori error equalities for problems with lower order terms (static and time-dependent reaction-diffusion problems), and use these equalities to derive two-sided error estimates for problems without lower order terms (diffusion and heat equation).

Implementation of Functional Approach to A Posteriori Error Control for Curvilinear Timoshenko Beams

► Olga Chistiakova
(St. Petersburg Polytechnic University, Russia)

Joint work with: M. Frolov (St. Petersburg Polytechnic University, Russia)

A new functional type a posteriori error estimate is obtained for the problems of bending of curvilinear Timoshenko beams under the action of traverse and axial forces and a bending moment. Similar estimates are known for Euler-Bernoulli beams (O. Mali, 2009-2011) and for straight Timoshenko beams (M. Frolov, 2010). The derived estimate is reliable, is applicable to any conforming approximation and can be used both for global accuracy control and for local error indication for subsequent mesh adaptations. Theoretical properties of the estimate are confirmed by results of numerical experiments, which show that it might be effectively applied to mathematical modeling in engineering.

An Adaptive Non-Symmetric Finite Volume and Boundary Element Coupling Method for a Fluid Mechanics Interface Problem

► Robert Schorr
(TU Darmstadt, Germany)

Joint work with: C. Erath (TU Darmstadt, Germany)

In this work we consider an interface problem often arising in transport problems: a coupled system of partial differential equations with one (elliptic) equation on a bounded domain and one equation (in this case the Laplace problem) on the complement, an unbounded domain. To solve this system we will use the non-symmetric coupling approach of Finite Volume Method and Boundary Element Method introduced in [2]. We extend this approach by introducing a residual error estimator and showing important properties such as reliability (the error estimator bounds the approximation error from above) and, under some restrictions on the mesh, also efficiency (the converse estimate). Additional assumptions even allow the construction of a robust error estimator. With this error estimator an adaptive algorithm is devised and tested on several numerical examples. The adaptive FVM-BEM coupling turns out to be an efficient method especially to solve problems from fluid mechanics, mainly because of the local flux conservation and the stable approximation of convection dominated problems.

The work of the second author is supported by the 'Excellence Initiative' of the German Federal and State Governments and the Graduate School of Computational Engineering at Technische Universität Darmstadt.

References

[1] C. Erath and R. Schorr: An adaptive non-symmetric finite volume and boundary element coupling method for a fluid mechanics interface problem. Preprint (2016), available online in May 2016.

[2] C. Erath, G. Of, and F.-J. Sayas: A non-symmetric coupling of the finite volume method and the boundary element method. Preprint (2015), available online: http://arxiv.org/abs/1509.00440.

[3] C. Erath: A posteriori error estimates and adaptive mesh refinement for the coupling of the finite volume method and the boundary element method. SIAM J. Numer. Anal. 51(3), 1777–1804 (2013).

Session III (3 talks)
Monday, July 4, 15:40–17:00

Time-harmonic Maxwell equations for radiation problems with mixed boundary conditions in exterior domains

► Frank Osterbrink

(Universität Duisburg-Essen, Germany)

We will talk about the weak solution theory for the time-harmonic Maxwell equations

$$\begin{aligned} -\operatorname{rot} H + i\omega\varepsilon E &= F \quad \text{in} \quad \Omega, & E \times n &= 0 \quad \text{on} \quad \Gamma_\tau, \\ \operatorname{rot} E + i\omega\mu H &= G \quad \text{in} \quad \Omega, & H \times n &= 0 \quad \text{on} \quad \Gamma_\nu, \end{aligned}$$

where $\Omega \subset \mathbb{R}^3$ is an exterior weak Lipschitz domain with boundary Γ, which is decomposed into two disjoint parts Γ_τ and Γ_ν.

Since we are working in an exterior domain, we will use the framework of polynomially weighted Sobolev spaces for the rotation and divergence. For the interesting physical case $\omega \in \mathbb{R} \setminus \{0\}$ the main tool is the principle of limiting absorption introduced by Eidus. We will present a decomposition result, which enables us to use well known results for the Helmholtz equation to prove the necassary a-priori-estimate and polynomial decay of eigenfunctions for the Maxwell equations.

Space-Time Methods for Optimal Control Models in Pedestrian Dynamics

► Monika Wolfmayr

(RICAM, Austria)

The complex behavior of large pedestrian groups has always fascinated researchers from various scientific fields. Starting with empirical observations, its research has continued with the development of different models in the field of applied physics and more recently applied mathematics. In this talk, we focus on optimal control models, which describe the evolution of a large pedestrian group trying to reach a specific target with minimal cost. We discuss different choices of cost functionals and the connection to the Hughes model for pedestrian flow. The proposed space-time method is based on the Benamou and Brenier formulation of optimal transport problems. We present its extension to more general cost functionals and optimal control problems and illustrate the dynamics with numerical simulations.

Computational Performance of Controllability Techniques for Time-Periodic Waves Using Discrete Exterior Calculus

► Tytti Saksa
(University of Jyväskylä, Finland)

Computational performance of Glowinski's controllability techniques [1, 2] for finding time-periodic solutions of a scalar wave equation is addressed. A controllability algorithm accelerates the solution process based on the idea of minimizing the difference between the initial conditions and the corresponding variables after one time period. Pauly and Rossi [3] formulated the controllability algorithm in terms of differential forms which can be discretized naturally using discrete exterior calculus. Numerical performance of the controllability algorithm is investigated by comparing two spatial discretizations, the first one being based on finite elements and the second one on discrete exterior calculus, for selected scattering problems of acoustic type.

References

[1] M. O. Bristeau, R. Glowinski, and J. Périaux. Controllability methods for the computa- tion of time-periodic solutions; Application to scattering. Journal of Computational Physics, 147(2):265–292, 1998.

[2] R. Glowinski and T. Rossi. A mixed formulation and exact controllability approach for the computation of the periodic solutions of the scalar wave equation. (I) controllability problem formulation and related iterative solution. C. R. Math. Acad. Sci. Paris, 343(7):493–498, 2006.

[3] D. Pauly and T. Rossi. Theoretical considerations on the computation of generalized time-periodic waves. Advances in Mathematical Sciences and Applications, 21(1):105–131, 2011.

Session IV (3 talks)
Monday, July 4, 17:20–18:20

Functional A Posteriori Error Estimate for the Nonlinear Poisson-Boltzmann Equation

► Svetoslav Nakov
(RICAM, Austria)

Joint work with: J. Kraus (Universität Duisburg-Essen, Germany) and S. Repin (Steklov Institute, St. Petersburg, Russia)

In this talk, we show a short derivation of the Poisson-Boltzmann equation and then the focus goes on deriving a functional a posteriori error estimate for the PBE. The advantage of the functional a posteriori error estimates based on the duality theory is that only the structure of the equation alone is exploited and therefore no global or local constants enter in the estimate. This is in contrast to other methods, e.g a residual based one, which depend on the particular triangulation. Therefore functional type a posteriori error estimates give not only an error indicator, but also a guaranteed bound on the error. We discuss two cases for the coefficient in the nonlinear term. Firstly, we derive an error estimate for a uniformly positive coefficient. Secondly, we obtain such error estimate when this coefficient is identically zero in the molecular part of the domain.

A Posteriori Error Bounds for Approximations of the Stokes Problem with Nonlinear Boundary Conditions

► Marjaana Nokka
(University of Jyväskylä, Finland)

We consider stationary model of Stokes problem with nonlinear slip and leak boundary conditons. This mathematical model is often used for simulation of blood veins affected by sclerosis. We present functional a posteriori estimates for the deviation between exact and approximate solutions in the energy norm that take into account these nonlinar boundary conditions.

Conservative Galerkin Methods for the Incompressible Navier-Stokes Equations

► Maxim Olshanskii
(University of Houston, USA)

In the talk we discuss conservation properties of Galerkin methods for the incompressible Navier-Stokes equations, without the divergence constraint strongly enforced. In typical discretizations such as the mixed finite element method, the conservation of mass is enforced only weakly, and this leads to discrete solutions which may not conserve energy, momentum, angular momentum, helicity, or vorticity, even though the physics of the Navier-Stokes equations dictate that they should. We aim to construct discrete formulations that conserve as many physical laws as possible without utilizing a strong enforcement of the divergence constraint. Doing so leads to a new formulation that conserves each of energy, momentum, angular momentum, enstrophy in 2D, helicity and vorticity.

Session I (1 talk)
Tuesday, July 5, 11:05–11:45

On Some Models of Thermo-Piezo-Electro-Magnetism

► Rainer Picard
(TU Dresden, Germany)

Joint work with: A. Mulholland (University of Strathclyde, UK), S. Trostorff (TU Dresden, Germany), and M. Waurick (University of Bath, UK)

Various linear models describing the interaction between thermodynamic, electromagnetic and elastic effects are discussed within a first order approach to the resulting coupled system. Well-posedness, i.e. existence, uniqueness of solutions and their continuous, causal dependence on the data, is established for associated initial boundary value problems covering a large class of materials. This is in part a report on joint work with A. Mulholland, S. Trostorff and M. Waurick, [1].

References

[1] A. J. Mulholland, R. Picard, S. Trostorff, and M. Waurick. On well-posedness for some thermo-piezoelectric coupling models. *Mathematical Methods in the Applied Sciences*, (currently available online), 2016.

[2] R. Picard. A Structural Observation for Linear Material Laws in Classical Mathematical Physics. *Math. Methods Appl. Sci.*, 32(14):1768–1803, 2009.

Session II (4 talks)
Tuesday, July 5, 14:00–15:20

An Enriched Discontinuous Galerkin Method for Resolving Eddy Current Singularities by P-Refinement

► Raffael Casagrande
(ETH Zürich, Switzerland)

Joint work with: R. Hiptmair (ETH Zürich, Switzerland)

The eddy current equation exhibits singular behavior in corners of conductors. This leads to a sub-optimal rate of convergence for both the h- and the p-finite element method (FEM) because singular functions are poorly approximated by polynomials. In 1973 Strang and Fix [1] included singular functions in the approximation space and showed that optimal convergence rates for h-refinement are obtained.

In this talk we consider a simple variant of this method: We enrich the polynomial approximation space with singular functions and increase the polynomial degree to achieve exponential convergence. The support of the singular basis functions is restricted to the elements adjacent to the corner(s) causing the singularity and there are no transition elements. Instead we use the Interior Penalty Method to couple the discontinuous basis functions.

Recent numerical results show exponential convergence and reveal that the method yields much better results than the Method of Strang/Fix (using p-refinement) for the same number of degrees of freedom.

References

[1] G. Strang, G. Fix. An Analysis of the Finite Element Method. Prentice-Hall, Englewood Cliffs, 1973

On Generalized Poisson-Nernst-Planck Equations with Inhomogeneous Boundary Conditions

► Anna Zubkova

(Karl-Franzens-Universität Graz, Austria)

Joint work with: V. A. Kovtunenko (Karl-Franzens-Universität Graz, Austria)

A time-dependent Poisson-Nernst-Planck system of nonlinear partial differential equations is considered. It is modeled in terms of the Fickian multiphase diffusion law coupled with thermodynamic principles. This system describes different electrokinetics in physical and biological sciences. The generalized model is supplied by volume and positivity constraints and quasi-Fermi electrochemical potentials depending on the pressure. We examine the nonlinear inhomogeneous transmission boundary conditions describing electro-chemical reactions on the boundary. We aim at a proper variational modeling, well-posedness and asymptotic analysis as well as homogenization of the model at the macro-scale level.

The work is supported by the Austrian Science Fund (FWF) in the framework of the research project P26147-N26: PION.

Finite Elements with Mesh Refinement for Linear Wave Propagation

► Fabian Müller
(ETH Zürich, Switzerland)

When dealing with propagation phenomena of acoustic or seismic waves, second-order linear wave equations provide a commonly used physical model. The method of lines is a popular simulation technique. There, the partial differential equation (PDE) is discretized in space first, followed by a time-stepping scheme solving the resulting ordinary differential equation. When using a Finite Element method (FEM) for the spatial discretization, the convergence order strongly depends on the regularity of the solution. When dealing with a PDE posed on a polygonal domain or in the presence of piecewise smooth coefficients ('wavespeeds'), the solution exhibits strongly singular behaviour in the neighbourhood of an isolated point set, implying slow convergence of the FEM on quasi-uniform meshes. Recently, we have proved that quasi-optimal convergence rates can be obtained again for the h-version of FEM with arbitrarily high local polynomial degree. We present results which are applicable to acoustic and elastic wave equations in homogeneous media, as well as the acoustic wave equation with piecewise constant wavespeeds.

An Adaptive Space-Time Boundary Element Method for the Wave Equation

► Marco Zank
(TU Graz, Austria)

For the discretisation of the wave equation by boundary element methods the starting point is the so-called Kirchhoff's formula, which is a representation formula by means of boundary potentials. In this talk different approaches to derive weak formulations of related boundary integral equations are considered. First, weak formulations based on the Laplace transform and second, space-time energetic formulations are introduced. In both cases coercivity is shown in appropriate Sobolev spaces. To derive an adaptive scheme an a posteriori error estimator based on the representation formula is used.

Finally, numerical examples for a one-dimensional spatial domain are presented and discussed.

Session III (3 talks)
Tuesday, July 5, 15:40–17:00

On Evolutionary Inclusions

► Sascha Trostorff
(TU Dresden, Germany)

We present a generalization of the framework of evolutionary equations introduced by Picard in 2009 to a nonlinear setting, by involving so-called maximal monotone operators. This generalization allows the treatment of nonlinear phenomena, like hysteresis in plasticity theory or frictional boundary conditions in contact problems.

Goal Functional Evaluations for Phase-Field Fracture Using PU-based DWR Mesh Adaptivity

► Thomas Wick
(RICAM, Austria)

In this presentation, a posteriori error estimation and goal-oriented mesh adaptivity are considered for phase-field fracture propagation. Goal functionals are computed with the dual-weighted residual method (DWR), which is realized by a recently introduced novel localization technique based on a partition-of-unity (PU). This technique is straightforward to apply as the weak residual is used. Influence of neighboring cells is gathered by the PU. Consequently, neither strong residuals nor jumps over element edges are required. Therefore, this approach facilitates the application of the DWR method to coupled (nonlinear) multiphysics problems such as fracture propagation. These developments then allow for a systematic investigation of the discretization error for certain quantities of interest. Specifically, our focus on the relationship between the phase-field regularization and the spatial discretization parameter in terms of goal functional evaluations is novel.

Computations of Elasto-Plasto-Damage Model

▶ Jan Valdman
(University of South Bohemia, Czechia)

Joint work with: T. Roubíček (Czech Academy of Sciences, Czechia)

This talk will report of results of two papers:

1. Tomáš Roubíček, Jan Valdman: Stress-driven solution to rate-independent elasto-plasticity with damage at small strains and its computer implementation. Mathematics and Mechanics of Solids (published online)

2. Tomáš Roubíček, Jan Valdman: Perfect plasticity with damage and healing at small strains, its modelling, analysis, and computer implementation. SIAM Journal on Applied mathematics 76, No. 1, 314-340 (2016)

The main focus or the talk is to explain computational issues and implementations details.

Session IV (3 talks)
Tuesday, July 5, 17:20–18:20

Direct Method for Determination of the Coefficients for Some Hyperbolic Equations

▶ Nikita Novikov
(Novosibirsk State University, Russia)

We consider the coefficient inverse problem for hyperbolic equations, like one-dimensional seismic inverse problem or two-dimensional acoustic inverse problem. We use the Gelfand-Levitan approach to reduce nonlinear inverse problems to families of linear integral equations. We present several numerical methods, based on the properties of Gelfand-Levitan equation, like Monte-Carlo method [1], stochastic iterative projection method [2], and method, based on the fast inversion of the Toeplitz matrix [3]. The results of the numerical experiments are presented.

References

[1] Kabanikhin, Sergey I. , Sabelfeld, Karl K. , Novikov, Nikita S. , Shishlenin, Maxim A. Numerical solution of an inverse problem of coefficient recovering for a wave equation by a stochastic projection methods, Monte Carlo methods and applications, Volume 21, Issue 3, p. 189–203.

[2] Kabanikhin, Sergey I. , Novikov, Nikita S. , Sabelfeld, Karl K. , Shishlenin, Maxim A. Numerical solution of the multidimensional Gelfand-Levitan equation, Journal of inverse and ill-posed problems, Volume 23, Issue 5, p. 439–450.

[3] Kabanikhin, Sergey I., Novikov, Nikita S., Oseledets, Ivan V., Shishlenin, Maxim A. Fast Toeplitz linear system inversion for solving two-dimensional acoustic inverse problem, Journal of Inverse and Ill-posed Problems, Volume 23, Issue 6, Pages 687–700.

Boundary Integral Representation for the Elastic Wave Equation

► Sarah Eberle

(Universität Tübingen, Germany)

We discuss numerical analysis aspects of the elastic wave equation in three-dimensions. For solving this equation numerically for non-convex domains, we construct the so-called Calderón operator, which couples the boundary integrals and the wave equation via the transmission conditions. The construction of this operator is based on the fundamental solution of the elastic wave equation as well as the corresponding potentials and boundary integrals. The major result of this work is the lemma which shows the positivity of the Calderón operator, since positivity is required to prove further important properties, e.g. the stability of the used numerical methods for discretization.

Inverse Point Source Location for the Helmholtz Equation

► Daniel Walter

(TU München, Germany)

Joint work with: K. Pieper, B. Tang, and P. Trautmann (TU München, Germany)

In this talk we consider the identification of a periodic accoustic point source from finitely many measurements. Given a system of N Helmholtz equations

$$\begin{aligned} -\omega_n^2 p_n - c^2 \Delta p_n = \hat{u}_n \delta_{\hat{x}} \quad & \text{in } \Omega, \\ i\omega_n p_n + c\partial_\nu p_n = 0 \quad & \text{on } \Gamma_{\text{art}}, \\ c\partial_\nu p_n = 0 \quad & \text{on } \Gamma_{\text{D}}, \end{aligned} \tag{1}$$

on $\Omega \subset \mathbb{R}^d$ with $\partial\Omega = \Gamma_{\text{art}} \cup \Gamma_{\text{D}}$, for known frequencies $\{\omega_n\}_{n=1}^N$ we aim at the reconstruction of the unknown position x and the vector $\hat{u}$ encoding the amplitude and the phase shift of the signal. For this, we consider the following convex relaxation

$$\begin{aligned} \min_{u \in \mathcal{M}(\Omega_c, \mathbb{C}^N)} \quad & \frac{1}{2} \sum_{k=1}^{K} |\vec{p}(x_k) - p_d^k|_{\mathbb{C}^N}^2 + \alpha \|u\|_{\mathcal{M}(\Omega_c, \mathbb{C}^N)}, \\ \text{subject to} \quad & -c^2 \Delta \vec{p} - \vec{\omega}^2 \vec{p} = u. \quad (+\text{BC}) \end{aligned} \tag{2}$$

where the source term is searched in the space of Radon measures on a closed control domain $\Omega_c \subset\subset \Omega$ and $\alpha > 0$ is a small regularization parameter. The observations on the state $\vec{p}$ are taken in K points $\{x_k\}_{k=1,2,...,K} \subset \Omega \setminus \Omega_c$. We provide existence and regularity results for the state equation with a measure valued right hand side and study wellposedness and optimality conditions for (2). Due to the finite dimenisonality of the observation optimal solutions won't be neither unique. We proof that among all minimizers there exists a special class of solutions consisting of finitely many Dirac-Deltas depending on the number of observations and frequencies and state an optimization algorithm guaranteeing convergence towars such minimizers. As an outlook we present a new approach to get rid off the assumption that $\{x_k\}_{k=1,2,...,K} \subset \Omega \setminus \Omega_c$ holds by considering weighted spaces of Radon measure. Numerical examples complete talk.

Session I (1 talk)
Wednesday, July 5, 11:05–11:45

On the Eddy Current Approximation for Non-Autonomous Maxwell's Equations

► Marcus Waurick
(University of Bath, UK)

In the talk, we provide a solution theory for both hyperbolic-type Maxwell's equations and a corresponding parabolic-type eddy current approximation (with vanishing dielectricity). Given a sequence of (space-time dependent) dielectricities converging uniformly to zero. We will prove that the corresponding solution operators for Maxwell's equations then converge in the strong operator topology to the solution operator of the eddy current approximation. The result is based on a continuous dependency result for evolutionary equations.

Session I (1 talk)
Thursday, July 5, 11:05–11:45

Damped Wave Systems on Networks: Exponential Stability and Uniform Approximations

► Herbert Egger
(TU Darmstadt, Germany)

We consider a damped linear hyperbolic system modelling the propagation of pressure waves in a network of pipes. Well-posedness is established via semigroup theory and the existence of a unique steady state is proven in the absence of driving forces. Under mild assumptions on the network topology and the model parameters, we show exponential stability and convergence to equilibrium. This generalizes related results for single pipes and multi-dimensional domains to the network context. Our proof of the exponential stability estimate is based on a variational formulation of the problem, some graph theoretic results, and appropriate energy estimates. The main arguments are rather generic and can be applied also for the analysis of Galerkin approximations. Uniform exponential stability can be guaranteed for the reasulting semi-discretizations under a mild compatibility condition on the approximation spaces. A particular realization by mixed finite elements is discussed. The theoretical results are illustrated by numerical tests in which also bounds for the decay rate are investigated.

Session II (4 talks)
Thursday, July 5, 14:00–15:20

Structure Preserving Model Reduction for Damped Wave Propagation on Networks

► Thomas Kugler
(TU Darmstadt, Germany)

We consider the model reduction of first order hyperbolic systems describing the damped propagation of waves on 1-d networks. The applications we have in mind cover electric transmission lines and the propagation of pressure waves in gas pipelines. These problems share many interesting physical properties, like conservation of charge or mass, passivity, a port-Hamiltonian structure, unique steady states, and exponential stability. We investigate the systematic dimension reduction by Galerkin projections onto low dimensional subspaces.

In order to guarantee all physical properties for the reduced models, we propose a particular construction of subspaces consisting of the following steps: We start with generating a subspace for the first solution component by a Krylov iteration for the corresponding second order system. This subspace is extended by a finite set of functions corresponding to the first solution component of underlying the stationary problem. A basis for the second solution component is then defined by an algebraic compatibility relation. To guarantee mass conservation, the spaces for the two solution components are finally augmented each by one additional function. The resulting pair of approximation spaces can be shown to yield a reduced order model with all the desired properties. Our construction is formulated in a functional space setting which additionally leads to results that are independent of the truth approximation for the underlying pde system. Both properties are illustrated by numerical tests.

Superconvergence and Postprocessing for Mixed Finite Element Approximations of the Wave Equation

► Bogdan Radu

(TU Darmstadt, Germany)

We consider the numerical approximation of the acoustic wave equation by mixed finite elements on unstructured grids. Optimal convergence of the discrete velocitiy error and superconvergence of the projected error in the pressure is established. Based on these results, we propose post processing strategies that allow to obtain pressure approximation that are one order better than the finite element approximation. Corresponding results are well-known for mixed finite element approximations of elliptic problems and the analysis is here extended to semi- and full discretizations of the hyperbolic problems under consideration. We will also consider mass lumping which is of importance for the efficient implementation of time stepping schemes. While the accuracy of the velocity approximation is in general reduced by one order if mass-lumping is applied, optimal convergence of the post-processed pressure can still be obtained even in the presence of mass-lumping. The optimal convergence of the post-processed pressures and the order reduction of the velocity approximation in the presence of mass-lumping are illustrated by numerical tests.

An Extended Midpoint Scheme for the Landau-Lifshitz-Gilbert Equation in Computational Micromagnetics

► Bernhard Stiftner
(TU Wien, Austria)

Joint work with: D. Praetorius (TU Wien, Austria) and M. Ruggeri (TU Wien, Austria)

Micromagnetic phenomena on a ferromagnetic sample $\Omega \subset \mathbb{R}^3$ are described by the time-dependent Landau-Lifshitz-Gilbert equation (LLG). Besides a geometric non-linearity, the reliable and effective numeric integration faces the following problems: first, the continuous solution $\boldsymbol{m} : \Omega \to \mathbb{R}^3$ satisfies a non convex side-constraint $|\boldsymbol{m}| = 1$; second, each time-step requires the computation of the so-called stray field and thus leads to a coupling with an elliptic PDE in full space. The discretization in space employs lowest-order Courant finite elements, where the side-constraint is satisfied in all vertices of the underlying discretization. The approximate stray field is computed via a FEM-BEM approach to cope with the unbounded domain. In our talk, we discuss the extension of the midpoint scheme proposed in [1], where only the so-called exchange contribution (Laplace operator) is treated implicitly, while the lower-order terms (including the computationally expensive stray field) are treated explicitly in time. The resulting scheme is still unconditionally convergent to weak solutions of LLG and formally preserves second-order accuracy in time. Numerical experiments rely on an appropriate coupling of the Netgen/NGSolve package [2] with the BEM++ library [3].

References

[1] Bartels, S., Prohl, A., 2008. Convergence of an implicit finite element method for the Landau-Lifshitz-Gilbert equation. SIAM J. Numer. Anal. 44 (4), 1405–1419 (electronic)

[2] Schöberl, J. NGSolve Finite Element Library. https://sourceforge.net/projects/ngsolve/

[3] Śmigaj, W., Betcke, T., Arridge, S., Phillips, J., Schweiger, M., 2015. Solving Boundary Integral Problems with BEM++. ACM Trans. Math. Software 41 (2), Art. 6, 40 pp.

Numerical Methods for P-Power Law Diffusion Problems

► Ioannis Toulopoulos
(RICAM, Austria)

Joint work with: T. Wick (RICAM, Austria)

In this work, we consider numerical methods for solving power-law diffusion problems, e.g. p-Laplace type problems. For the space discretization we use continuous Galerkin finite element methods (FE) with high order polynomial spaces. For the solution of the resulting nonlinear system we employ different Newton methods, such as residual-based and error-oriented globalization techniques. In addition, we also transform the original problem into a saddle point problem using an augmented Lagrangian (ALG) decomposition technique.

Assuming sufficient regularity for the solution, we derive high order interpolation and error estimates in relevant quasi-norms. We mainly focus on a systematic comparison of first and second order finite element approximations in order to confirm our theoretical findings. Our second goal is a very detailed comparison of two different Newton methods: a residual-based procedure and an error-oriented procedure. Lastly, we discuss the solution of the produced ALG saddle point problem. We discretize it using a FE methodology and then we present two iterative methods for solving the resulting nonlinear algebraic system. The first iterative method is the classical ALG1 iterative method, which is usually used in the literature. It can be interpreted as a variant of the Uzawa algorithm, where the Lagrange multiplier is separately updated. The second proposed iterative method can be characterized as a monolithic approach where all the unknown variable are simultaneously computed in one step. All proposed methods are compared with respect to computational cost and convergence rates in several examples.

This talk is based on a joint work with Thomas Wick, [1]. We gratefully acknowledge the financial support of this research work by the Austrian Science Fund (FWF) under the grant NFN S117-03.

References

[1] I. Toulopoulos and T. Wick (2016), Numerical methods for power-law diffusion problems, RICAM- Report, No. 2016-11, http://www.ricam.oeaw.ac.at/publications/ricam-reports

Session III (4 talks)
Thursday, July 5, 15:40–17:00

Robust multigrid methods for Isogeometric Analysis

► Stefan Takacs
(RICAM, Austria)

Joint work with: C. Hofreither (University of Linz, Austria)

In this talk, we will discuss how to set up multigrid methods for linear systems arising from the discretization of a partial differential equation with an isogeometric discretization. As a model problem, we consider a Poisson equation, which is discretized with B-splines of maximum smoothness. The main focus of this talk is set on the construction of smoothers such that the convergence properties of the multigrid solver do not deteriorate if the polynomial degree p is increased.

This is done based on the recent paper [1], where a p-robust approximation error estimate and a p-robust inverse inequality are shown for a subspace of the whole spline spaces. In one dimension, this subspace is the space of all splines, whose odd derivatives vanish on the boundary. So, for typical problems, that space is almost as large as the whole spline space. Based on that result, it is possible to extend the inverse inequality to the whole spline space, by adding a harmonic extension from the boundary to the interior.

Based on that knowledge, we have proposed in [2] to modify the idea of mass-Richardson smoothers by adding an appropriate boundary condition. Recently ([3]), we could propose a different smoother, which is based on an additive Schwarz type splitting of the degrees of freedom based on the mentioned subspace. In the talk, convergence analysis and numerical experiments are presented. Both, the analysis and the experiments, confirm that the proposed method behaves optimal in both, the grid size and the polynomial degree p.

References

[1] S. Takacs and T. Takacs. Approximation error estimates and inverse inequalities for B-splines of maximum smoothness. Mathematical Models and Methods in Applied Sciences, 26(7). 2016.

[2] C. Hofreither, S. Takacs and W. Zulehner. A Robust Multigrid Method for Isogeometric Analysis using Boundary Correction. To appear in Computer Methods in Applied Mechanics and Engineering. 2016.

[3] C. Hofreither and S. Takacs. A Robust Multigrid Method for Isogeometric Analysis based on an Schwarz type splitting. In preparation.

Dual-Primal Isogeometric Tearing and Interconnecting Solvers for Continuous and Discontinuous Galerkin IgA Equations

▶ Christoph Hofer
(Johannes Kepler Universität Linz, Austria)

Joint work with: U. Langer (Johannes Kepler Universität Linz, Austria)

In this talk, we construct and investigate fast solvers for large-scale linear systems of algebraic equations arising from isogeometric analysis (IgA) of diffusion problems with heterogeneous diffusion coefficient on multipatch domains. In particular, we investigate the adaption of the Dual-Primal Finite Element Tearing and Interconnecting (FETI-DP) method to IgA, called Dual-Primal IsogEometric Tearing and Interconnecting (IETI-DP) method. The use of open knot vectors is very crucial since in this case we can still distinguish between basis functions corresponding to the boundary and to the interior of the patches (subdomains). We consider the cases where we have matching and non-matching meshes on the interfaces. In the latter case we use a discontinuous Galerkin (dG) method to couple the different patches. This requires a special extension of the IETI-DP method to the dG-IgA formulation. We use ideas from the finite element case in order to formulate the corresponding IETI-DP method, called dG-IETI-DP. We design the dG-IETI-DP method in such a way that it can be seen as a IETI-DP method on an extended discrete interface space. We present numerical results for complicated two and three dimensional domains. We observe a quasi-optimal behavior of the condition number κ of the preconditioned system with respect to the mesh-size h and the patch-size H. More precisely, this condition number κ behaves like $O((1 + \log(H/h))^2)$, and is robust with respect to jumping diffusion coefficients.

An Adaptive hp-XFEM Method for a Hardy Problem Featuring an Inverse Square Point Potential

▶ Robert Gruhlke
(Technische Universität Berlin, Germany)

Joint work with: K. Schmidt (Technische Universität Berlin, Germany)

We consider a partial differential equation with the operator $-\Delta - \frac{\lambda}{|\mathbf{x}|^2}$ in two dimensions, which depends on a parameter λ. To answer the question of existence and uniqueness in presence of the inverse square potential, we use Hardy inequalities [3] and hereby extend the existing theory in classic Sobolev-spaces $H_0^1(\Omega)$ to spaces $H_{\Gamma_0}^1(\Omega)$, allowing for non-zeros traces. In particular we introduce geometrical scenarios, in which the best Hardy constant can be recovered. The inverse square potential cannot be considered as an lower order perturbation of the Laplace operator, hence yielding to several problems from both, the theoretical and numerical point of view. Based on the analysis in Kondrat'ev spaces we are able to give an explicit decomposition of the solution in locally conical domains. As a conclusion the inverse square operator can cause arbitrary worse singular solutions in the sense of $H^{1+\epsilon}(\Omega)$-regularity, where $\epsilon \to 0$, when λ tends to the Hardy constant.

Although it is clear, that standard uniform FEM approaches are not suitable, the use of *hp*-adaptive finite element method also yields to multiple difficulties related to approximation quality and the coercivity constant. To recover good approximation properties we apply an eXtended *hp*-adaptive FEM that is enriching the *hp*-adaptive FEM space with the first singularities. This strategy allows for exponential convergence rates bounded away from zero independent of λ. The introduced problem of numerical quadrature in the XFEM framework are then solved with the help of a generalized Duffy transformation [4]. Moreover, we discuss the challenges for a-posteriori error estimation with the inverse square potential.

References

[1] R. Gruhlke. Analysis and Numerics of a Hardy problem in two dimension featuring an inverse square point potential. Master thesis, 2016.

[2] R. Gruhlke and K. Schmidt. An apdative hp-XFEM method for a Hardy problem featuring an inverse square point potential. In preparation, 2016.

[3] G. H. Hardy, J. E. Littlewood, and G. Polya. Inequalities. Cambridge university press, 1952.

[4] S. E. Mousavi and N. Sukumar. Generalized Duffy transformation for integrating vertex singularities. Computational Mechanics, 45(2–3): 127–140, 2010.

A Finite Element Approach to Solving a Mathematical Model for Tumour Evolution

► Joe Eyles

(University of Sussex, UK)

This talk considers a model for in vitro tumour evolution, in which the tumour is large and has been unable to grow blood vessels. Three formulations of the model will be presented, along with three finite element schemes. These include both fitted and unfitted sharp interface schemes, and a diffuse interface scheme. Some analytical results will be introduced. The results of a number of simulations will also be presented.

Session IV (3 talks)
Thursday, July 5, 17:20–18:20

Riccati Based Feedback Stabilization to Trajectories for Parabolic Equations

► Duy Phan
(RICAM, Austria)

Joint work with: S. S. Rodrigues (RICAM, Austria)

The problem which we address here is the *local* exponential stabilization to trajectories for parabolic systems, for $t \in (0, +\infty)$ in the form

$$\partial_t y - \nu\Delta y + F(y) + \nabla \cdot G(y) + f + \sum_{i=1}^{M} u_i \Phi_i = 0, \qquad y|_\Gamma = g,$$

or in the form

$$\partial_t y - \nu\Delta y + F(y) + \nabla \cdot G(y) + f = 0, \qquad y|_\Gamma = g + \sum_{i=1}^{M} u_i \Psi_i,$$

where F and G may be nonlinear functions and vector functions respectively, and $u : (0, +\infty) \longrightarrow \mathbb{R}^M$ is a control function. That is, given a positive constant $\lambda > 0$ and a solution $\hat{y}(t) = \hat{y}(t, \cdot)$ of the uncontrolled system (with $u = 0$), our goal is to find u such that the solution $y(t) := y(t, \cdot)$ of the system, supplemented with the initial condititon

$$y(0) := y(0, x) = y_0(x),$$

is defined on $[0, +\infty)$ and satisfies, for a suitable Banach space X,

$$|y(t) - \hat{y}(t)|_X^2 \leq C\mathrm{e}^{-\lambda t} |y(0) - \hat{y}(0)|_X^2, \quad \text{provided} \quad |y(0) - \hat{y}(0)|_X < \epsilon.$$

We give conditions on the family of controllers $\{\Phi_i\}$, respectively $\{\Psi_i\}$, for the existence of a stabilizing control. Then we compute the feedback stabilizing controller by solving a suitable differential Riccati equation. The results of some numerical simulations are presented, both for internal and boundary controls, showing that the presence of the feedback controller can avoid instability observed in the case of free dynamics.

References

[1] D. Phan and S. S. Rodrigues, 2016. Stabilization to trajectories for parabolic equations. In preparation.

Time Fractional Diffusion Equation

► Sarah-Lena Bonkhoff
(Technische Universität Graz, Austria)

In the last years fractional partial differential equations are gaining more and more interest since they are a useful approach for the description of recently investigated phenomena in physics.

We consider the time fractional diffusion equation which is a modification of the standard diffusion equation. Therefore, fractional order derivatives are introduced and replace the first order time derivative of the standard diffusion equation. This class of parabolic problems are non-local in time and represent a memory effect in the diffusing material.

As a starting point of the numerical investigation we derive the variational formulation of the time fractional diffusion equation in the space-time domain and we consider a mathematical analysis in appropriate function spaces.

A Sharp Korn's Inequality and Related Finite Elements

► Qingguo Hong
(Universität Duisburg-Essen, Germany)

Joint work with: Y. Lee (Texas State University)

In this talk, a sharp Korn's inequality is presented. A detailed proof of it will be shown. This Korn's inequality is easy to verify and a lot of nonconforming elements will be considered. Further, we can construct some new elements that guided by the Korn's inequality.

ABSTRACTS:
SCHAFBERG SPECIAL SESSION

Inverse Inequality Estimates with Symbolic Computation

► *Part One*: Christoph Koutschan
(RICAM, Austria)

► *Part Two*: Cristian-Silviu Radu
(Johannes Kepler Universität Linz, Austria)

Joint work with: U. Langer (Johannes Kepler Universität Linz, Austria)

We present a symbolic computation approach to solve a generalized eigenvalue problem coming from an inverse inequality in finite element methods. More precisely, we aim at evaluating the corresponding determinant using the holonomic ansatz (Zeilberger, 2007), which is a powerful tool that translates the determinant problem into summation identities. However, it turns out that this method does not succeed on the problem at hand. As a solution we present a variation of the original holonomic ansatz that is applicable to a larger class of determinants, including the one we are dealing with here. With the help of computer algebra, in particular with symbolic summation techniques, we obtain an explicit closed form for the determinant, which turned out to be crucial for the precise estimation of the largest eigenvalue.

Sharp Estimates of Constants in Functional Inequalities and A Posteriori Error Estimates for PDEs

► Sergey Repin
(Steklov Institute, St. Petersburg, Russia)

The talk will contain an overview of methods developed for getting two-sided bounds of eigenvalues. Main attention is paid on constants in Poincaré type inequalities for functions and traces of functions and to the constant in the LBB condition. We discuss applications to projection type estimates and to a posteriori error majorants of the functional type.

LIST OF PARTICIPANTS

Immanuel Anjam
Universität Duisburg-Essen, Germany
Fakultät für Mathematik
immanuel.anjam@uni-due.de

Sarah-Lena Bonkhoff
Technische Universität Graz, Austria
Institute of Computational Mathematics
Bonkhoff@tugraz.at

Raffael Casagrande
Eidgenössische Technische Hochschule Zürich, Switzerland
Seminar for Applied Mathematics
raffael.casagrande@sam.math.ethz.ch

Olga Chistiakova
St. Petersburg Polytechnic University, Russia
Department of Applied Mathematics
chistiakova.olga@gmail.com

Martin Costabel
Université de Rennes 1, France
IRMAR, Institut Mathématique
costabel@univ-rennes1.fr

Monique Dauge
Université de Rennes 1, France
IRMAR, Institut Mathématique
Monique.Dauge@univ-rennes1.fr

Sarah Eberle
Eberhard Karls Universität Tübingen, Germany
Numerical Analysis Group
eberle@na.uni-tuebingen.de

Herbert Egger
Technische Universität Darmstadt, Germany
Fakultät für Mathematik
egger@mathematik.tu-darmstadt.de

Joe Eyles
University of Sussex, UK
Department of Mathematics
J.Eyles@sussex.ac.uk

Robert Gruhlke
Technische Universität Berlin, Germany
Institut für Mathematik
gruhlke@math.tu-berlin.de

Christoph Hofer
Johannes Kepler Universität Linz, Austria
Institute of Computational Mathematics
christoph.hofer@jku.at

Qingguo Hong
Universität Duisburg-Essen, Germany
Fakultät für Mathematik
qingguo.hong@uni-due.de

Christoph Koutschan
Austrian Academy of Sciences, Austria
Johann Radon Institute for Computational and Applied Mathematics
christoph.koutschan@ricam.oeaw.ac.at

Thomas Kugler
Technische Universität Darmstadt, Germany
Fakultät für Mathematik
kugler@mathematik.tu-darmstadt.de

Johannes Kraus
Universität Duisburg-Essen, Germany
Fakultät für Mathematik
johannes.kraus@uni-due.de

Svetlana Matculevich
Austrian Academy of Sciences, Austria
Johann Radon Institute for Computational and Applied Mathematics
svetlana.matculevich@oeaw.ac.at

Svetoslav Nakov
Austrian Academy of Sciences, Austria
Johann Radon Institute for Computational and Applied Mathematics
svetoslav.nakov@ricam.oeaw.ac.at

Marjaana Nokka
University of Jyväskylä, Finland
Department of Mathematical Information Technology
marjaana.nokka@jyu.fi

Maxim Olshanskii
University of Houston, USA
Department of Mathematics
molshan@math.uh.edu

Frank Osterbrink
Universität Duisburg-Essen, Germany
Fakultät für Mathematik
frank.osterbrink@uni-due.de

Kees Oosterlee
CWI - Center for Mathematics and Computer Science,
Amsterdam, Netherlands
and
Delft University of Technology, Netherlands
Delft Institute of Applied Mathematics
c.w.oosterlee@cwi.nl

Dirk Pauly
Universität Duisburg-Essen, Germany
Fakultät für Mathematik
dirk.pauly@uni-due.de

Duy Phan
Austrian Academy of Sciences, Austria
Johann Radon Institute for Computational and Applied Mathematics
duy.phan-duc@ricam.oeaw.ac.at

Rainer Picard
Technische Universität Dresden, Germany
Fakultät Mathematik und Naturwissenschaften
rainer.picard@tu-dresden.de

Bogdan Radu
Technische Universität Darmstadt, Germany
Fakultät für Mathematik
radu@gsc.tu-darmstadt.de

Cristian-Silviu Radu
Johannes Kepler Universität Linz, Austria
Research Institute for Symbolic Computation
sradu@risc.jku.at

Sergey Repin
Russian Academy of Sciences, St. Petersburg, Russia
Steklov Mathematical Institute
and
University of Jyväskylä, Finland
Department of Mathematical Information Technology
repin@pdmi.ras.ru

Tytti Saksa
University of Jyväskylä, Finland
Department of Mathematical Information Technology
tytti.saksa@jyu.fi

Michael Schomburg
Universität Duisburg-Essen, Germany
Fakultät für Mathematik
michael.schomburg@uni-due.de

Robert Schorr
Technische Universität Darmstadt, Germany
Graduate School of Computational Engineering
schorr@gsc.tu-darmstadt.de

Daniel Sebastian
Universität Duisburg-Essen, Germany
Fakultät für Mathematik
daniel.sebastian@stud.uni-duisburg-essen.de

Bernhard Stiftner
Technische Universität Wien, Austria
Institute for Analysis and Scientific Computing
bernhard.stiftner@tuwien.ac.at

Stefan Takacs
Austrian Academy of Sciences, Austria
Johann Radon Institute for Computational and Applied Mathematics
stefan.takacs@ricam.oeaw.ac.at

Ioannis Toulopoulos
Austrian Academy of Sciences, Austria
Johann Radon Institute for Computational and Applied Mathematics
ioannis.toulopoulos@oeaw.ac.at

Sascha Trostorff
Technische Universität Dresden, Germany
Fakultät Mathematik und Naturwissenschaften
sascha.trostorff@tu-dresden.de

Jan Valdman
University of South Bohemia, Czechia
Institute of Mathematics and Biomathematics
and
Czech Academy of Sciences, Czechia
Institute of Information Theory and Automation
jan.valdman@gmail.com

Daniel Walter
Technische Universität München, Germany
Centre for Mathematical Sciences
walter@ma.tum.de

Marcus Waurick
University of Bath, UK
Department of Mathematical Sciences
M.Waurick@bath.ac.uk

Wolfgang Wendland
Universität Stuttgart, Germany
Institut für Angewandte Analysis und numerische Simulation
wolfgang.wendland@mathematik.uni-stuttgart.de

Thomas Wick
Austrian Academy of Sciences, Austria
Johann Radon Institute for Computational and Applied Mathematics
thomas.wick@ricam.oeaw.ac.at

Monika Wolfmayr
Austrian Academy of Sciences, Austria
Johann Radon Institute for Computational and Applied Mathematics
monika.wolfmayr@ricam.oeaw.ac.at

Marco Zank
Technische Universität Graz, Austria
Institut für Numerische Mathematik
zank@math.tugraz.at

Anna Zubkova
Karl-Franzens-Universität Graz, Austria
Institut für Mathematik und Wissenschaftliches Rechnen
anna.zubkova@uni-graz.at

Walter Zulehner
Johannes Kepler Universität Linz, Austria
Institut für Numerische Mathematik
zulehner@numa.uni-linz.ac.at