AANMPDE-6-13

Strobl, Austria, July 8–12 2013
6th Workshop on Analysis and Advanced Numerical Methods for Partial Differential Equations
(for Junior Scientists)

AANMPDE-6-13: Monday, July 8, 2013

13:00–13:30 Registration

13:45–13:55 Opening

BLACKBOARD TALK. CHAIR: JAN VALDMAN

14:00-14:40	*Immanuel Anjam*	Functional a posteriori error equalities for conforming mixed approximations of elliptic problems

SESSION I. CHAIR: JAN VALDMAN

14:40-15:00	*Svetlana Matculevich*	On computable estimates of the derivation between the approximate and exact solution of the evolutionary reaction-diffusion equation
15:00-15:20	*Marjaana Nokka*	On a posteriori error bounds for approximations of the Oseen problem generated by the Uzawa algorithm
15:20-15:40	*Tytti Saksa*	Stability analysis of axially moving viscoelastic panels interacting with surrounding flowing air and modelled by an integro-differential equation

15:40-16:00 Coffee Break

SESSION II. CHAIR: SERGIY NESENENKO

16:00-16:20	*Qingguo Hong*	A multigrid algorithm for a discontinuous Galerkin method for the Stokes equations
16:20-16:40	*Christoph Augustin*	Strongly scalable numerical solvers for multiphysics models in computational cardiology
16:40-17:00	*Nadir Bayramov*	Finite element methods for transient convection-diffusion equations with small diffusion

17:00-17:20 Coffee Break

SESSION III. CHAIR: RAINER PICARD

17:20-17:40	*Marijke Grimmonprez*	A new numerical scheme for a one-dimensional semilinear parabolic problem with two-point boundary conditions
17:40-18:00	*Karel Van Bockstal*	A nonlocal parabolic and hyperbolic model for type-I superconductors

AANMPDE-6-13: Tuesday, July 9, 2013

Key Note Lecture. Chair: Olli Mali

09:45-10:45	*Ronald H.W. Hoppe*	Optimal control and optimization of biological micro-electro-mechanical systems (BioMEMS)

Blackboard Talk. Chair: Olli Mali

10:50-11:30	*Dirk Pauly*	On the Maxwell constants in 3D

11:30-14:00 Lunch Break

Session IV. Chair: Ulrich Langer

14:00-14:20	*Walter Zulehner*	Operator preconditioning for mixed methods with applications to biharmonic problems
14:20-14:40	*Monika Wolfmayr*	A robust and optimal AMLI preconditioned MINRES solver for time-periodic parabolic optimal control problems
14:40-15:00	*Huidong Yang*	Numerical simulation for some fluid-structure interaction problems
15:00-15:20	*Stefan K. Kleiss*	Guaranteed and Sharp a Posteriori Error Estimates in Isogeometric Analysis

15:20-15:40 Coffee Break

Session V. Chair: Sebastian Bauer

15:40-16:00	*Bernhard Kiniger*	A transformation approach in shape optimization
16:00-16:20	*Moritz Keuthen*	Moreau-Yosida regularization in shape optimization with geometric constraints
16:20-16:40	*Jan Valdman*	Computations of combined elastoplastic-damage model of rupture of lithospheric faults

16:40-17:00 Coffee Break

Session VI. Chair: Ronald H.W. Hoppe

17:00-17:20	*Benjamin Müller*	A least squares finite element method for hyperelastic materials
17:20-17:40	*Steffen Münzenmaier*	A least squares finite element method for coupled Stokes-Darcy flows
17:40-18:00	*Stephen E. Moore*	Discontinuous Galerkin isogeometric analysis for elliptic PDEs on manifolds

AANMPDE-6-13: Wednesday, July 10, 2013

KEY NOTE LECTURE. CHAIR: SVETLANA MATCULEVICH

09:45-10:45	*Rainer Picard*	On the structure of electrodynamical multiphysics problems

BLACKBOARD TALK. CHAIR: SVETLANA MATCULEVICH

10:50-11:30	*Lucy Weggler*	Generalization of tangential trace spaces of $\mathbf{H}^1(\Omega)$ for curvilinear Lipschitz polyhedral domains Ω

14:00–19:00 Excursion

AANMPDE-6-13: Thursday, July 11, 2013

KEY NOTE LECTURE. CHAIR: IRWIN YOUSEPT

09:45-10:45	*Ludmil Zikatanov*	Multilevel hierarchies and multilevel methods on graphs

BLACKBOARD TALK. CHAIR: IRWIN YOUSEPT

10:50-11:30	*Sebastian Bauer*	Radiative friction in electrodynamic matter-field interactions

11:30-14.00 Lunch Break

KEY NOTE LECTURE. CHAIR: NICOLE MARHEINEKE

14:00-15:00	*Sergey Repin*	Guaranteed and computable estimates of constants in embedding inequalities and applications to a posterioiri error estimates

15:00-15:20 Coffee Break

SESSION VII. CHAIR: WALTER ZULEHNER

15:20-15:40	*Sergiy Nesenenko*	Homogenization of rate-dependent inelastic models of monotone type with nonlinear hardening effects
15:40-16:00	*Johannes Lankeit*	The minimization of matrix logarithms - On a fundamental property of the unitary factor in the polar-decomposition
16:00-16:20	*Stefan Schiessl*	Asymptotic analysis and numerical methods for viscous jet spinning

16:20-16:40 Coffee Break

Session VIII. Chair: Ludmil Zikatanov

16:40-17:00	*Yi Lu*	Model order reduction via Greedy-POD for parametrized transport equations
17:00-17:20	*Carmen Rodrigo*	Design of efficient geometric multigrid methods for saddle point problems with applications in poroelasticity
17:20-17:40	*Irwin Yousept*	Quasilinear magnetostatic field optimal control problem

ABSTRACTS

Key Note Lecture I
Tuesday, July 9, 9.45–10.45

Optimal control and optimization of biological micro-electro-mechanical systems (BioMEMS)

► Ronald H.W. Hoppe
(University of Houston, USA & University of Augsburg, Germany)

We consider the optimal control and design of devices and systems that rely on surface acoustic wave driven micro ows and are used in clinical diagnostics, pharmacology, and forensics for hybridization in genomics, cell sorting, and enantiomer separation. The state equations often represent multi-scale, multi-physics processes described by systems of PDEs such that we are faced with PDE constrained optimization problems. We address the associated optimality systems and consider discretizations by suitable finite element methods with respect to simplicial triangulations of the computational domains. In order to decrease the computational complexity of the resulting large-scale optimization problems, we apply model order reduction such as balanced truncation combined with domain decomposition techniques and derive explicit error bounds for the modeling error.
The results are based on joint work with H. Antil, T. Franke, M. Heinkenschloss, C. Linsenmann, D. Sorensen, A. Wixforth, and K. Zeleke.

Key Note Lecture II
Wednesday, July 10, 9.45–10.45

On the Structure of Electrodynamical Multiphysics Problems

► Rainer Picard
(Technical University, Dresden, Germany)

Maxwell's equations will be embedded into a general class of well-posed linear evolutionary problems, which turns out to be spacious enough for the discussion of coupling phenomena of electromagnetic waves with other (linearized) physical phenomena (multiphysics). It is demonstrated that these problems share a common structure, which is in the simplest case of the form

$$(\partial_0 M_0 + M_1 + A)\, U = F,$$

where ∂_0 denotes the time-derivative, A is a skew-selfadjoint operator and M_0, M_1 are suitable bounded operators in a Hilbert space H. The complexity of physical phenomena appear as encoded in the corresponding material law

$$M\left(\partial_0^{-1}\right) = M_0 + \partial_0^{-1} M_1.$$

The usefulness of this structural perspective is illustrated by applications to various coupled systems.

References

[1] R. Picard and D. F. McGhee. *Partial Differential Equations: A unified Hilbert Space Approach*, volume 55 of *De Gruyter Expositions in Mathematics*. De Gruyter. Berlin, New York. 518 p., 2011.

[2] R. Picard. Mother Operators and their Descendants, Technical report, TU Dresden, arXiv:1203.6762v2, 2012. In press: JMAA (2013), doi:10.1016/j.jmaa2013.02.004

Key Note Lecture III
Thursday, July 11, 9.45–10.45

Multilevel hierarchies and multilevel methods on graphs

► Ludmil Zikatanov
(Pennsylvania State University, USA)

Motivated by the increasing importance of large-scale networks typically modeled by graphs we study properties associated with the popular graph Laplacian. We exploit its mixed formulation based on its natural factorization as product of two operators. The goal is to construct a coarse version of the mixed graph Laplacian operator with the purpose to construct two-level, and by recursion, a multilevel hierarchy of graphs and associated operators. In many situations in practice having a coarse (i.e., reduced dimension) model that maintains some inherent features of the original large-scale graph and respective graph Laplacian offers potential to develop efficient algorithms to analyze the underlined network modeled by this large-scale graph. One possible application of such a hierarchy is to develop multilevel methods that have the potential to be of optimal complexity. One result that we will report is the construction of a projection from the edge-space onto a properly constructed coarse edge-space associated with the edges of the coarse graph. This projection commutes with the discrete divergence operator, and the pair of coarse edge-space and coarse vertex-space offer the potential to construct two-level, and by recursion, multilevel methods for the mixed formulation of the graph Laplacian. The performance of one two-level method with overlapping Schwarz smoothing and correction based on the constructed coarse spaces for solving such mixed graph Laplacian systems is illustrated on couple of examples. We further discuss applications using Algebraic Multilevel Iteration Methods for graph Laplacians and multilevel methods for mimetic finite differences.
The presentation is based on several ongoing works joint with Panayout Vassilevski, Johannes Kraus, Yao Chen, James Brannick, Paola Antonietti and Marco Verani.

Key Note Lecture IV
Thursday, July 11, 14.00–15.00

Guaranteed and computable estimates of constants in embedding inequalities and applications to a posterioiri error estimates

► Sergey Repin
(Steklov Mathematical Institute, St. Petersburg, Russia)

In this talk, we first discuss new type Poincare estimates for functions having zero traces on the boundary of a Lipschitz domain (or on measurable part of the boundary). We obtain exact constants in these inequalities for certain classes of etalon domains (rectangulars and right triangles). Next, we show that using these results it is easy to estimate errors caused by simplification of data of a boundary value problem. Finally, we briefly discuss numerical methods that can be used for getting lower bounds of the smallest positive eigenvalue of an elliptic operator (which leads to guaranteed upper bounds of embedding constants).

Blackboard Talk I
Monday, July 8, 14.00–14.40

Functional a posteriori error equalities for conforming mixed approximations of elliptic problems

► Immanuel Anjam
(University of Jyväskylä, Finland)

We show, for a certain class of elliptic PDEs, that the functional type a posteriori error estimation technique enables us to obtain the exact error of a conforming approximation, even though the exact solution is not known. Here the approximation is understood as the pair $(\tilde{u}, \tilde{p})$, where $\tilde{u}$ is the approximation for the primal variable u, and $\tilde{p}$ is the approximation for the dual variable p. The error is measured in a certain "combined" norm which measures the error in both the primal and dual variables.

Blackboard Talk II
Tuesday, July 9, 10.50–11.30

On the Maxwell constants in 3D

► Dirk Pauly
(University Duisburg - Essen, Germany)

It is well known that in 2D and for bounded Lipschitz domains the Maxwell constants are bounded from below and above by the Poincare constants. We will prove that this result extends to 3D for bounded and convex domains.

Blackboard Talk III
Wednesday, July 10, 10.50–11.30

Generalization of tangential trace spaces of $\mathbf{H}^1(\Omega)$ for curvilinear Lipschitz polyhedral domains Ω

► Lucy Weggler
(University of Saarland, Germany)

For curvilinear Lipschitz polyhedral domains Ω, explicit characterizations of the tangential trace spaces of $\mathbf{H}^1(\Omega)$ are presented. These extend the original characterizations given by Buffa and Ciarlet which hold on Lipschitz polyhedral domains with plane faces. The tangential trace spaces of $\mathbf{H}^1(\Omega)$ are fundamental for the definition, analysis and intuitive understanding of the trace spaces of $\mathbf{H}(\mathbf{curl}, \Omega)$ and therefore, more general characterizations of the latter are obtained at the same time.

Blackboard Talk IV
Thursday, July 11, 10.50–11.30

Radiative friction in electrodynamic matter-field interactions

► Sebastian Bauer
(University Duisburg - Essen, Germany)

We study the dynamics of many charges interacting with the Maxwell field. The particles are modeled by means of non-negative distribution functions f^+ and f^- representing two species of charged matter with positive and negative charge, respectively. If their initial velocities are small compared to the speed of light, c, then in lowest order, the Newtonian or classical limit, their motion is governed by the Vlasov-Poisson system. We investigate higher order corrections with an explicit control on the error terms. The Darwin order correction, order $|v/c|^2$, has been proved previously. In this talk we are concerned with dissipative corrections due to radiation damping, which are of order $|v/c|^3$ relative to the Newtonian limit. If all particles have the same charge-to-mass ratio, the dissipation would vanish at that order.

Session I (3 talks)
Monday, July 8, 14.40–15.40

On computable estimates of the derivation between the approximate and exact solution of the evolutionary reaction–diffusion equation

► Svetlana Matculevich
(University of Jyväskylä, Finland)

Our research is concerned with a posterioiri estimates for the evolutionary reaction-diffusion problem with mixed Dirichlet–Robin boundary condition. Let $\Omega \in \mathbb{R}^n$ be a bounded connected domain with Lipschitz boundary $\partial\Omega$, which consists of two measerable non-intersecting parts Γ_D and Γ_R associated with Dirichlet and Robin boundary conditions, respectively. Let $Q_T := \Omega \times (0,T)$, $T > 0$. denote the space-time cylinder, and $S_T := \partial\Omega \times [0,T] = (\Gamma_D \cup \Gamma_R) \times [0,T] = S_D \cup S_R$ its lateral surface.
We consider the classical evolutionary reaction–diffusion boundary value problem: Find $u(x,t) \in \overline{Q}_T$ satisfying the following system:

$$\begin{aligned} u_t - \nabla \cdot A\nabla u + \lambda u &= f, && (x,t) \in Q_T, && (1)\\ u(x,0) &= \varphi, && x \in \Omega, && (2)\\ u &= u_0 = 0, && (x,t) \in S_D, && (3)\\ A\nabla u \cdot n + \sigma u &= g, && (x,t) \in S_R, && (4) \end{aligned}$$

where $\varphi(x) \in L^2(\Omega)$, $f(x,t) \in L^2(Q_T)$, $g(x,t) \in L^\infty(S_R) \bigcap L^2(0,T;\ H^{\frac{1}{2}}(\partial\Omega))$. We assume that $\lambda \in \mathbb{R}$ is a non-negative constant and $\sigma = \sigma(s,t) \in L^\infty(S_R)$ satisfies the inequality $|\sigma| \leq \mu$, where $\mu > 0$. Also, we assume that the operator $A = (a_{ij})_{i,j=1}^{d\times d} : Q_T \to \mathbb{R}^d_{sym}$ is symmetric and satisfies the condition $\nu_1|\xi|^2 \leq A(x,t)\xi \cdot \xi \leq \nu_2|\xi|^2, \quad \xi \in \mathbb{R}^n, \quad 0 < \nu_1 < \nu_2 < \infty$.
Assume that $v \in H^1(Q_T)$ is the certain function such that $v(x,t) = 0$ on S_D. We compare it with the exact solution in terms of the measure

$$\begin{aligned} [u-v]_{(\nu,\chi,\zeta,\varrho)} = \nu \| \nabla(u-v) \|^2_{Q_T} + \chi\|\sqrt{\lambda}(u-v)\|^2_{Q_T} + \\ + \zeta\|(u-v)(x,T)\|^2_\Omega + \varrho\|\sqrt{\sigma}(u-v)\|^2_{S_R}. \end{aligned} \quad (5)$$

where δ, γ, ϵ, and χ are positive numbers, and

$$\| \tau \|^2_{Q_T} = \int\limits_{Q_T} A^{-1}\tau \cdot \tau \,\mathrm{d}x\mathrm{d}t, \ \|\tau\|^2_A = \int\limits_\Omega A\tau \cdot \tau \,\mathrm{d}x\mathrm{d}t, \ \|\tau\|^2_{A^{-1}} = \int\limits_\Omega A^{-1}\tau \cdot \tau \,\mathrm{d}x\mathrm{d}t.$$

Therefore, to deduce computable and realistic bounds of (5) and confirm their efficiency by numerical experiments.

References

[1] A. V. Gaevskaya and S. I. Repin, A posteriori error estimates for approximate solutions of linear parabolic problems, Springer, Differential Equations, 41 (7): 970–983, 2005.

[2] S. I. Repin, Estimates of deviations from exact solutions of initial-boundary value problem for the heat equation, Rend. Mat. Acc. Lincei, 13 (9): 121–133, 2002.

[3] S. Repin and S. Sauter, Functional a posteriori estimates for the reaction-diffusion problem, C. R. Acad. Sci. Paris, 343 (1): 349–354, 2006.

On a posteriori error bounds for approximations of the Oseen problem generated by the Uzawa algorithm

► Marjaana Nokka

(University of Jyväskylä, Finland)

joint work with: S. Repin (Steklov Mathematical Institute, St. Petersburg, Russia)

We present functional a posteriori estimates for the deviation between exact and approximate solutions in the energy norm to the Oseen problem. We adapt the majorant (the upper deviation estimate) to the well known Uzawa method and obtain a simple form of the majorant. We demonstrate results by numerical examples.

Stability analysis of axially moving viscoelastic panels interacting with surrounding flowing air and modelled by an integro-differential equation

► Tytti Saksa

(University of Jyväskylä, Finland)

joint work with: J. Jeronen (University of Jyväskylä, Finland), N. Banichuk (University of Jyväskylä, Finland, Russian Academy of Sciences, Moscow, Russia)

We study the physics of transportation of materials having viscoelastic characteristics, high transport speeds, a small thickness and a large surface area. A model including both material viscoelasticity and the fluid-structure interaction between the travelling material and the surrounding flowing fluid is introduced. A web (continuum) travelling between two fixed supports is considered, modelling the web as a Kelvin-Voigt type viscoelastic panel and the air flow as a free stream potential flow. The case of the free stream potential flow obstructed by the travelling panel is analyzed using the analytical solution for the aerodynamic

reaction pressure, and one dynamic integro-differential equation describing the system behaviour is obtained. Stability of the system is studied with the help of its eigenfrequencies (eigenvalues). Some numerical examples are given.

Session II (3 talks)
Monday, July 8, 16.00–17.00

A multigrid algorithm for a discontinuous Galerkin method for the Stokes equations

► Qingguo Hong
(RICAM, Austrian Academy of Sciences, Linz, Austria)

joint work with: J. Kraus (RICAM, Linz, Austria), J. Xu (Pennsylvania State University, USA), L. Zikatanov (Pennsylvania State University, USA)

In this talk, a multigrid algorithm for discontinuous Galerkin (DG) H(div)-conforming discretizations of the Stokes equations is presented. Using the augmented Uzawa method to solve the linear system, a linear elasticity problem need to be solved eciently. A varying V-cycle is designed to solve the elasticity problem since the bilinear form from DG disretizations are nonnested. The independence of the Poisson ratio parameter and the mesh size is proved in the talk which shows that the multigrid algorithm is robust and optimal.

Strongly scalable numerical solvers for multiphysics models in computational cardiology

► Christoph Augustin
(Medical University of Graz, Austria)

joint work with: G. Plank (Medical University of Graz, Austria)

Anatomically realistic and biophysically detailed multiscale computer models of cardiovascular tissues like the heart or arterial vessels are playing an increasingly important role in advancing our understanding of integrated cardiac function in health and disease. However, such detailed multiphysics simulations are computationally vastly demanding. While current trends in high performance computing (HPC) hardware promise to alleviate this problem, exploiting the potential of such architectures remains challenging for various reasons. On one hand, strongly scalable algorithms are necessitated to achieve a sufficient reduction in execution time by engaging a large number of cores, and, on the other hand, alternative acceleration technologies such as graphics processing units (GPUs) are playing an increasingly important role which imposes further constraints on design and implementation of solver codes.

We discuss two different parallel approaches, the finite element tearing and interconnecting (FETI) method and a proper domain decomposition algebraic multigrid (AMG), to solve the problems arising from the simulation of the electrical and mechanical behavior of cardiovascular tissues. Scalability results for these multiphysics simulations will be presented. We will also discuss challenges that need to address with regard to highly scalable parallel implementations as well as numerical difficulties that may occur regarding physiological simulations.

Finite element methods for transient convection-diffusion equations with small diffusion

▶ Nadir Bayramov
(RICAM, Austrian Academy of Sciences, Linz, Austria)

joint work with: J. Kraus (RICAM, Linz, Austria)

Transient convection-diffusion or convection-diffusion-reaction equations, with in general small or anisotropic diffusion, are considered. A specific exponential fitting scheme, resulting from finite element approximation, is applied to obtain a stable monotone method for these equations. Error estimates are discussed for this method and a comparison to the more commonly known SUPG method is drawn. Numerical results are also presented for both methods.

References

[1] J. Xu, L. Zikatanov, *A monotone finite element scheme for convection-diffusion equations*, Mathematics of Computation, Volume 68, 228 (1999), pp. 1429–1446.

[2] N. Bayramov, J. Kraus, *On the stable solution of transient convection-diffusion equation with application to optimal control* (in preparation)

Session III (2 talks)
Monday, July 8, 17.20–18.00

A new numerical scheme for a one-dimensional semilinear parabolic problem with two-point boundary conditions

► Marijke Grimmonprez
(Ghent University, Belgium)

joint work with: M. Slodička (Ghent University, Belgium)

Modelling of physical systems in transport theory is usually based on mass balance. Mathematical description frequently leads to an appropriate partial differential equation (PDE). If this process takes place in a bounded domain, then the governing PDE must be accompanied by suitable boundary conditions (BCs) describing the behaviour of the unknown quantity outside the area of consideration.
Researchers have already worked (under appropriate assumptions on the data functions) on solution methods for problems involving local boundary conditions. On the other hand investigation of problems with various types of so-called nonlocal boundary conditions is a hot topic nowadays, because multipoint boundary-value problems for ordinary differential equations have many applications in modelling and analysing problems arising from electric power networks, electric railway systems, telecommunication lines and also in chemistry and analysing kinetic reaction problems. However, there are only few papers devoted to time dependent problems along with multipoint BCs.
The problem we have studied describes a transient space-dependent heat equation in $\Omega = (a, b)$ with two controllers located at the interior points c, d, where

$$a < c < d < b.$$

The physical application is controlled cooling of a rod. The role of both controllers is to adjust the boundary data to the measured temperature, i.e.

$$u(a) = u(c), \qquad u(b) = u(d). \tag{6}$$

The combination of a transient heat equation with (6) is to our knowledge new in literature. The main difficulty is the fact that we cannot prove that the governing (steady-state) differential operator is coercive, due to the nonlocal BCs. Unfortunately, most solution methods rely on the coercivity of the operator. That's why we first developed a new solution method for the steady-state differential problem. This solution method is based on the principle of linear superposition. Secondly, we combined Rothes method for the time discretization and the principle of linear superposition to design a numerical scheme for the approximation of a solution to a semilinear parabolic equation accompanied with the two-point BCs (6). Finally, we carried out some numerical experiments to support our results. In this experiments, the space discretization is based on the Finite Element Method (FEM) using first order Lagrange polynomials.

A nonlocal parabolic and hyperbolic model for type-I superconductors

► Karel Van Bockstal
(Ghent University, Belgium)

joint work with: M. Slodička (Ghent University, Belgium)

A vectorial nonlocal linear parabolic and hyperbolic problem with applications in superconductors of type-I is studied. A superconductive material of type-I occupies a bounded domain $\Omega \subset \mathbb{R}^3$ with a Lipschitz continuous boundary $\partial\Omega$. The full Maxwell's equations ($\delta = 1$) and quasi-static Maxwell's equations ($\delta = 0$) for linear materials are considered. They can be written as

$$\begin{aligned} \nabla \times \boldsymbol{H} &= \boldsymbol{J} + \delta\partial_t \boldsymbol{D} = \boldsymbol{J} + \delta\varepsilon\partial_t \boldsymbol{E}; \\ \nabla \times \boldsymbol{E} &= -\partial_t \boldsymbol{B} = -\mu\partial_t \boldsymbol{H}. \end{aligned} \tag{7}$$

The current density $\boldsymbol{J}$ is supposed to be the sum of a normal and a superconducting part, that is $\boldsymbol{J} = \boldsymbol{J}_n + \boldsymbol{J}_s$. The normal density current $\boldsymbol{J}_n$ is required to satisfy Ohm's law $\boldsymbol{J}_n = \sigma\boldsymbol{E}$. The nonlocal representation of the superconductive current $\boldsymbol{J}_s$ by Eringen [1] is considered. This representation identifies the state of the superconductor, at time t, with the magnetic field $\boldsymbol{H}(\cdot, t)$ and is given by the linear functional

$$\begin{aligned} &\boldsymbol{J}_s(\boldsymbol{x}, t) = \int_\Omega \sigma_0 \left(|\boldsymbol{x} - \boldsymbol{x}'|\right)(\boldsymbol{x} - \boldsymbol{x}') \times \boldsymbol{H}(\boldsymbol{x}', t)\mathrm{d}\boldsymbol{x}' =: -(\mathcal{K}_0 \star \boldsymbol{H})(\boldsymbol{x}, t), \\ &(\boldsymbol{x}, t) \in \Omega \times (0, T). \end{aligned}$$

Taking the curl of (7) results into the following parabolic ($\delta = 0$) and hyperbolic ($\delta = 1$) integro-differential equation

$$\delta\varepsilon\mu\partial_{tt}\boldsymbol{H} + \sigma\mu\partial_t\boldsymbol{H} + \nabla \times \nabla \times \boldsymbol{H} + \nabla \times (\mathcal{K}_0 \star \boldsymbol{H}) = \boldsymbol{0}. \tag{8}$$

A new convolution kernel is derived, namely $\nabla \times \boldsymbol{J}_s = -\mathcal{K} \star \boldsymbol{H}$ when $\boldsymbol{H}$ is divergence free. The positive definiteness of the kernel $\mathcal{K}$ is shown. Taking into account the previous consideration, equation (8) can be rewritten as

$$\delta\varepsilon\mu\partial_{tt}\boldsymbol{H} + \sigma\mu\partial_t\boldsymbol{H} - \Delta\boldsymbol{H} + \mathcal{K} \star \boldsymbol{H} = \boldsymbol{0}.$$

The well-posedness of both problems is discussed under low regularity assumptions and the error estimates for various time-discrete schemes (based on backward Euler approximation) are established.

References

[1] Eringen, A.C.. *Electrodynamics of memory-dependent nonlocal elastic continua*, volume 25 of *J. Math. Phys.*. 3235–3249 p., 1984.

Session IV (4 talks)
Tuesday, July 9, 14.00–15.20

Operator preconditioning for mixed methods with applications to biharmonic problems

► Walter Zulehner
(Johannes Kepler University, Linz, Austria)

Operator preconditioning is a well-established approach for constructing efficient iterative methods for solving discretized (systems of) partial differential equations. It is based on the mapping properties of the involved operators on the continuous level and their implications for the discrete level. We will illustrate this for mixed formulations of biharmonic problems and will derive efficient preconditioners for biharmonic problems on non-convex polygonal domains.

A robust and optimal AMLI preconditioned MINRES solver for time-periodic parabolic optimal control problems

► Monika Wolfmayr
(Johannes Kepler University, Linz, Austria)

joint work with: U. Langer (Johannes Kepler University, Linz, Austria & RICAM, Linz, Austria), J. Kraus (RICAM, Linz, Austria)

In this talk, we will consider an optimal control problem with a parabolic time-periodic partial differential equation appearing in its PDE constraints. In electromagnetics, such kind of problems arise from the optimal control of two-dimensional eddy current problems. In order to solve the optimal control problem, we state its optimality system and discretize it by the multiharmonic finite element method leading to a system of linear algebraic equations that decouples into smaller systems, which can be solved totally in parallel. In [1], we construct preconditioners for these systems which yield robust and fast convergence rates for the preconditioned minimal residual method with respect to all parameters. These block diagonal preconditioners are practically implemented by the algebraic multilevel iteration method presented in [2]. The diagonal blocks of the preconditioners consist of a weighted sum of stiffness and mass matrices. In [3], we discuss and prove the robustness and optimality of the AMLI method for solving these reaction-diffusion type problems discretized by the finite element method. Moreover, the multiharmonic finite element analysis of time-periodic parabolic optimal control problems can also be found in [4], where different variational settings are investigated and estimates of the complete discretization error are derived.

References

[1] Kollmann, M., Kolmbauer, M., Langer, U., Wolfmayr, M., Zulehner, W. *A Finite Element Solver for a Multiharmonic Parabolic Optimal Control Problem*, volume 65(3) of *Comput.Math. Appl.*. 469-486 p., 2013.

[2] Kraus, J. *Additive Schur Complement Approximation and Application to Multilevel Preconditioning*, volume 34(6) of *SIAM J. Sci. Comput.*. A2872-A2895 p., 2012.

[3] Kraus, J., Wolfmayr, M. *On the Robustness and Optimality of the Algebraic Multilevel Method for Reaction-Diffusion Type Problems*, submitted.

[4] Langer, U., Wolfmayr, M. *Multiharmonic Finite Element Analysis of a Time-Periodic Parabolic Optimal Control Problem*, submitted.

Numerical Simulation for Some Fluid-Structure Interaction Problems

► Huidong Yang

(RICAM, Austrian Academy of Sciences, Linz, Austria)

joint work with: U. Langer (Johannes Kepler University, Linz, Austria & RICAM, Linz, Austria)

In the talk, we will present recent results on numerical simulation for some fluid-structure interaction (FSI) problems coupling elasticity models with an incompressible fluid model. We will discuss the stabilized finite element discretization, and stable time discretization techniques for both sub-problems. A domain decomposition method is employed to solve such a class of multi-field problems. We will focus on the robustness and efficiency of sub-problem solvers handling nearly incompressibility, multi-layered property, etc. Finally some numerical results will be presented.

Guaranteed and Sharp a Posteriori Error Estimates in Isogeometric Analysis

► Stefan K. Kleiss

(RICAM, Austrian Academy of Sciences, Linz, Austria)

joint work with: S. Tomar (RICAM, Linz, Austria)

The potential and the performance of isogeometric analysis (IGA), introduced in [1], have been well-studied for applications from many fields over the last years, see the monograph [2]. Though not a pre-requisite, most of the studies of

IGA are based on non-uniform rational B-splines (NURBS). Since the straightforward implementation of NURBS leads to a tensor-product structure, local mesh refinement methods are subject of active current research. Despite the fact that adaptive mesh refinement is closely linked to the question of reliable a posteriori error estimation, the latter is still in its infancy stage in isogeometric analysis.
Functional-type a posteriori error estimates, see the recent monograph [3] and the references therein, which have also been studied for a wide range of problems, provide reliable and sharp error bounds, which are fully computable and do not contain any generic, un-determined constants.
We present functional-type a posteriori error estimates in isogeometric analysis. By exploiting the properties of NURBS, we present efficient computation of these error estimates. The numerical realization and the quality of the computed error distribution are addressed. The potential and the limitations of the proposed approach are illustrated using several computational examples.
References:
[1] T.J.R. Hughes, J. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. Comput. Methods Appl. Mech. Engrg., 194(39-41):4135âĂŞ4195, 2005.
[2] T.J.R. Hughes, J. Cottrell, and Y. Bazilevs. Isogeometric Analysis: Toward Integration of CAD and FEA. Wiley, Chichester, 2009.
[3] S. Repin. A Posteriori Estimates for Partial Differential Equations. Walter de Gruyter, Berlin, Germany, 2008.

Session V (3 talks)
Tuesday, July 9, 15.40–16.40

A transformation approach in shape optimization

► Bernhard Kiniger
(Technical University of Munich, Germany)

joint work with: B. Vexler (Technical University of Munich, Germany)

We consider a model shape optimization problem with a tracking-type functional. The state variable solves an elliptic equation on a domain whose boundary can be parametrized via a control function. We use a transformation to reformulate the problem on a fixed reference domain, show the existence of an optimal solution and introduce the Lagrangian. We show higher regularity of the optimal control and the corresponding state which allows for the representation of the derivative of the reduced cost functional as a boundary integral. We will also put some emphasis on the minimal Sobolev regularity of the control function which is needed to ensure that all these computations can be carried out.

Moreau-Yosida regularization in shape optimization with geometric constraints

► Moritz Keuthen
(Technical University of Munich, Germany)

joint work with: M. Ulbrich (Technical University of Munich, Germany)

In the context of shape optimization with geometric constraints we employ the method of mappings (perturbation of identity) to obtain an optimal control problem with a nonlinear state equation on a fixed reference domain. The Lagrange multiplier associated with the geometric shape constraint has a low regularity (similar to state constrained problems), which we circumvent by penalization and continuation scheme. We employ a Moreau-Yosida-type regularization and assume a second-order condition to hold. The regularized problems can then be solved with a semismooth Newton method and we study the properties of the regularized solutions and the rate of convergence towards a solution of the original problem. A model for the value function in the spirit of HintermÃijller and Kunisch is introduced and used in an update strategy for the regularization parameter. The theoretic findings are supported by numerical tests.

Computations of combined elastoplastic-damage model of rupture of lithospheric faults

► Jan Valdman
(Technical University, Ostrava, Czech Republic & UTIA, Academy of Sciences of the Czech Republic, Prague, Czech Republic)

Based on the paper of Roubicek et al. concerning a rupture of lithospheric faults, we are interested in 2D computations using the methods of finite elements. Key mechanical ingredients are rate-dependent plasticity and material damage/healing. This is a joint work with Tomas Roubicek (Prague).

Session VI (3 talks)
Tuesday, July 9, 17.00–18.00

A least squares finite element method for hyperelastic materials

▶ Benjamin Müller
(University of Hannover, Germany)

joint work with: G. Starke, J. Schröder, A. Schwarz, K. Steeger

Deformation processes of solid materials are omnipresent and can be described by systems of partial differential equations in continuum mechanics. In this talk we present a least squares finite element method based on the momentum balance and the constitutive equation for hyperelastic materials. Our approach is motivated by a well-studied least squares formulation for linear elasticity. This method is generalized to an approach which takes physical as well as geometrical nonlinearities into account. The novelty of our approach is that, in addition to the displacement $\mathbf{u}$, we consider the full first Piola-Kirchhoff stress tensor $\mathbf{P}$ and compute both simultaneously.
In the discrete formulation we use quadratic Raviart-Thomas elements for the stress tensor and continuous quadratic elements for the displacement vector. For the minimization of the nonlinear least squares functional, the Gauss-Newton method with backtracking line search is used.
We emphasize the advantages of our approach in comparison to other solution methods and demonstrate the performance of our method for some examples using a plane strain configuration, a constitutive equation of Neo-Hooke/ Mooney-Rivlin type and adaptive refinement.

A least squares finite element method for coupled Stokes-Darcy flows

▶ Steffen Münzenmaier
(University Duisburg-Essen, Germany)

joint work with: G. Starke, J. Schröder, A. Schwarz, K. Steeger

The coupled problem for an instationary generalized Newtonian Stokes flow in one domain and a generalized Newtonian Darcy flow in a porous medium is studied in this work. Both flows are treated as a first order system in a stress-velocity formulation for the Stokes problem and a volumetric flux-hydraulic potential formulation for the Darcy problem. The coupling along an interface is done by using the well known Beavers-Joseph-Saffman interface condition. A least-squares finite element method is used for the numerical approximation of the solution. It is shown that under some assumptions on the viscosity the least-squares functional corresponding to the nonlinear first order system is an efficient and reliable error estimator which allows for adaptive refinement of the triangulations. The adaptive refinement is examined in a numerical example where boundary singularities are present. Due to the nonlinearity of the problem a Gauss-Newton method is used to iteratively solve the problem leading to a sequence of well-posed variational problems. It is shown that the variational problems arising in the Gauss-Newton method are well-posed.

Discontinuous Galerkin isogeometric analysis for elliptic PDEs on manifolds

► Stephen E. Moore

(RICAM, Austrian Academy of Sciences, Linz, Austria)

joint work with: U. Langer (Johannes Kepler University, Linz, Austria & RICAM, Linz, Austria)

Isogeometric analysis uses the same class of basis functions for both, representing the geometry of the computational domain and approximating the solution. In practical applications, geometrical patches are used in order to get flexibility in the geometrical representation. This patch representation corresponds to a domain decomposition.
In this talk, we will present a discontinuous Galerkin (DG) method that allows for discontinuities only along the subdomain (patch) boundaries. The required smoothness is obtained by the DG terms associated with the boundary of the subdomains. The construction and corresponding discretization error analysis of such DG scheme will be presented for elliptic PDEs in a 2D as well as on open and closed surfaces.

Session VII (3 talks)
Thursday, July 11, 15.20–16.20

Homogenization of rate-dependent inelastic models of monotone type with nonlinear hardening effects

► Sergiy Nesenenko

(Technical University of Darmstadt,Germany)

Using the periodic unfolding method and the monotone operator methods we derive the homogenized equations for the quasi-static initial boundary value problem with internal variables, which model the deformation behavior of viscoplastic materials with a periodic microstructure. The free energy associated with the problem is allowed to be positive semi-definite.

The minimization of matrix logarithms - On a fundamental property of the unitary factor in the polar-decomposition

► Johannes Lankeit
(University Duisburg - Essen, Germany)

We show that the unitary factor U_p in the polar decomposition of $Z = U_p H$ is the minimizer for both $\|\mathrm{Log}(Q^* Z)\|$ and $\|\mathrm{sym}_*(\mathrm{Log(Q^*Z)})\|$ over unitary $Q \in U(n)$ for any given invertible matrix $Z \in \mathbb{C}^{n\times n}$. We prove this to be true for any unitarily invariant norm in any dimension. As important tools we use a generalized Bernstein trace inequality and the theory of majorization.

Asymptotic analysis and numerical methods for viscous jet spinning

► Stefan Schiessl
(Friedrich-Alexander University of Erlangen-Nuremberg, Germany)

The dynamics of slender viscous jets can be described by asymptotic Cosserat rod and string models. The string models are asymptotic limits of the rod in terms of a vanishing slenderness parameter. Whereas the validity range of the strings is restricted due to a singular perturbance, the rod model is generally applicable. It consists of partial and ordinary differential equations whose numerical treatment is challenging in view of large elongations. Adaptivity strategies are required.

Session VIII (3 talks)
Thursday, July 11, 16.40–17.40

Model order reduction via Greedy-POD for parametrized transport equations

► Yi Lu
(Friedrich-Alexander University of Erlangen-Nuremberg, Germany)

Many problems in natural sciences and engineering can be modeled in terms of parametrized partial differential equations (PPDEs), where the parameters can be material-, geometry- or control-parameters. To simulate and optimize PPDEs, lot of repeated computations running quickly at different parameters are necessary. To satisfy the time requirements, model order reduction techniques are applied. In this talk we present the method of Greedy Proper Orthogonal Decomposition (Greedy-POD) and its application for the parametrized transport equations.

Design of efficient geometric multigrid methods for saddle point problems with applications in poroelasticity

► Carmen Rodrigo
(University of Zaragoza, Spain)

joint work with: F.J. Gaspar, F.J. Lisbona, Y. Notay, C.W Oosterlee

In this work, we focus on the multigrid solution of saddle point problems, namely the Stokes equations and the poroelasticity equations. We present efficient geometric multigrid methods for this type of problems in which the choice of smoothing operators (smoothers) and algorithm parameters is guided by the local Fourier analysis (LFA). We estimate the spectral radius of the k-grid operator using LFA in order to obtain quantitative measures for the error reduction and to accurately predict the asymptotic convergence factor of the method.
A key ingredient in designing an optimal multigrid solver is the choice of the smoother. The smoothers that we consider are mainly the obtained via a coupled relaxation approach or the distributive relaxation approach. They are tuned specifically for the indefinite matrices corresponding to the saddle point problems of interest. The LFA of such smoothers and the corresponding geometric multigrid methods require non-standard techniques, and they are used to adjust the parameters of the underlying multigrid method and increase its efficacy.

Quasilinear magnetostatic field optimal control problem

► Irwin Yousept
(Technical University of Darmstadt, Germany)

Strong material parameter dependence on electromagnetic fields is a well-known physical phenomenon. In the context of magnetism, for instance, there is a wide variety of ferromagnetic materials whose physical properties can be significantly influenced by magnetic fields. The governing PDEs for such phenomena feature quasilinear curl-curl equations. In this talk, recent mathematical and numerical results on the optimal control of such issues are presented.

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