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A single peak of the Rosensweig instability

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Abstract

A single peak formed on the free surface of a ferrofluid subject to an overcritical magnetic field is investigated both theoretically and experimentally with respect to its static and dynamic properties. Using a pertubation analysis up to fifth order in the surface deflection the static profile of the surface in the non-linear regime is obtained for the one-dimensional situation. If the static magnetic field is superimposed with an oscillating part an intricate pattern of subharmonic response of the peak is found experimentally. A simple, analytically tractable model is introduced which displays qualitative agreement with the experimental results. © 1999 Published by Elsevier Science B.V. All rights reserved.

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1. Introduction

A thorough theoretical analysis of the Rosensweig instability in a ferrofluid including the detailed shape of the surface beyond the linear stability analysis is quite complicated due to the dependence of the magnetic field on the profile of the free surface. Previous theoretical investigations of the non-linear regime have been mainly focussed on the developing two dimensional pattern of peaks (see Refs. [1,2] for reviews). In contrast we are interested here in the static and dynamic properties of one individual peak as produced in recent experiments [3].

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2. The static profile in the one-dimensional situation

We consider a magnetic fluid of density ρ, surface tension σ and relative permeability μ_r subject to an external magnetic field $\boldsymbol{H}$. In a one-dimensional system the free surface of the fluid is a line described by $z = \Omega(x)$ and determined by the pressure equilibrium at the surface given by Refs. [1,5]

$$p_{\mathrm{atm}} = \rho g \Omega - \sigma \frac{\Omega''}{[1 + (\Omega')^2]^{3/2}} - \frac{\mu_0(\mu_r - 1)}{2} H^2 - \frac{\mu_0}{2} M_n^2. \tag{1}$$

Here the prime denotes differentiation with respect to x, H is the magnitude of the magnetic field immediately below the surface (i.e. *inside* the fluid) and M_n is the component of the magnetization

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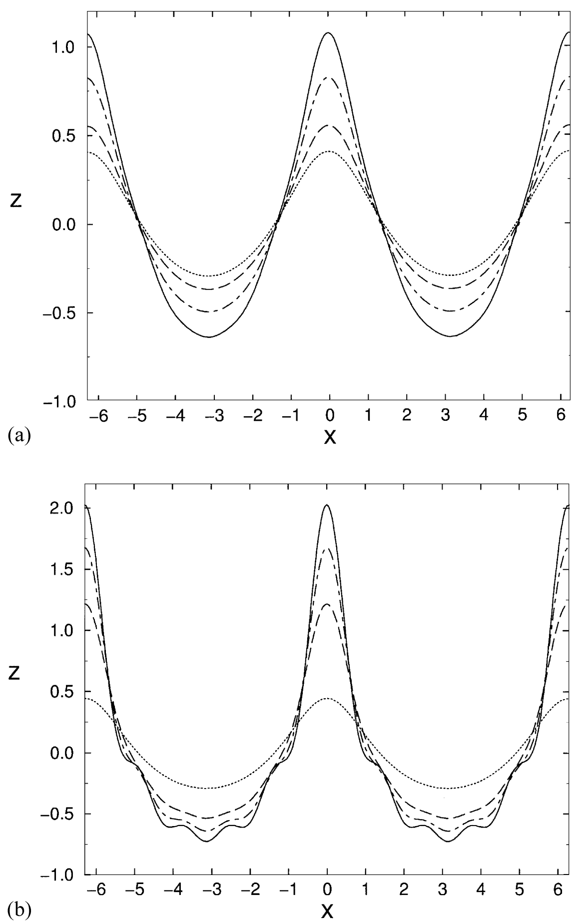

Fig. 1. Static profiles of the surface deflection in fifth-order pertubation theory. (Left) Case of a forward bifurcation ($\mu = 3$). Magnetic field strength $H/H_c = 1.01$ (dotted), 1.02 (dashed), 1.05 (dashed-dotted) and 1.1 (full). (Right) Case of a backward bifurcation ($\mu = 4.2$). Magnetic field strength $H/H_c = 0.995$ (dotted), 1.005 (dashed), 1.025 (dashed-dotted) and 1.05 (full).

normal to the surface. The magnetic field is determined by the static Maxwell equations.

To our knowledge no exact solution for the static profile for overcritical fields exists. In order to find an approximate expression we use the Galerkin ansatz

$$\Omega(x) = a\cos(qx) + a^2 b_2 \cos(2qx) + a^3 b_3 \cos(3qx) + a^4 b_4 \cos(4qx) + a^5 b_5 \cos(5qx) \quad (2)$$

which is a generalization of the ansatz used in Ref. [4]. Next, we solve the magnetic field problem as a power series in a up to terms of order a^5. The pressure condition [1] is then used to determine the coefficients $b_2, \cdots, b_5$ giving rise to the profiles shown in Fig. 1. The figures show the clear tendency from cosine shaped deflections near criticality to pronounced asymmetric peaks for larger external fields. We also reproduce the result of Ref. [4] that the bifurcation of the flat surface is forward for $\mu < 3.53$ and backward for $\mu > 3.53$. However, since we included terms up to fifth order we are for the first time able to determine the saturation value for the amplitude a for certain values of $\mu > 3.53$ (cf. Fig. 1b).

3. Subharmonic response in an oscillating field

In the case of the superposition of a static and an oscillating magnetic field a single ferro-fluid peak shows a variety of subharmonic responses. As an extremely simplified model for which a theoretical analysis becomes feasible we approximate the peak by a half-ellipsoid. Using well-known results for the magnetization of an ellipsoid [5] we are then able to derive the following approximate equation for the time dependence of the rescaled height h of a peak with base radius R as function of the magnetic field:

$$\frac{d^2h}{dt^2} = \frac{1(\mu_r + 1)}{h(\mu_r - 1)} B^2 \left[\left(\frac{B_e}{B} \right)^2 - 1 \right] - 1 - \frac{1}{R^2}. \quad (3)$$

Here $B = B_0 + \Delta B \cos(\omega t)$ denotes the total magnetic field where B_0 is the static part and ΔB the amplitude of the oscillating field, respectively. The quotient

$$\frac{B_e}{B} = \frac{1 + (\mu_r - 1)(1 - \beta/2)}{1 + (\mu_r - 1)\beta(1 - \beta/2)} \quad (4)$$

gives the ratio between the magnetic field B_e at the top of the peak and the external field B where β is a geometry factor depending on the eccentricity of the ellipsoid. In the static situation ($d^2h/dt^2 = 0$) this simple model correctly yields a backward bifurcation to $h > 0$ at a critical field B_c. To investigate the dynamics Eq. (3) is solved numerically for

$\mu_r = 2.15$, $R = \pi/2$, and different strengths of the static and oscillating field. At fixed frequency and strength of the oscillating field the peak response shows three different types of behaviour with increasing static field. For small values of B_0 no peak appears, for medium values of B_0 the height oscillates between $h_{min} = 0$ and $h_{max} > 0$, and for large static fields the peak height oscillates between two non-zero values $h_{min} > 0$ and $h_{max} > h_{min}$. The comparison with the experimental results is focused on the second type of behaviour for which the different periods of the peak oscillations are determined by Poincaré sections. At low frequency of the oscillating field, $\omega = 25.4$ Hz, the peak response is always harmonic independent of the strength of B_0 and ΔB as found in the experiment (see Fig. 5a in Ref. [3]). At a medium frequency, $\omega = 101.8$ Hz, and a fixed value of ΔB the peak starts with a harmonic response at small B_0 followed by an amazing variety of period doublings, triplings etc. when B_0 is increased [6]. The appearance of higher periods is in qualitative agreement with the experiment (see Fig. 6a in Ref. [3]) whereas the pattern formed by the domains of higher periods disagrees with the pattern found experimentally. Hence despite its extreme simplicity the model is able to qualitatively reproduce both important static (backward bifurcation) as well as dynamic features of the experimental situation.

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