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Maximal growth rate at the Rosensweig instability: theory, experiment, and numerics

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We investigate the growth of a pattern of liquid crests emerging in a layer of magnetic liquid when subjected to a magnetic field oriented normally to the fluid surface. After a step like increase of the magnetic field, the temporal evolution of the pattern amplitude is measured by means of a Hall-sensor array. The extracted growth rate is compared with predictions from linear stability analysis by taking into account the nonlinear magnetization curve $M(H)$. The remaining discrepancy can be resolved by numerical calculations via the finite element method. By starting with a finite surface perturbation it can reproduce the temporal evolution of the pattern amplitude and the growth rate.

"In the beginning was the word, ..." [1], which reads in the greek original Ἐν ἀρχῇ ἦν ὁ λόγος, ... The word ὁ λόγος, however, has a plethora of different meanings, including also "way, manner" which is "modus" in latin, "mode" in english. We may therefore translate John, 1.1: "In the beginning was the *mode*, ...", and indeed at the beginning of an evolving pattern stands an unstable mode [2]. As long as the amplitude of the mode is small, its wave number and growth rate can be calculated by linear stability analysis. In this way the early stage of pattern formation has been investigated in many different systems [2]. In the following we examine the growth of the Rosensweig or normal field instability [3]. It is observed in a layer of magnetic fluid [4], when a critical value B_c of the vertical magnetic induction is surpassed. For a sudden increase of the magnetic induction B the growth rate of the fastest growing mode $\hat{\omega}_{2,m}$ was recently calculated in detail [5, 6] to follow the equation

$$\hat{\omega}_{2,m} = c_1\sqrt{\hat{B}} + c_2\hat{B}\ . \tag{1}$$

Here $\hat{B} = (B - B_c)/B_c$ denotes the scaled overcritical induction, and $c_1 = 1.24$ and $c_2 = 0.94$ the calculated parameters taking into account the measured nonlinear magnetization curve $M(H)$ of the fluid. In the following we report an experimental and numerical test of those predictions.

In order to measure the temporal evolution of the growing amplitudes of the surface pattern we utilize a linear array of Hall sensors [7], mounted beneath the bottom of the vessel. Details of the experimental setup and the method of measuring can be found in Ref. [6]. On the basis of the time recorded magnetic profiles, we determine the amplitude from the root-mean-square

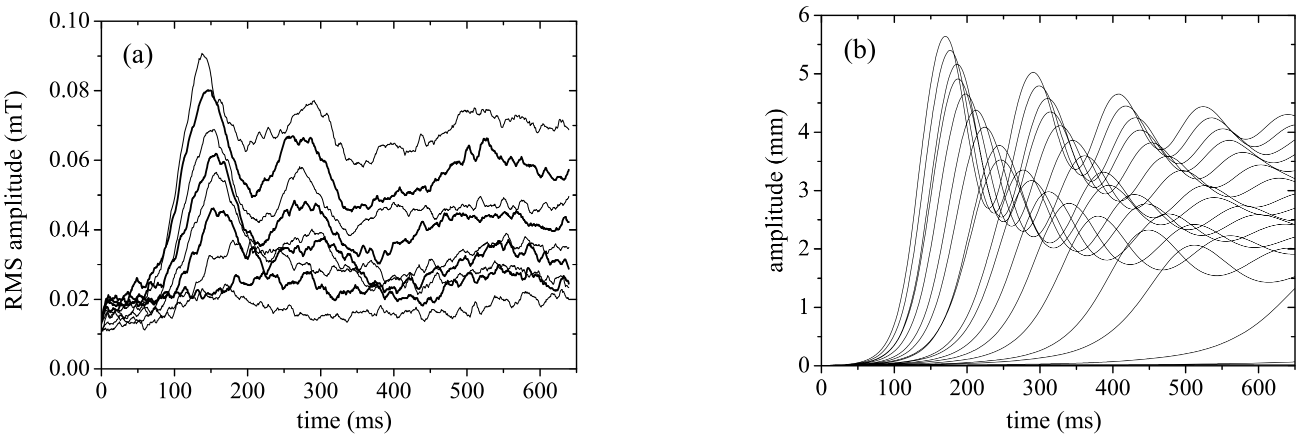

Fig. 1 Plots of the time-resolved amplitudes for the fluid EMG 909 (Ferrotec Co) with $B_c = 25.7\mathrm{mT}$. (a) Measurements for jumps from a subcritical induction $\hat{B} = -0.16$ to increasing supercritical inductions $\hat{B} = 0.006$, 0.02, 0.048, 0.091, 0.112, 0.119, 0.148, 0.162, and 0.19. For clearer appearance, the plotted lines are smoothed by averaging ten neighboring points of the original data set. (b) Numerical results from $\hat{B} = 0.008$ to $\hat{B} = 0.253$. The first 10 curves are separated by $\Delta\hat{B} = 0.01$, the remaining ones by $\Delta\hat{B} = 0.02$.

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value (RMS) of that data. Figure 1(a) displays the results for different supercritical inductions. The offset of the amplitudes result from the noise of the Hall sensors. Subfigure (b) shows the corresponding outcome of the numerical simulations [6,8] by a finite element approach. It is based on a coupled system of nonlinear governing equations, namely the Maxwell equations in the magnetic liquid and its surroundings, the Navier-Stokes equations in the magnetic liquid, and the Young-Laplace equation on the free surface. Both plots show after a jump-like increase of $\hat{B}$ at time $t = 0$ ms a drastic increase in the surface height, followed by an oscillatory relaxation towards the final stage in the pattern forming process.

To determine the growth rate from these data we fit the first phase of growth in the amplitude with an exponential function $y(t) = y_0 + A\exp(\omega_2 t)$, where y_0 denotes an offset and A the amplitude of the exponential growth. The measured growth rate is multiplied by the capillary time t_c of the magnetic fluid yielding the dimensionless variable $\hat{\omega}_2$. The experimental (numerical) results are plotted as open squares (full triangles) in Fig. 2, respectively. Both data sets can be fitted well with Eq. (1), but differ in the parameters c_1 and c_2 from the theoretical prediction, which is shown as dashed and long-dashed line. This strong deviation between theory and experiment, even if taking into account the nonlinear magnetization curve of the fluid, occurs because the experiment starts with a finite amplitude due to the discontinuity of the magnetization at the edge of the vessel whereas the theory assumes a infinitesimal perturbation. The numerics is taking into account a starting with a finite amplitude. This results in a convincing agreement with the experimental data.

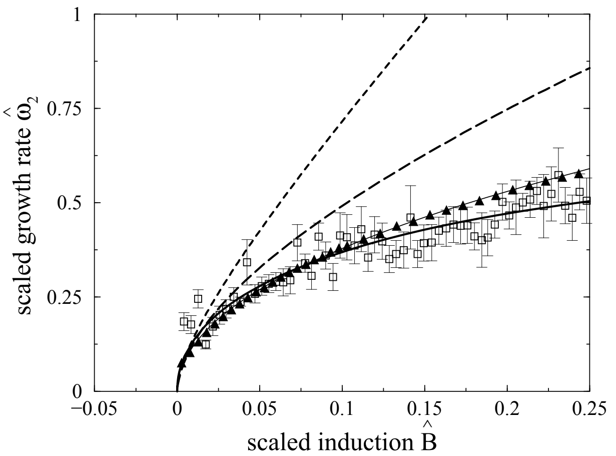

Fig. 2 Plot of the scaled growth rate $\hat{\omega}_2$ versus the scaled induction $\hat{B}$ for the magnetic fluid EMG 909. The open squares give the experimental values with the corresponding errors. A fit for those data using the approximation Eq. (1) yields the thick solid line. Using a linear law of magnetization and an infinite thickness of the layer, the dashed line shows the theoretical result. The results with a nonlinear law of magnetization and a finite thickness of $h = 5$ mm are indicated by the long-dashed line. From the numerical simulations the resulting growth rate is given by the filled triangles. A fit to these results with Eq. (1) gives the thin solid line.

To reduce the discrepancy between experiment and theory in future measurements, it will be necessary to reduce, to the highest possible extent, the effect of the lateral boundaries on the growth of the unstable mode by choosing improved experimental and computational conditions (e.g. the size of the container). Moreover we expect an improvement of the accuracy by a radioscopic measurement of the growth rate with a two-dimensional x-ray detector [9], which has a much better spatially resolution than the one-dimensional Hall sensor array, but which is becoming feasible only for a slow evolution of highly viscous magnetic fluids. For fluids with low viscosity alternatively a magneto-optic detection of the growth rate, utilizing the faraday effect, may become feasible.

To conclude, we have experimentally, theoretically, and numerically investigated the growth rate during the first stage of pattern formation in the Rosensweig instability. Despite the use of a nonlinear law of magnetization there remains a discrepancy between predictions of the linear stability analysis and experimental data. In contrast, the experimental data are confirmed by numerical simulations using a nonlinear magnetization curve together with a finite initial surface undulation.

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