Journal of Magnetism and Magnetic Materials 252 (2002) 194–196
journal of magnetism and magnetic materials
www.elsevier.com/locate/jmmm

Thermal convection of magnetic fluids in a cylindrical geometry

Adrian Lange*

ITP, Universität Magdeburg, Postfach 4120, D-39016 Magdeburg, Germany

Abstract

The thermal convection in a layer of magnetic fluid confined in a two-dimensional cylindrical geometry is studied. The critical external induction for the onset of thermal convection is determined for dilute and non-dilute magnetic fluids. The detected difference between both thresholds allows to test experimentally whether a test fluid is a dilute one or not.

Keywords: Thermal convection; Magnetic fluids; Hydrodynamic stability

1. Introduction

Thermal convection in magnetic fluids (MFs) is typically studied in a geometry of a horizontally extended layer which is simultaneously subjected to a vertical temperature gradient and a vertical magnetic field [1,2]. The analysis of a cylindrical geometry is motivated by a recently observed novel convective instability. In Refs. [3,4] a horizontal layer of MF between two glass plates is locally heated by a focused laser beam. The beam passes perpendicularly through the layer in the presence of a homogeneous vertical magnetic field. The absorption of the light by the fluid generates a temperature gradient and subsequently a refractive index gradient which enhances the beam divergence. As a result, a stationary diffraction pattern of concentric rings is observed for zero magnetic field. Above a certain threshold of the magnetic field, the circular rings are replaced by polygonally shaped patterns. These patterns are interpreted as 'fingerprints' of vertical convection columns [4].

2. Model

If one assumes that the experimental temperature distribution is purely axis-symmetrical and that the influence of the upper and lower boundaries is rather small, the setup can be modeled by the two-dimensional configuration of two concentric rings. The inner ring of radius R_1 has the temperature T_1 and the outer ring of radius R_2 is held at the temperature $T_0 < T_1$. The whole system is subjected to a homogeneous vertical magnetic field, where the susceptibility of the MF between the rings is matched with that of the ring material. The susceptibility of the MF is given by $\chi = \chi_L(1 + \beta\chi_L)$, where χ_L is the susceptibility according to Langevins theory. The coefficient of the quadratic term was determined in different microscopic models which all provide the same value $\beta = \frac{1}{3}$.

The system is governed by the equation of continuity, the Navier–Stokes equations, and the equation of heat conduction for the MF which are in nondimensional form

$$\operatorname{div} \bar{\mathbf{v}} = 0, \tag{1}$$

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + (\bar{\mathbf{v}}\,\mathrm{grad})\bar{\mathbf{v}} = P(-\mathrm{grad}\,\bar{p} + \Delta\bar{\mathbf{v}}) + M\frac{\chi_L^2\{1 + \beta[3\chi_L(1 + \beta\chi_L) - 1]\}}{(1+\chi)^3}\frac{\mathrm{grad}\,\bar{T}}{\bar{T}}, \tag{2}$$

*Corresponding author. Max-Planck-Institut Für Physik Komplexer Systeme, Noethnitzer Strasse 38, D-01187 Dresden, Germany. Fax: +49-391-6711-205.

E-mail address: adlange@mpipks-dresden.mpg.de (A. Lange).

0304-8853/02/$ - see front matter
PII: S 0 3 0 4 - 8 8 5 3 (0 2) 0 0 6 5 6 - X

$$\frac{\partial \bar{T}}{\partial t} + (\bar{\mathbf{v}}\,\mathrm{grad})\bar{T} = \Delta \bar{T}, \tag{3}$$

where the Prandtl number $P = \nu/\kappa$ characterizes the fluid and the magnetization number $M = B_{\mathrm{ext}}^2(R_2 - R_1)^2/(\mu_0\rho\kappa^2)$ tunes the external excitation. Denoting ν as kinematic viscosity, the velocity $\mathbf{v} = (u, v)$ is scaled with $\kappa/(R_2 - R_1)$, time with $(R_2 - R_1)^2/\kappa$, temperature with $(T_1 - T_0)$, and pressure p with $\rho\kappa\nu/(R_2 - R_1)^2$. Δ and grad are the corresponding differential operators in the plane cylindrical coordinates $\bar{r}$ and ϕ. Rigid boundary conditions are assumed for the velocity at the inner and outer radius, $\bar{u} = \partial_{\bar{r}}\bar{u} = 0$ at $\bar{r} = \eta/(1-\eta)$ and $\bar{r} = 1/(1-\eta)$, where the radii ratio is given by $\eta = R_1/R_2$. The temperature is assumed to be constant at each boundary, $\bar{T}(\bar{r} = \eta/(1-\eta)) = \bar{T}_1$ and $\bar{T}(\bar{r} = 1/(1-\eta)) = \bar{T}_0$.

The particular form of the Kelvin force density (last term in Eq. (2)) and the disregard of diffusion phenomena are debated in Ref. [5]. Since the Kelvin force is the only destabilizing force present in the system, one has to determine which profile leads to a potentially unstable stratification in the fluid. For heating at the inner radius, the r-component of the Kelvin force density has to act inwards and its absolute value has to increase with increasing distance from the origin. These restrictions have to be fulfilled by the quiescent conductive state which is given by $\bar{\mathbf{v}}_G = 0$ and $\bar{T}_G = \bar{T}_0 + (\bar{T}_1 - \bar{T}_0)\ln[\bar{r}(1-\eta)]/\ln\eta$. Applying the above condition to the Kelvin force density in Eq. (2) entails that the radii ratio η has to be larger than the critical value $\eta_c = \bar{T}_0/\bar{T}_1$. For realistic temperatures T_1 above a room temperature of $T_0 = 300\,\mathrm{K}$, this condition is met only in a narrow gap (see Fig. 1). Therefore, terms as $\partial_{\bar{r}}(\partial_{\bar{r}} + 1/\bar{r})$ are approximated by $\partial_{\bar{r}}^2$ and the new variable $\zeta = \bar{r} - \eta/(1-\eta)$ is introduced.

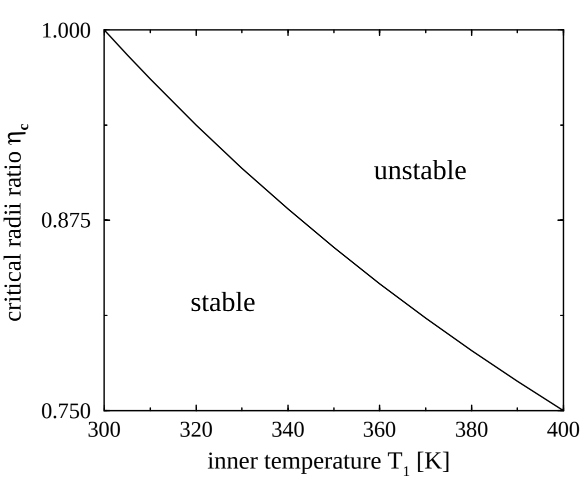

Fig. 1. Region of potentially unstable and stable force profiles for a fixed outer temperature of $T_0 = 300\,\mathrm{K}$.

3. Results and discussion

In the frame of a linear stability analysis, all small disturbances from the ground state are decomposed into normal modes, i.e. into components of the form $[\bar{u}, \bar{p}, \bar{T}] = e^{n\bar{t}}\cos(l\phi)[\bar{u}(\zeta), \bar{p}(\zeta), \bar{T}(\zeta)]$ and $\bar{v} = e^{n\bar{t}}\sin(l\phi)\bar{v}(\zeta)$, respectively. The nondimensional growth rate is denoted by n and l is the azimuthal wave number. For marginal stability, $n \equiv 0$, the resulting differential equation for the temperature $\bar{T}$ and the $\bar{r}$-component of the velocity, $\bar{u}$, are

$$\left(\frac{\partial^2}{\partial\zeta^2} - \alpha^2\right)^2 \bar{u} - \frac{\alpha^2}{l^2}\left(\frac{\partial^2}{\partial\zeta^2} - \alpha^2\right)\bar{u} = -\alpha^2 \frac{M}{P} f_\chi \frac{\bar{T}}{\bar{T}_G^2}\frac{\partial \bar{T}_G}{\partial\zeta}, \tag{4}$$

$$\left(\frac{\partial^2}{\partial\zeta^2} - \alpha^2\right)\bar{T} = \bar{u}(\bar{T}_0 - \bar{T}_1), \tag{5}$$

where

$$\alpha = (1-\eta)l/\eta$$

and

$$f_\chi = \chi_{\mathrm{L}}^2[6\beta^2\chi_{\mathrm{L}}^3(1+\beta\chi_{\mathrm{L}}) + 4\chi_{\mathrm{L}}^2\beta(1-4\beta) + \chi_{\mathrm{L}}(1-10\beta) + 2\beta - 2]/(1+\chi)^4.$$

Eq. (5) can be solved analytically, whereas Eq. (4) can only be approximately solved by the Galerkin method (for details see Ref. [6]). For the calculations fluid parameters of EMG 901 are used: $\rho = 1.53 \times 10^3\,\mathrm{kg\,m^{-3}}$, $\nu = 6.54 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$, $\chi_{\mathrm{L}} = 3$, and $\kappa = 4.2 \times 10^{-8}\,\mathrm{m^2\,s^{-1}}$. The temperature at the outer radius of $R_2 = 5\,\mathrm{cm}$ is fixed at $T_0 = 300\,\mathrm{K}$. The inner radius is given by $\eta = 1.01\eta_c$. With these data the critical

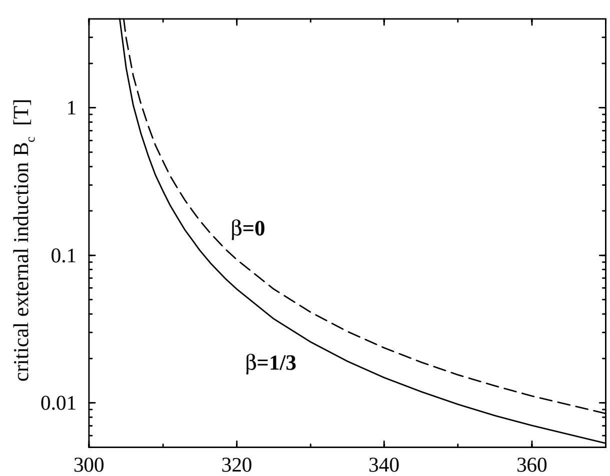

Fig. 2. Critical external induction B_c versus inner temperature T_1 for a room temperature of $T_0 = 300\,\mathrm{K}$. The inclusion of a quadratic term in the susceptibility with $\beta = \frac{1}{3}$ (solid line) results in a lower threshold for the onset of convection than in the dilute case, $\beta = 0$ (long-dashed line). Fluid parameters of the magnetic fluid EMG 901 (see text) were used.

external induction B_c is calculated in dependence of the inner temperature T_1 (Fig. 2).

Decreasing the temperature difference from $\Delta T = 70$ to 4 K causes a dramatic increase in the critical induction of about three orders of magnitude. With decreasing temperature difference the critical radii ratio grows, i.e. the allowed gap becomes more narrow. Since the convection rolls prefer the same length scale in r and ϕ-direction, much more rolls have to be driven in a very small gap. The energy for this effort comes from the external induction which is why it amplifies drastically for small ΔT (Fig. 2).

The inclusion of a quadratic term in the susceptibility with $\beta = \frac{1}{3}$ results in a lower threshold for the onset of convection than in the dilute case, $\beta = 0$ (Fig. 2). The difference between the thresholds is nearly the same value, $B_c(\beta = \frac{1}{3}) \simeq 0.63 B_c(\beta = 0)$, for all tested temperatures $304\,\mathrm{K} \leqslant T_1 \leqslant 370\,\mathrm{K}$. This clear and measurable difference opens a very good opportunity to decide whether a test sample is a dilute fluid or not. Just by measuring the threshold for the onset of convection in the proposed model system the answer can be given.

References

[1] B.A. Finlayson, J. Fluid Mech. 40 (1970) 753.
[2] L. Schwab, U. Hildebrandt, K. Stierstadt, J. Magn. Magn. Mater. 39 (1983) 113;
L. Schwab, K. Stierstadt, J. Magn. Magn. Mater. 65 (1987) 315;
L. Schwab, J. Magn. Magn. Mater. 85 (1990) 199.
[3] T. Du, W. Luo, Appl. Phys. Lett. 72 (1998) 272.
[4] W. Luo, T. Du, J. Huang, Phys. Rev. Lett. 82 (1999) 4134.
[5] A. Lange, J. Magn. Magn. Mater. 241 (2002) 327.
[6] A. Lange, Phys. Fluids 14 (2002) 2059.