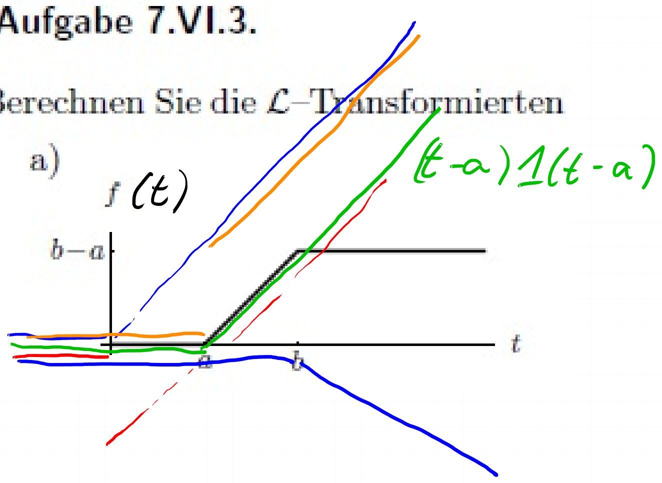
Ma 3: Laplace-Transformation (LT) - Anwendung Rechenregeln LT

Aufgabe 7.VI.3.

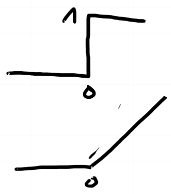
Berechnen Sie die $\mathcal{L}$–Transformierten

a)



$$f(t) = (t-a)\,1(t-a) + (t-b)\,1(t-b)$$

$$F(s) = e^{-sa}\frac{1}{s^2} + e^{-sb}\cdot\frac{1}{s^2}$$



$\mathbf{1}(t) \circ\!\!-\!\!\bullet \frac{1}{s}$

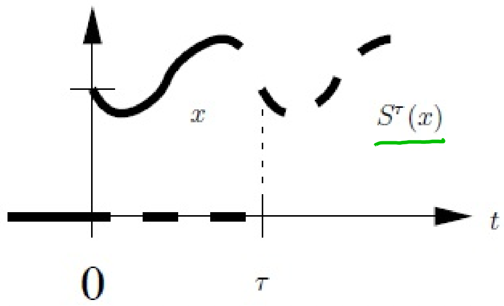
$\underbrace{t\,\mathbf{1}(t)}_{r(t)} \circ\!\!-\!\!\bullet \frac{1}{s^2}$

$S^a(r(t)) \circ\!\!-\!\!\bullet e^{-sa}\frac{1}{s^2}$

$-S^b(r(t)) \circ\!\!-\!\!\bullet -e^{-sb}\frac{1}{s^2}$

Verschiebungssatz: $\mathcal{L}\left(f(t-t_0)\,\mathbb{1}(t-t_0)\right)(s) = e^{-t_0 s}\,\mathcal{L}(f)(s)$, $Re(s)>\alpha$, $t_0 \geq 0$

wobei $f(t)=0$ für $t<0$ gelte.



x: Gegeben durch $x(t)\ \mathbf{1}(t)$

$S^\tau(x)$: Gegeben durch $x(t-\tau)\ \mathbf{1}(t-\tau)$

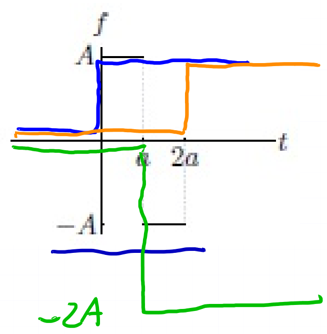
Quelle: LV Systemtheorie

$x(t) \circ\!\!-\!\!\bullet X(s)$

$S^\tau(x(t)) = x(t-\tau)\quad (\tau>0) \circ\!\!-\!\!\bullet e^{-s\tau}X(s)$

c) Rechteckimpuls

$$f(t) = \begin{cases} A & 0<t<a \\ -A \quad \text{für} & a<t<2a \\ 0 & t>2a \end{cases}$$



$$f(t) = A\cdot 1(t) - 2A\,1(t-a) + A\,1(t-2a)$$

$$F(s) = A\left(\frac{1}{s} - 2\,e^{-sa}\frac{1}{s} + e^{-2sa}\frac{1}{s}\right)$$