Dualität

Vektorraum

$\boxed{V}$

(15.1)

Dualraum

$\boxed{V^*}$

Linearformen $f: V \to \mathbb{K}$

Basis:

$\boxed{B = (v_1, \dots, v_n)}$

(15.2)

duale Basis

$\boxed{B^* = (v_1^*, \dots, v_n^*)}$

$v_i^*(v_j) := \delta_{ij} = \begin{cases} 1 & (i=j) \\ 0 & (i \neq j) \end{cases}$

Galoisverbindung

$\boxed{X \subseteq V}$ $\longmapsto$ Annulator $\boxed{X^\circ := \{ f \in V^* \mid \forall v \in X : f(v) = 0 \}}$

$\boxed{Y \subseteq V^*}$ $\longmapsto$ Annulator $\boxed{Y^\circ := \{ v \in V \mid \forall f \in Y : f(v) = 0 \}}$

(15.6)
(15.5.)

$X \subseteq V$ Galois-Hüllen $Y \subseteq V^*$

$\dim X = k \iff \dim X^\circ = n-k$

$\dim Y^\circ = n-k \iff \dim Y = k$

Interpretation als orthogonales Komplement $(X, Y \subseteq V = \mathbb{K}^n)$

$X^\circ = X^\perp, \; Y^\circ = Y^\perp := \{ x \in \mathbb{K}^n \mid \forall y \in Y \; x * y = 0 \}$ ($*$ Skalarprodukt)

lin. Abb.

$\boxed{\varphi: V \to W}$

(15.4)

duale Abb.

$\boxed{\varphi^d: W^* \to V^*}$

$\varphi^d(g) := g \circ \varphi$

$$\begin{array}{ccc} V & \xrightarrow{\varphi} & W \\ & {\scriptstyle g\circ\varphi} \searrow & \swarrow {\scriptstyle g \in W^*} \\ & \mathbb{K} & \end{array}$$