LAG ~~18.159~~ 18.17

zu 18.17

Fläche $\Phi: x_3 = \lambda_1 x_1^2 + \lambda_2 x_2^2$

Signatur	Vorzeichen λ_1	λ_2	geometrische Eigenschaften
1. Fall: $(2,0,0)$	>0	>0	positiv definit (15.4, 15.7) $x_1 = 0 \Rightarrow \boxed{x_3 = \lambda_2 x_2^2}$ Parabel nach oben $x_2 = 0 \Rightarrow \boxed{x_3 = \lambda_1 x_1^2}$ Parabel nach oben Paraboloid  Schnitt mit Ebene $x_3 = c$ ergibt Ellipse
2. Fall $(1,1,0)$	>0 analog <0	<0 >0	negativ definit (15.4, 15.7) $x_1 = 0 \Rightarrow \boxed{x_3 = \lambda_2 x_2^2}$ Parabel nach unten $x_2 = 0 \Rightarrow \boxed{x_3 = \lambda_1 x_1^2}$ Parabel nach oben Sattelfläche (hyperbolisches Paraboloid)
3. Fall $(0,2,0)$	<0	<0	wie 1. Fall aber nach unten geöffnetes Paraboloid
4. Fall $(1,0,1)$	>0 analog $=0$	$=0$ >0	$x_1 = 0 \Rightarrow x_3 = 0$ (Gerade x_2-Achse) $x_2 = 0 \Rightarrow \boxed{x_3 = \lambda_1 x_1^2}$ Parabel nach oben ausgeartetes Paraboloid: parabolischer Trog 
5. Fall $(0,1,1)$	<0	$=0$	wie 4. Fall aber nach unten geöffneter parabolischer Trog
6. Fall $(0,0,2)$	$=0$	$=0$	$x_3 = 0$ d.h. $\Phi = x_1 x_2$-Ebene 

aus Buch:
D. C. LAY:
Linear Algebra and its Applications (1994)

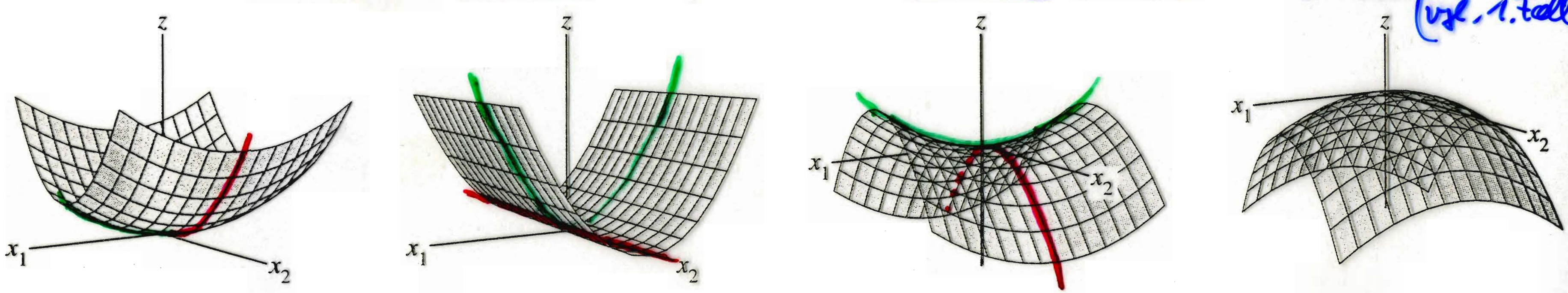
zu 18.17

Classifying Quadratic Forms

When A is an $n \times n$ matrix, the quadratic form $Q(\mathbf{x}) = \mathbf{x}^T A \mathbf{x}$ is a real-valued function with domain $\mathbb{R}^n$. We distinguish several important classes of quadratic forms by the type of *values* they assume for various $\mathbf{x}$'s.

Figure 4 displays the graphs of four quadratic forms. For each point $\mathbf{x} = (x_1, x_2)$ in the domain of a quadratic form Q, a point (x_1, x_2, z) is plotted, where $z = Q(\mathbf{x})$. Notice that except at $\mathbf{x} = \mathbf{0}$, the values of $Q(\mathbf{x})$ are all positive in Fig. 4(a) and all negative in Fig. 4(d). The horizontal cross sections of the graphs are ellipses in Figs. 4(a) and 4(d), and hyperbolas in 4(c).

1. Fall Signatur (2,0,0) — 4. Fall (1,0,1) — 2. Fall (1,1,0) — 3. Fall (0,2,0) (vgl. 1. Fall)



(a) $z = 3x_1^2 + 7x_2^2$ (b) $z = 3x_1^2$ (c) $z = 3x_1^2 - 7x_2^2$ (d) $z = -3x_1^2 - 7x_2^2$

FIGURE 4 Graphs of quadratic forms.

The simple 2×2 examples in Fig. 4 illustrate the following definitions.

DEFINITION

A quadratic form Q is

a. **positive definite** if $Q(\mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{0}$,

b. **negative definite** if $Q(\mathbf{x}) < 0$ for all $\mathbf{x} \neq \mathbf{0}$,

c. **indefinite** if $Q(\mathbf{x})$ assumes both positive and negative values.